Streamwise scalings of a wind turbine operated with different inflows and tip speed ratios

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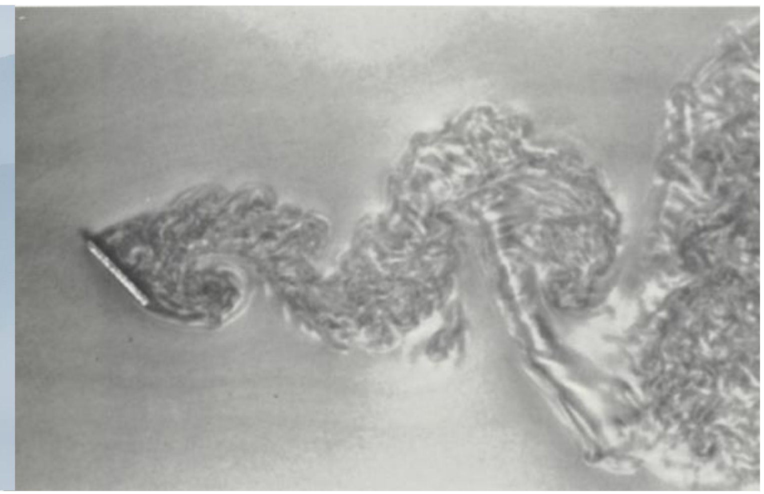
EERA DeepWind conference

Trondheim, Norway, 18-20 January 2023

Motivation: turbulent wakes

Turbulent wakes are ubiquitous in wind energy

A flow that has puzzled researchers for more than half a century



Cantwell, Annual Review of Fluid Mechanics 13.1 (1981): 457-515.

Measured results are still inconclusive or contradictive

Johansson et al, Physics of Fluids 15.3 **(2003)**

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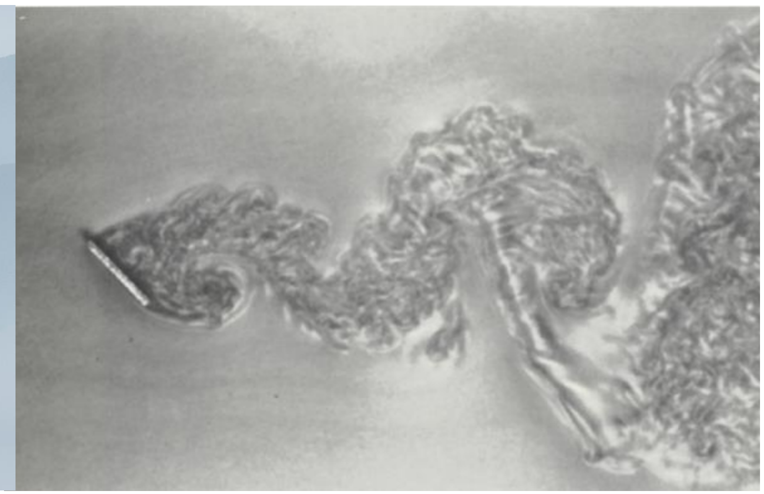


Cantwell, Annual Review of Fluid Mechanics 13.1 (1981): 457-515.

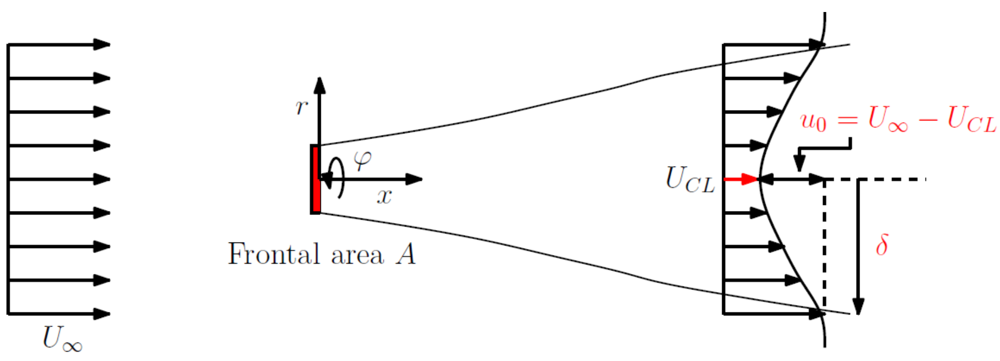
The recently discovered non-equilibrium energy cascade may also force to restate the assumptions/models used in applications

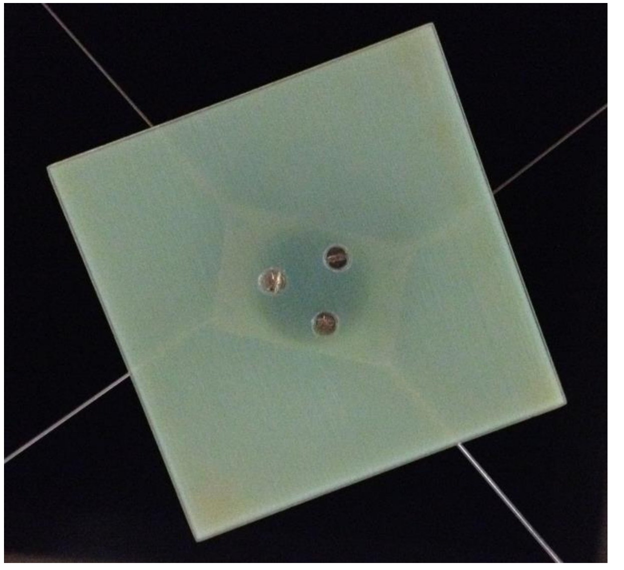
Vassilicos ARFM 2015

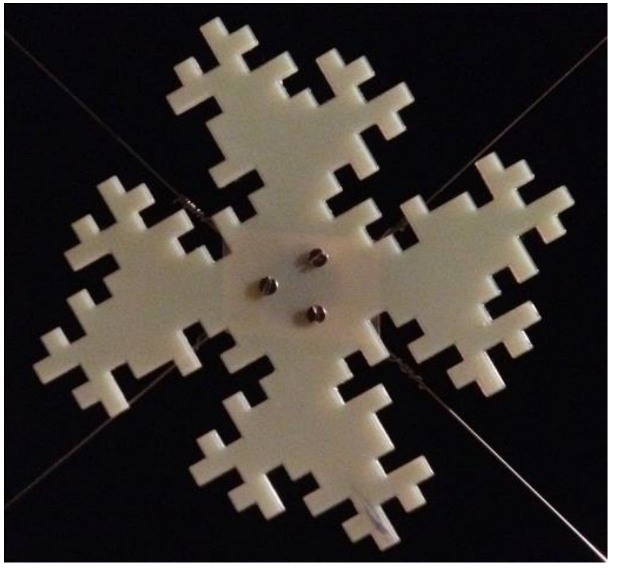
Motivation: turbulent wakes

The far wake of bluff bodies has been found to have different functional laws according to the produced turbulence

Different energy cascades result in difference streamwise scalings







Nedić et al, Physical review letters 111.14 **(2013)**: 144503.
Dairay Obligado & Vassilicos JFM (2015)
Obligado Dairay & Vassilicos PRF (2016)

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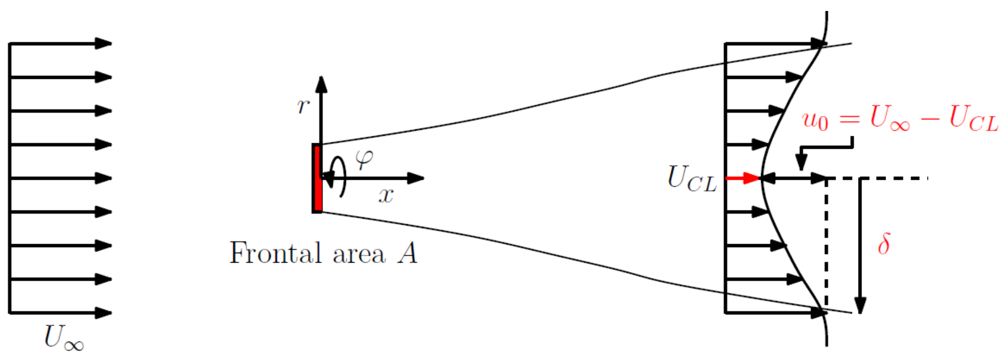

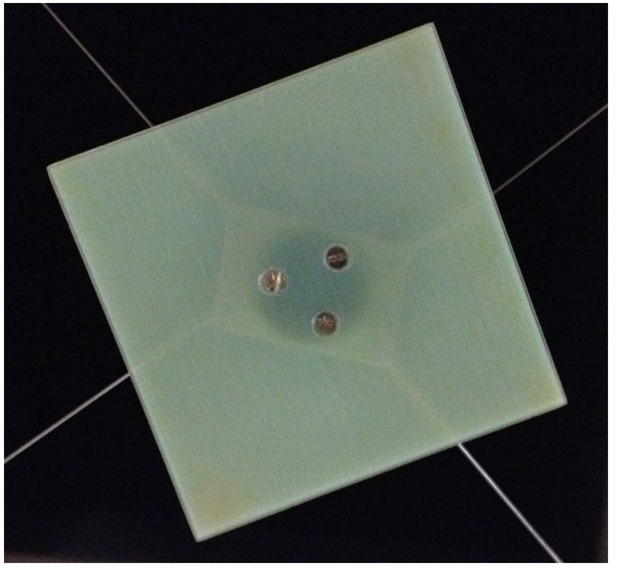

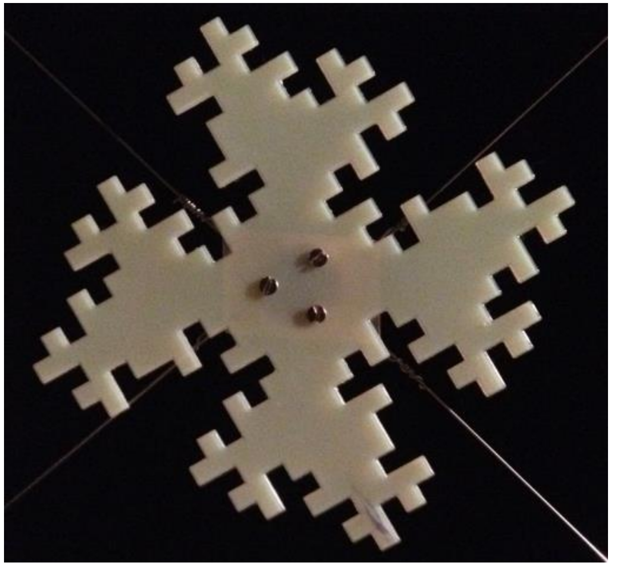

The Townsend-George theory relates the streamwise scalings with the energy cascade

Townsend 1980
George 1989

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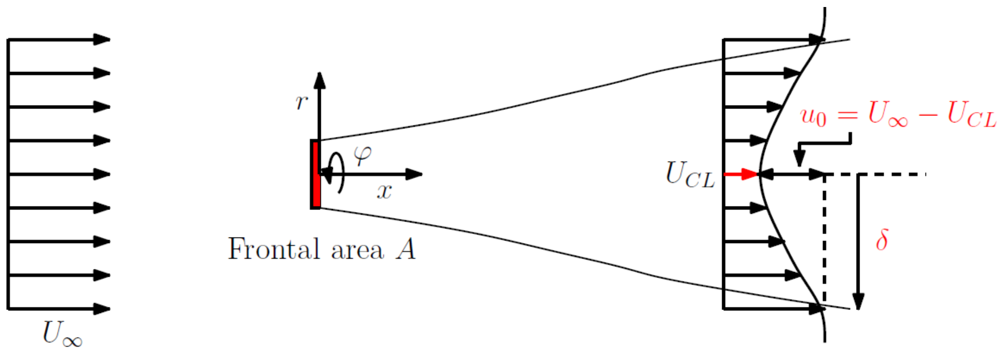

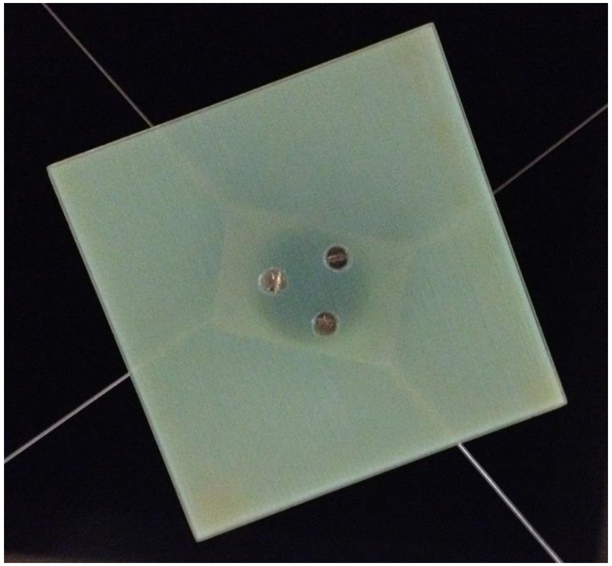

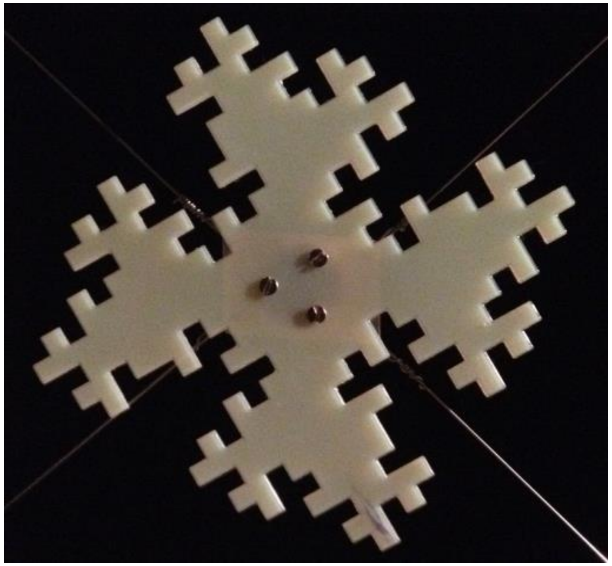

Energy cascade: Richardson Kolmogorov

$$u_0(x) \sim (x - x_0)^{-2/3}$$

$$\delta(x) \sim (x - x_0)^{1/3}$$

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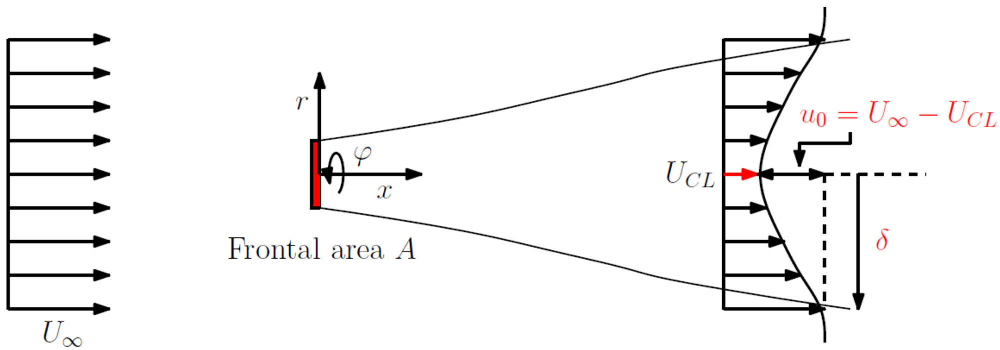
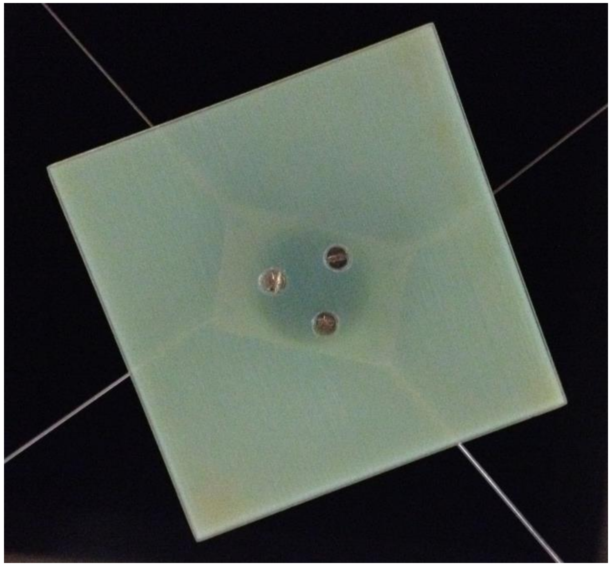
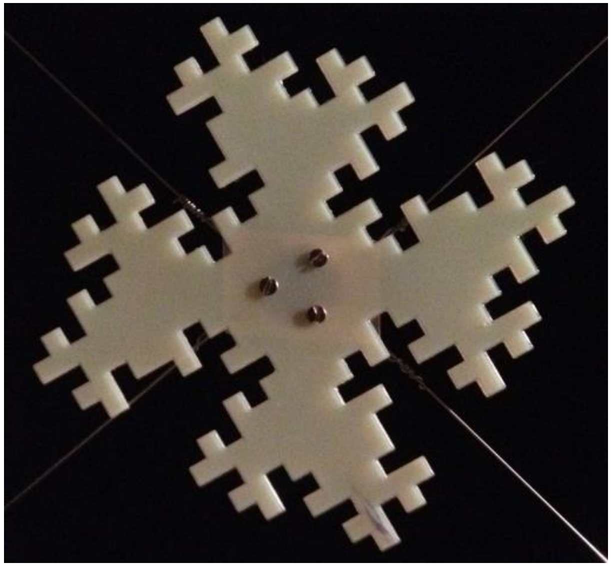


Energy cascade: **non equilibrium**

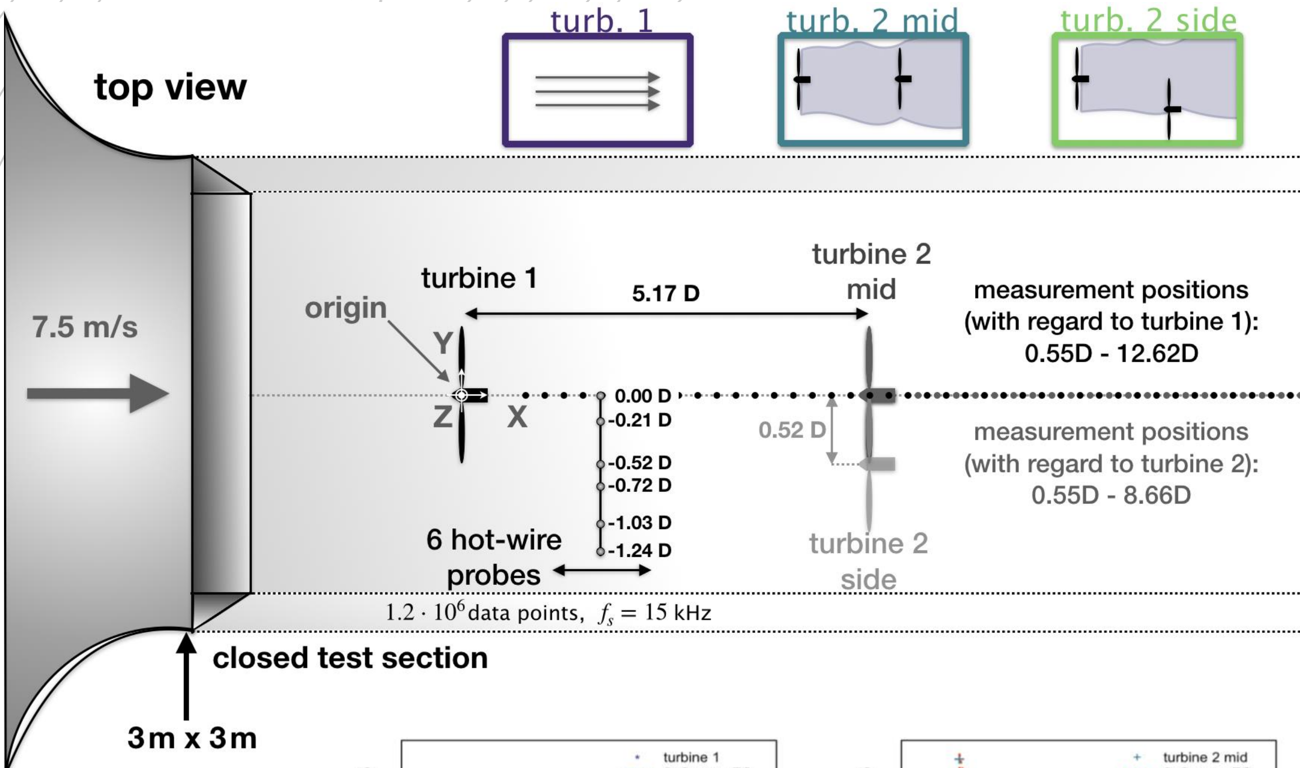
$$u_0(x) \sim (x - x_0)^{-1}$$

$$\delta(x) \sim (x - x_0)^{1/2}$$





What about wind turbines?

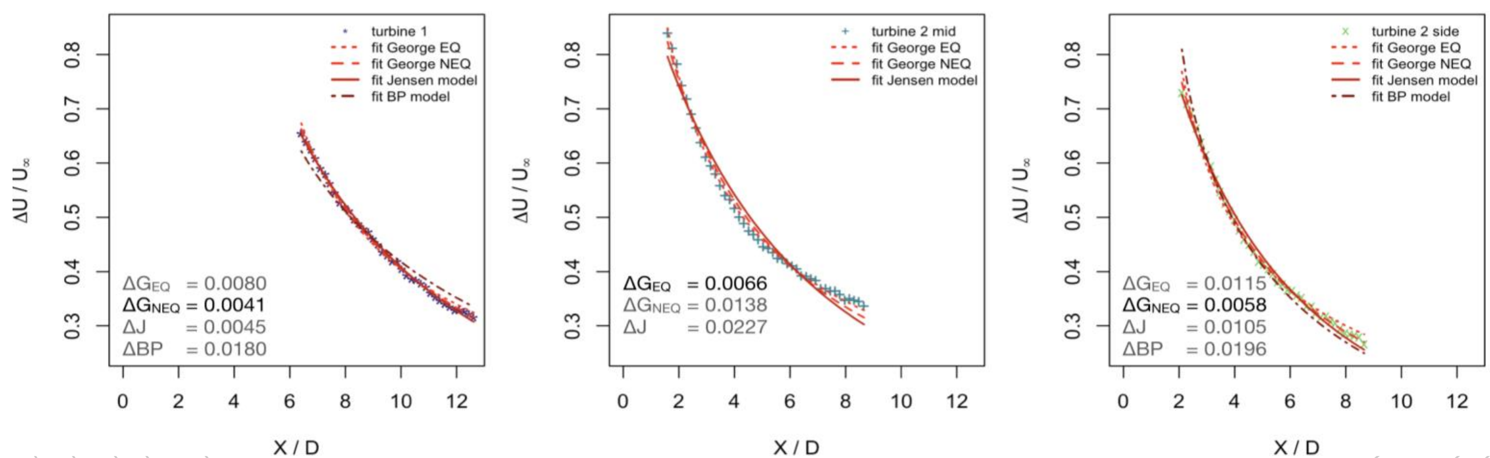


Wind turbine wakes and interactions

$1 < x/D < 12$

Consistent with the **Richardson-Kolmogorov cascade** for x<10D ?

What happens further downstream?

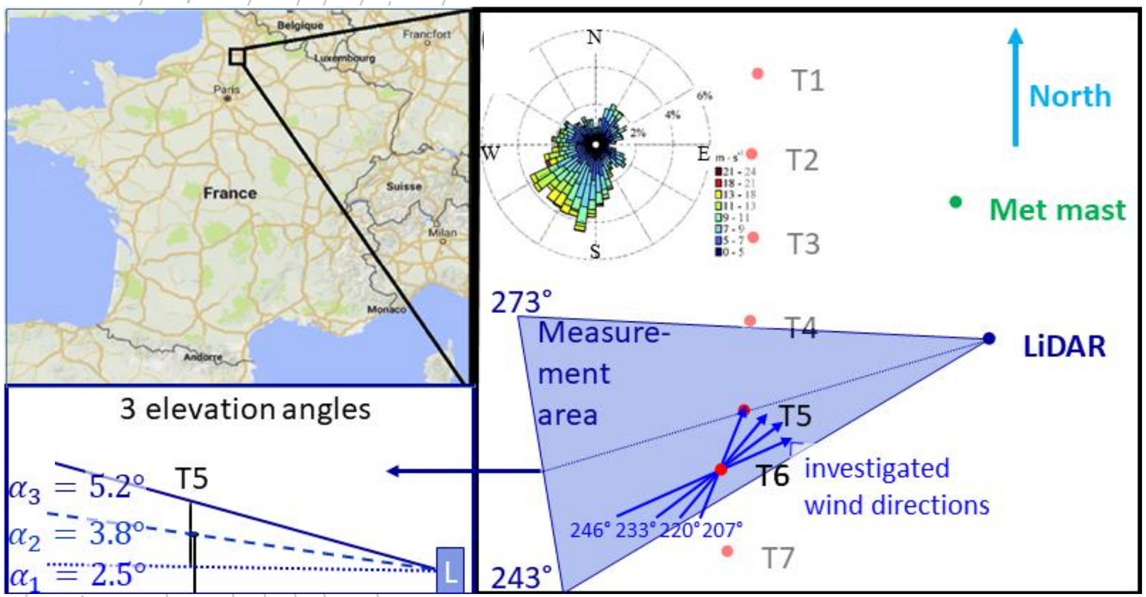


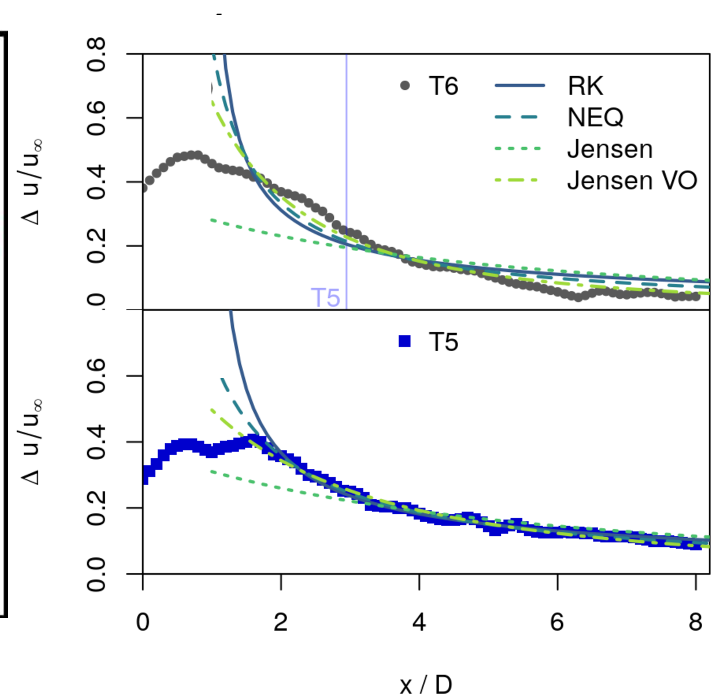
Neunaber, Peinke & Obligado, WES (2022)

What about wind turbines?

The **Townsend-George** theory seems to be useful to study wind turbine wakes

Tests in LIDAR measurements on a wind farm





Neunaber, Obligado, Peinke & Aubrun, Journal of Physics CS (2021)

Objectives

1) Verify the validity of the Townsend-George theory in the far wake of scaled wind turbines

2) Test different operating conditions: Reynolds number (Re_D) and tip speed ratio (TSR)

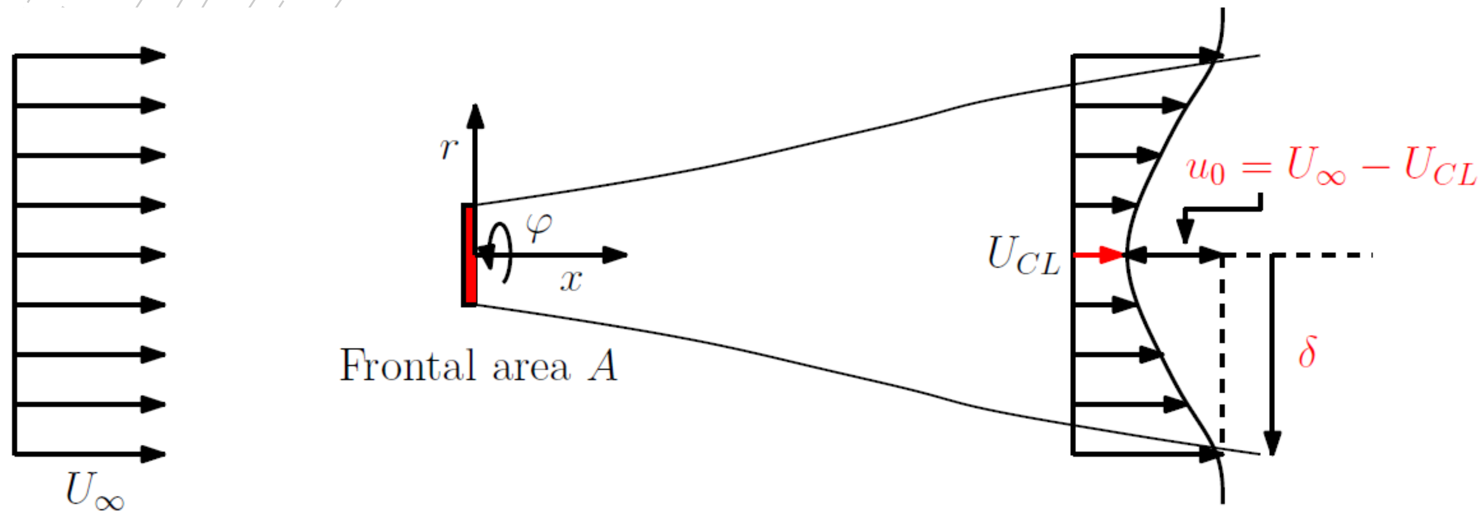
3) Compare with standard engineering models

Experimental strategy

- Test a scaled wind turbine as far downstream as possible
- Reynolds numbers up to 2.9×10^5
- Quantify the energy cascade (not discussed here)

Turbulent axisymmetric wakes

Theoretical background



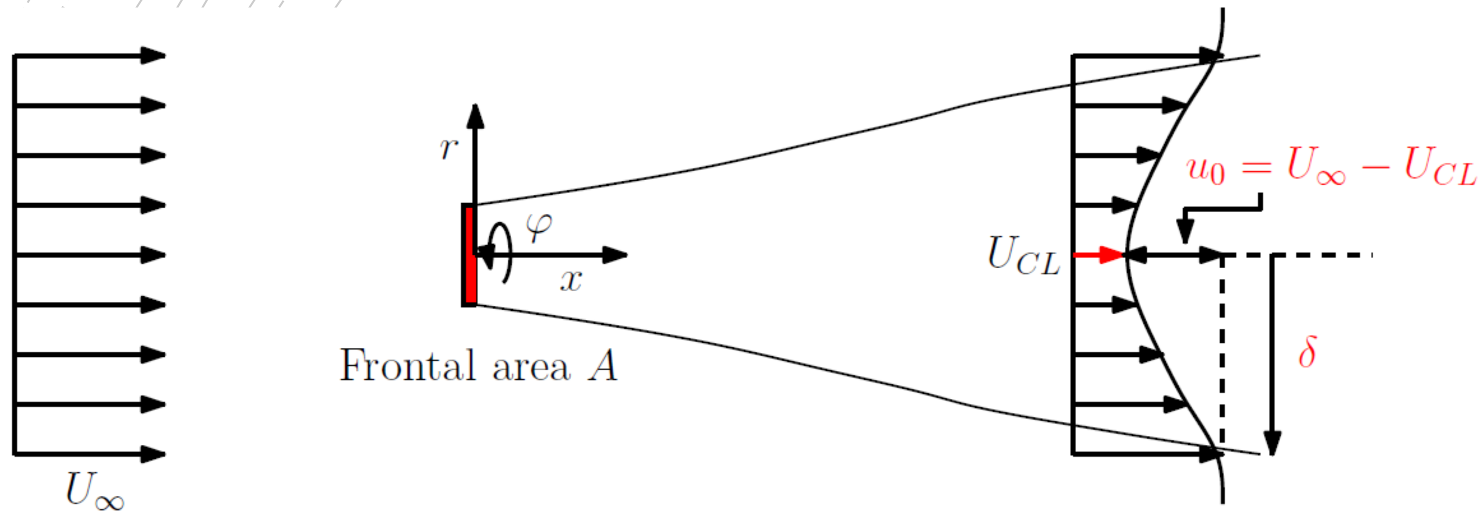
Reference length $L_b = \sqrt{A}$

Freestream velocity U_∞

Centreline velocity deficit $u_0(x) = U_\infty - U(x, r = 0)$

Turbulent axisymmetric wakes

Theoretical background



Momentum thickness

$$\theta^2 = \frac{1}{U_\infty^2}\int_0^\infty U_\infty (U_\infty - U) r dr$$

Wake width

$$\delta^2 = \frac{1}{u_0}\int_0^\infty (U_\infty - U) r dr$$

Momentum conservation

$$U_\infty \theta^2 = u_0 \delta^2$$

Experimental setup

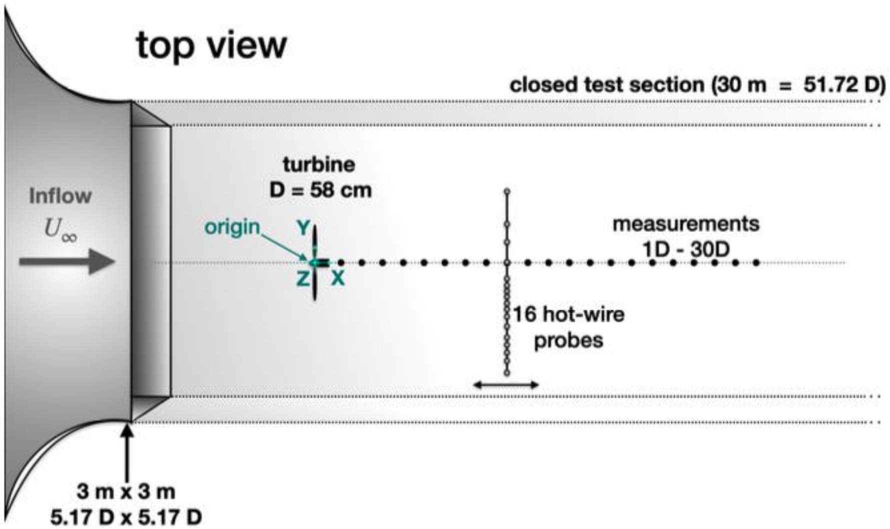
Experimental campaign in Oldenburg wind tunnel (test section: $3 \times 3 \times 30m$^3)

Mowito .6 turbine ($D = 58$ cm) and 24 synchronized hot wires

- Laminar inflow
- Optimal and non-optimal TSR
- Up to 33D downstream

I. Neunaber, M. Hölling & M. Obligado, Energies (2022)

Experimental setup

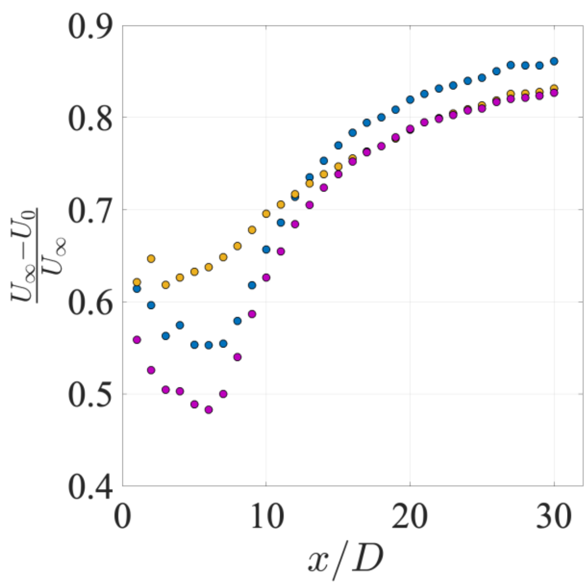


- Horizontal profiles every 1D up to 30D
- 2 different Re_D
- Different TSR by changing the blade pitch angle

Case	U_∞ (m/s)	Re_D	γ	c_T	Thrust (N)	TSR
1	7.5	$Re_D = 2.9 \times 10^5$	γ_o	0.70	6.37	5.31
2	7.5	$Re_D = 2.9 \times 10^5$	$\gamma_o + 6°$	0.34	3.44	4.53
3	5.0	$Re_D = 1.9 \times 10^5$	γ_o	0.73	2.95	5.25

Results: averaged quantities

- In the far wake, case 1 experiences a better recovery
- Cases 2 & 3 collapse onto a single curve
- The evolution of δ is strongly dependent on the TSR



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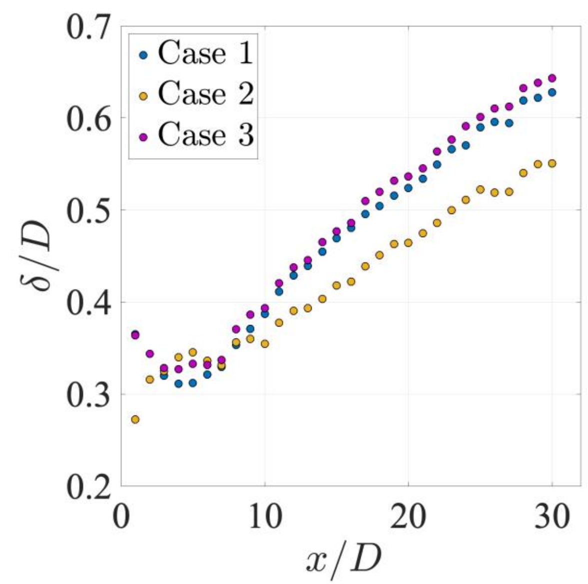
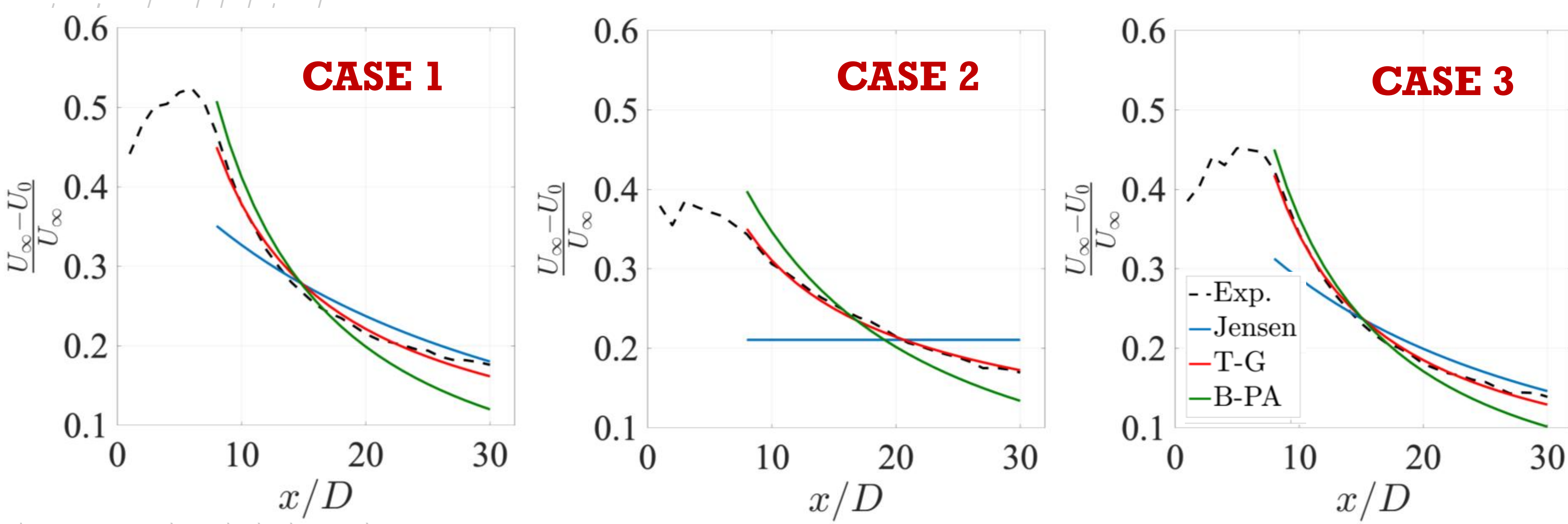


$$U_\infty \theta^2 = u_0 \delta^2$$

Results: streamwise scalings (no virtual origin)

- We tested the Townsend-George model together with the Jensen and Bastankhah-Porté-Agel ones

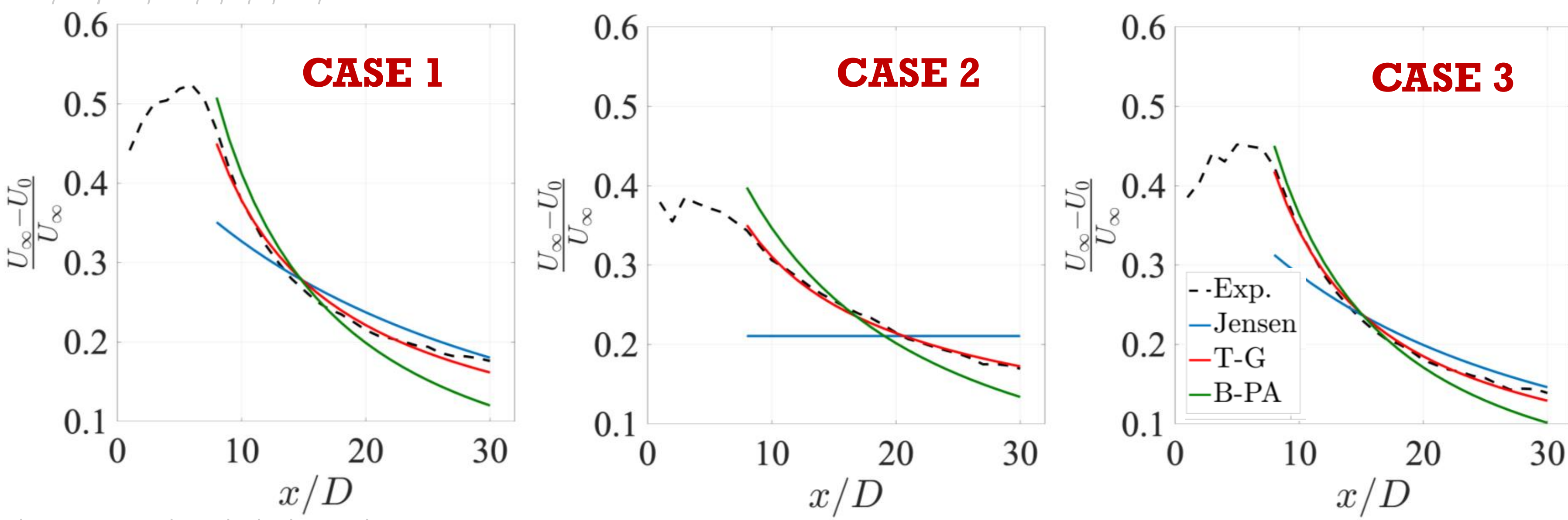
- All three cases are properly modelled by power laws
$(8 < x/D < 30)$



Results: streamwise scalings (no virtual origin)

- All three cases are properly modelled by power laws ($8 < x/D < 30$)

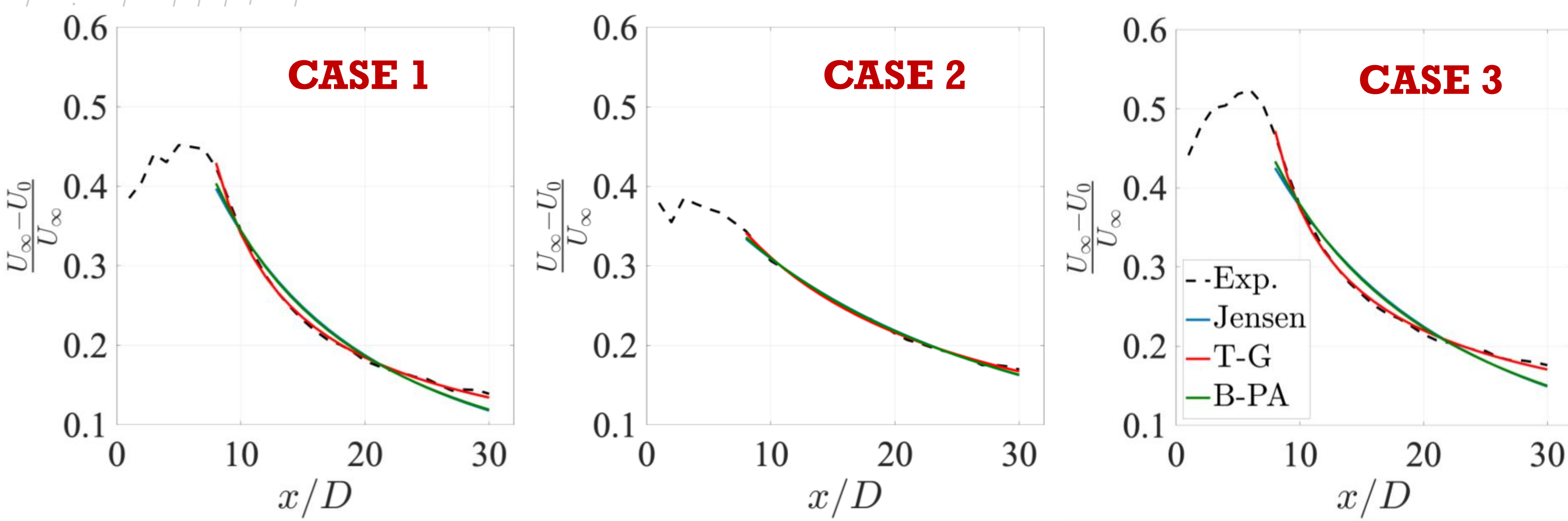
Case	Jensen	B-PA	T-G	
	k_j	k_{BP}	A_{TG}	α
1	0.0126	0.0145	2.65	−0.89
2	2.6×10^{-7}	0.0074	1.07	−0.55
3	0.0105	0.0131	2.25	−0.77



Results: streamwise scalings (with virtual origin)

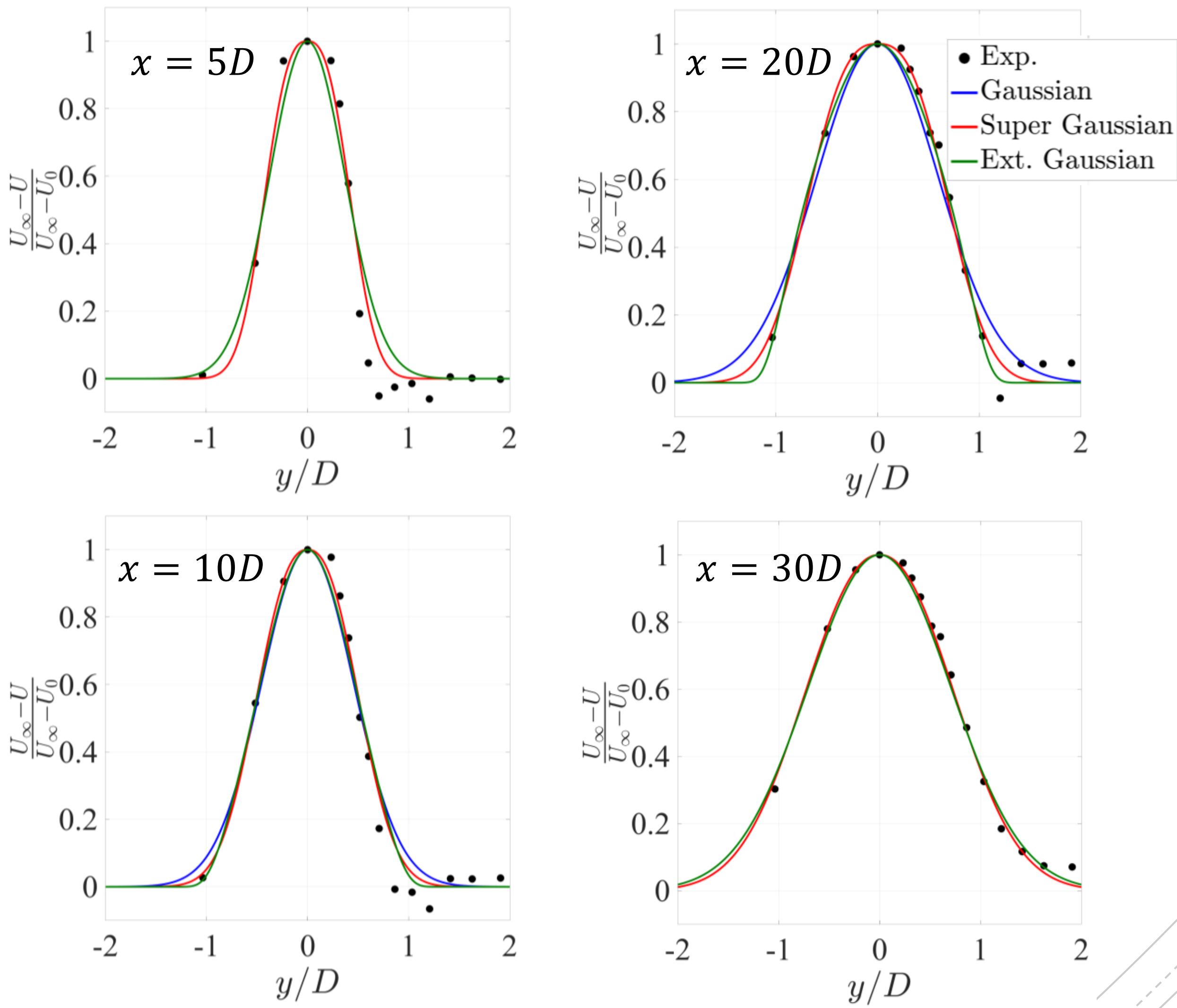
The addition of a virtual origin significantly increases the accuracy of all models in the far wake

Case	Jensen		B-PA		T-G		
	k_j	x_0	k_{BP}	x_0	A_{TG}	α	x_0
1	0.02	6.35	0.012	−3.21	1.3	−0.69	3.03
2	0.0078	21.14	0.0048	−7.99	4.28	−0.88	−9.61
3	0.0166	6.17	0.0094	−5.25	4.53	−0.89	−0.51



Results: radial profiles

- The radial profiles are not fully resolved
- An acceleration ring is found near the wind turbine
- The profile is close to a super-Gaussian near the wind turbine and to a Gaussian further downstream



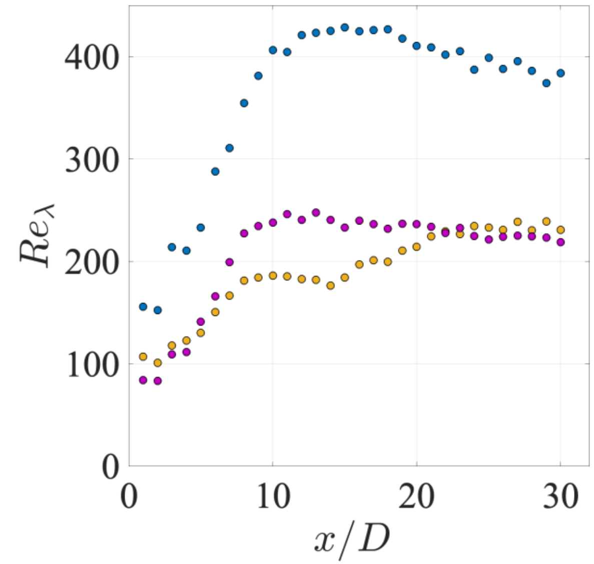
Super-Gaussian:

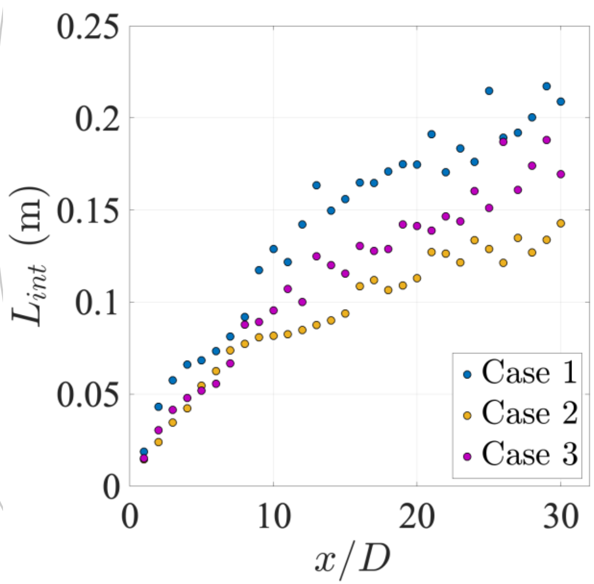
$$\frac{U_\infty - U}{U_\infty - U_0} = a\exp(-b(y/\delta)^n)$$

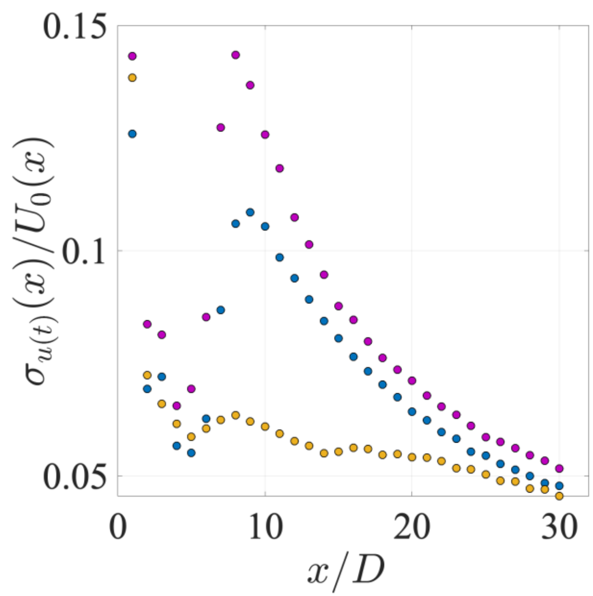
Blondel & Cathelain, Wind Energy Sci. 2020

Results: turbulence quantities

- The turbulent flow was characterized at the centreline
- All parameters are strongly affected by the operating conditions





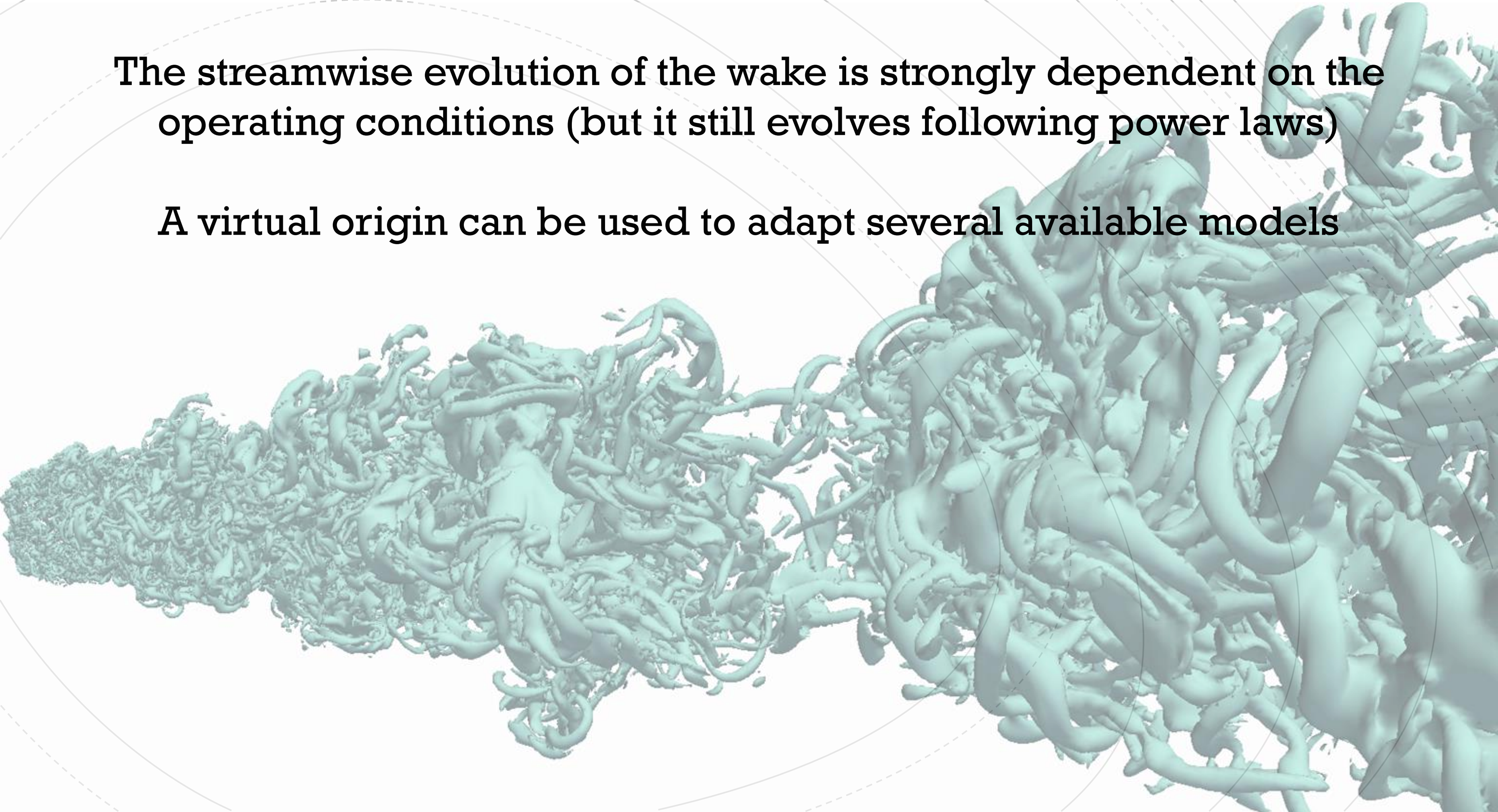


Final remarks

The Townsend-George theory can be applied to wind turbines within a streamwise range relevant for applications

The streamwise evolution of the wake is strongly dependent on the operating conditions (but it still evolves following power laws)

A virtual origin can be used to adapt several available models



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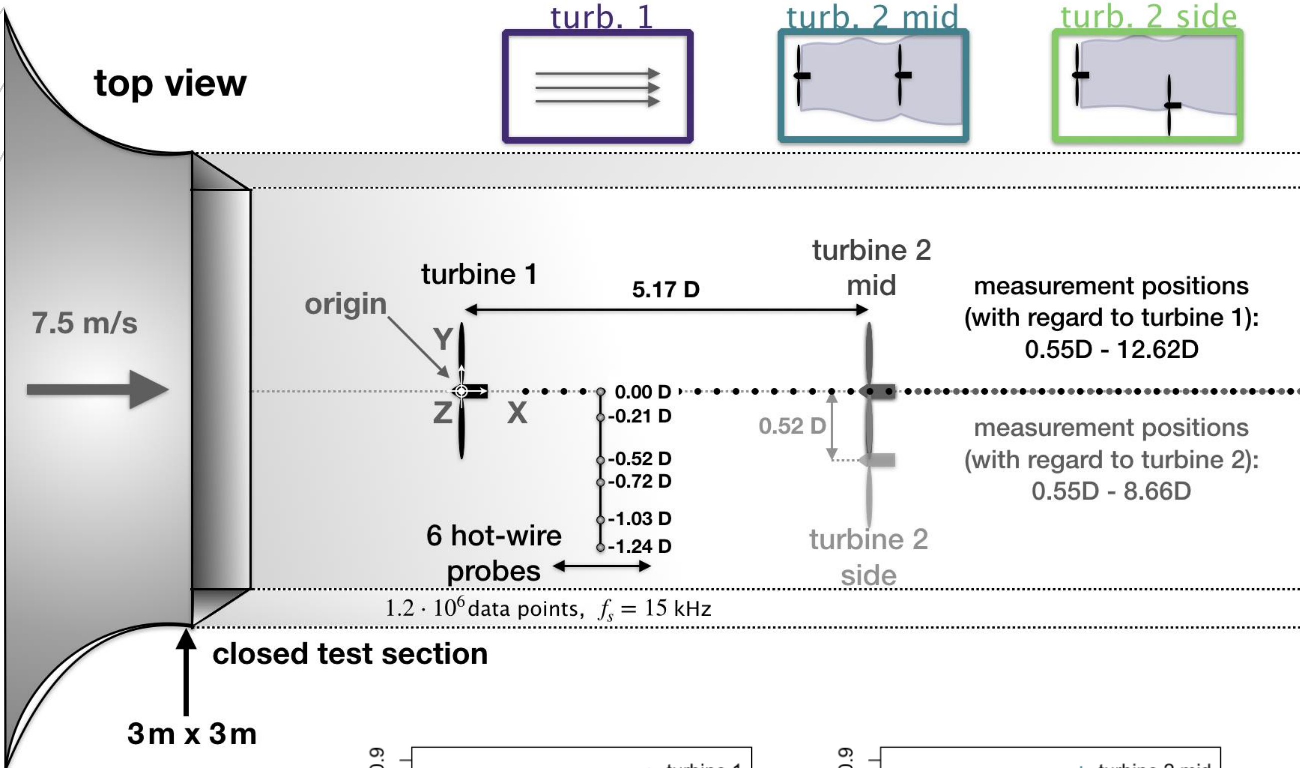

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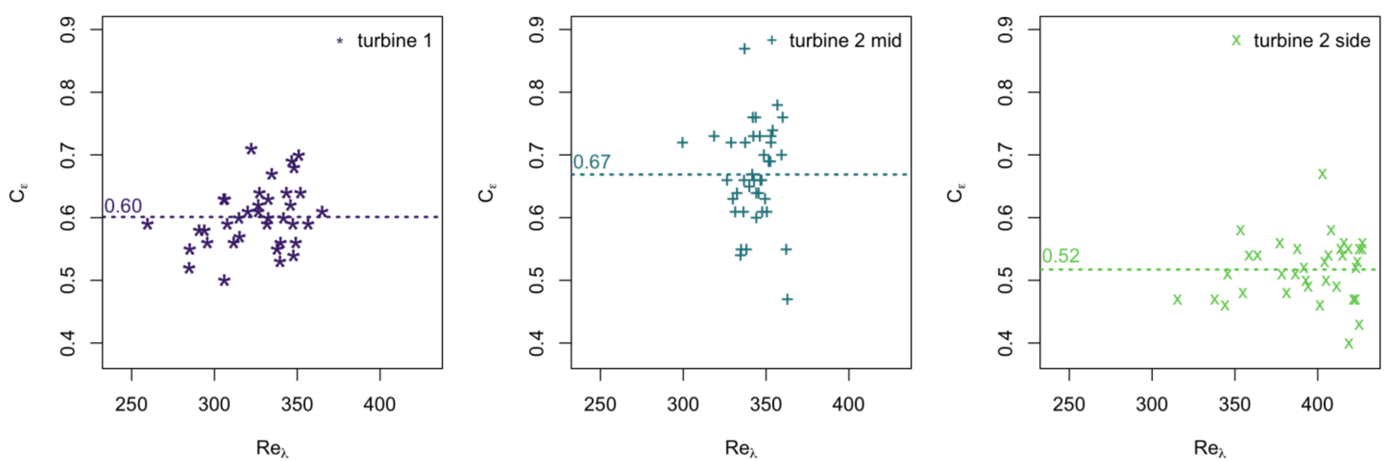

Neunaber, Peinke & Obligado, WES (under review)

Results: streamwise scalings

Townsend-George:

$$\Delta U(x) = A_{TG} U_\infty ((x - x_0)/\theta)^\alpha$$

Jensen:

$$\frac{\Delta U}{U_\infty} = \frac{1 - \sqrt{1 - c_T}}{(1 + 2k_j x/D)^2},$$

Bastankhah–Porté-Agel

$$\frac{\Delta U}{U_\infty} = \underbrace{\left(1 - \sqrt{1 - \frac{c_T}{8\left(k_{BP} \cdot x/D + 0.2\sqrt{\beta}\right)^2}}\right)}_{\text{centerline velocity deficit}} \cdot \underbrace{\exp\left(-\frac{(z/D)^2 + (y/D)^2}{2\left(k_{BP} \cdot x/D + 0.2\sqrt{\beta}\right)^2}\right)}_{\text{Gaussian velocity profile}},$$

Case	Jensen	Jensen VO	B-PA	B-PA VO	T-G	T-G VO
1	93.9	97.5	99.3	98.1	99.5	99.8
2	95.7	99.5	99.8	99.6	99.4	99.9
3	91.7	95.4	98.8	96.3	98.8	99.8