Optimization with probabilistic constraints of complex systems
Application to the design of an offshore wind turbine

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Problem

- Minimize the **cost** of the mooring system of a floating offshore wind turbine (FOWT) [1].

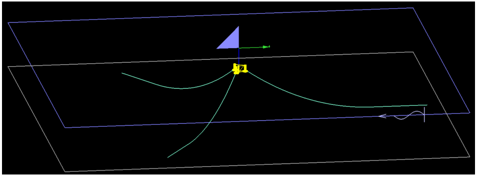

Figure: Mooring system of a FOWT

- Constraints on the **surge**, the **tension** in the mooring lines and the accumulated damage (**fatigue**).
- Uncertainties on model parameters: **random vector** X_p.
- Sea elevation: **time-dependent stochastic process** $\eta(t)$.

Optimization problem with reliability constraints

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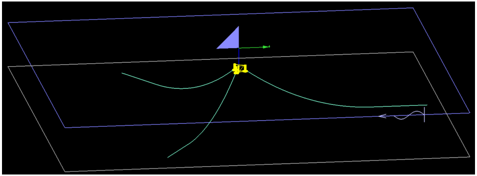

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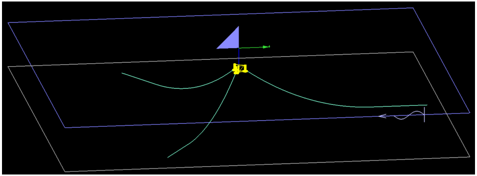

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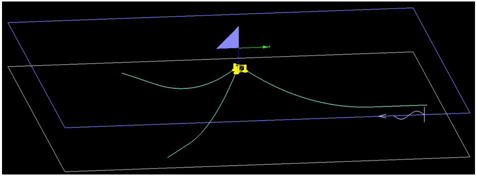

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Mathematical formulation

Time-dependent Reliability-Based Design Optimization:

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\begin{aligned}
&\min_{d \in \Omega_d} \quad cost(d) \quad \text{such that} \\
&\mathbb{P}_{X_p,\eta}\left(\max_{[0,T]} \mathcal{S}\left(d, X_p; t\right) > \mathcal{S}_{\max}\right) && < 10^{-4} \\
&\mathbb{P}_{X_p,\eta}\left(\min_{[0,T]} \mathcal{T}^l\left(d, X_p; t\right) < 0\right) && < 10^{-4} \quad , l = 1,2,3 \\
&\mathbb{P}_{X_p,X_R,\eta}\left(\int_0^T \mathcal{D}^l\left(d, X_p; t\right) \mathrm{d}t > X_R\right) && < 10^{-4} \quad , l = 1,2,3
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with $T = 1$ year.

Difficulty

Estimation of the failure probabilities at each iteration of the optimization algorithm: one realization of $\mathcal{S}$ and $\mathcal{T}^l$ require one expensive simulation.

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Reformulation of the extreme and integral based constraints

Linear equation of motion + $\eta(t)$ piece-wise stationary Gaussian
$\Rightarrow \mathcal{S}$ and $\mathcal{T}^l$ are piece-wise stationary Gaussian processes.

- **Extreme-based constraints**: Extreme Value Theory

$$\mathbb{E}_{X_p}\left[\mathbb{P}_\eta\left(\max_{[0,T]} \mathcal{S}\left(d, X_p; t\right) > \mathcal{S}_{\max}\right)\right]$$

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- **Integral-based constraints**: Ergodic theory + Dirlik [2] method

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Adaptive Kriging for Expectation Constraints Optimization (AK-ECO)

Problem to solve:

$$\min_{d \in \Omega_d} \quad cost(d) \quad \text{such that}$$
$$\mathbb{E}_{X_p}\left[F\left(\sum_{j=1}^{7} M\left(d, X_p, s^j\right)\right)\right] < 10^{-4} \tag{1}$$

Overview of AK-ECO

- Create a metamodel of M: Gaussian process regression (kriging)
- Carry out cycles of optimization until convergence of the design point. Each cycle is composed of:
  - a local enrichment of the metamodel of M
  - a resolution of problem (1) (estimating the expectation with Monte Carlo and replacing M with its current metamodel)

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Comparaison of AK-ECO with reference methods in RBDO

d : (added length, mass, position connection).

	K1600	SORA[3]	Stieng[4]	AK-ECO
$d^{\min}$	(0.86, 108.9, 0)	(2, 141.2, 0)	(2, 140.6, 0)	(0.95, 109.5, 0)
$cost(d^{\min})$	0.282	0.588	0.582	0.287
$p_{\mathcal{S}}(d^{\min})$	1.0×10^{-4}	0	0	0.9×10^{-4}
$p_{\mathcal{T}^l}(d^{\min})$, $l = 1, 2, 3$	0	0	0	0
$p_{\mathcal{D}^l}(d^{\min})$, $l = 1, 2$	0	0	0	0
$p_{\mathcal{D}^3}(d^{\min})$	1.0×10^{-4}	0.2×10^{-4}	0.2×10^{-4}	0.9×10^{-4}
# code calls	1600	16394	5754	305

- AK-ECO finds a reliable optimum with few calls to the expensive code.

[3] Du and Chen, *Sequential Optimization and Reliability Assessment Method for Efficient Probabilistic Design*, Journal of Mechanical Design, 2004

[4] Stieng. *Optimal design of offshore wind turbine support structures under uncertainty*. PhD thesis, Norwegian University of Science and Technology, 2019.

Thank you for your attention!

My PhD thesis is available on:

https://tel.archives-ouvertes.fr/tel-03500604/document