NTNU
Norwegian University of Science and Technology

# Simplified wake modelling for wind farm load prediction

Jacobus B de Vaal
Institute for Civil and Environmental Engineering
Norwegian University of Science and Technology

# Background & Motivation

- Dynamic wake meandering is a proven tool for estimating power production, fatigue- and extreme loads in aeroelastic wind farm simulations.
  1) 2003 : Original idea
  2) 2010 : Implementation for aeroelastic codes
  3) 2013 : Validation (max. influence wake summation)
  4) 2015 : Validation (max. influence & linear wake summation)
  5) 2019 : Included in IEC 61400-1 standard

- Aeroelastic simulations not desirable to do early in design phase


1) Madsen, H. A., et al., *A New Method for Prediction of Detailed Wake Loads*", Proceedings of the IEA Joint Action of Wind Turbines 16th Symposium, 2003.
2) Madsen, H. A., et al., *Calibration and Validation of the Dynamic Wake Meandering Model for Implementation in an Aeroelastic Code*, Journal of Solar Energy Engineering, 2010.
3) Larsen, T. J., et al., *Validation of the Dynamic Wake Meander Model for Loads and Power Production in the Egmond aan Zee Wind Farm*, Wind Energy, Volume 16, Issue 4, 2013. pp. 605–624.
4) Larsen, T. Jet al., *Wake effects above rated wind speed. An overlooked contributor to high loads in wind farms*, EWEA 2015 Conference, 2015.

# Objectives

To develop an *engineering* wind farm model:

- Uses same building blocks as DWM for aeroelastic simulations
  - Velocity deficit model
  - Wake meandering
  - Wake added turbulence
  - Wake summation method (independent of operating conditions)

- Gives accurate estimates for:
  - mean inflow velocity (i.e. power production and mean loads)
  - inflow turbulence intensity (i.e. indication of fatigue loads)

# Single wake deficit model

- Based on axis-symmetric RANS equations for a stationary wake with thin shear layer approximation:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\varepsilon\frac{\partial u}{\partial r}\right)$$

$$\frac{\partial u}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}(rv) = 0$$

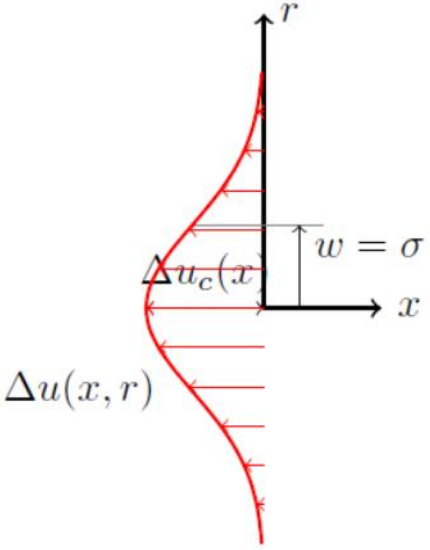
- 1D Gaussian velocity deficit model:[1, 2]

$$u(x,r) = u_0 - \Delta u(x,r)$$

$$\Delta u(x,r) = \Delta u_c(x)\exp\left(-\frac{r^2}{2w^2}\right)$$

$$w = C_T D^2 / \left[8\left(1 - \left(\frac{u_0}{u_c}\right)^2\right)\right]$$

$$\frac{du_c}{dx} = \frac{8\varepsilon}{C_T D^2}\left(\frac{u_0}{u_c}\right)\left[\left(\frac{u_0}{u_c}\right)^3 - \left(\frac{u_0}{u_c}\right)^2 - \left(\frac{u_0}{u_c}\right) + 1\right]$$




1) Anderson, M. *Simplified solution to the eddy-viscosity wake model*, Renewable Energy Systems Ltd., 2009.
2) Gunn, K. *Improvements to the Eddy Viscosity Wind Turbine Wake Model*, Journal of Physics: Conference Series, IOP Publishing, 2019.

# Single wake deficit model (cont.)

- Eddy viscosity model:[2-4]
  - Ambient contribution: $\varepsilon_a = \kappa^2 u_0 z_h I_a$
  - Wake contribution: $\varepsilon_w(x) = kw\Delta u_c(x)$
  - Combined: $\varepsilon(x) = f(x)\varepsilon_w(x) + \varepsilon_a$

$$f(x) = \begin{cases} 0.65 + \left(\frac{x/D - 4.5}{23.32}\right)^{\frac{1}{3}} & , \ x/D < 5.5 \\ 1 & otherwise \end{cases}$$

- Initial velocity deficit:[4]
  - Magnitude: $\frac{\Delta u_{c,0}}{u_0} = C_T - 0.05 - (16\, C_T - 0.5)\,\frac{I_a}{1000}$
  - Location: $x/D = 2$


2) Gunn, K. *Improvements to the Eddy Viscosity Wind Turbine Wake Model*, Journal of Physics: Conference Series, IOP Publishing, 2019.
3) Lange, B. et al. *Modelling of Offshore Wind Turbine Wakes with the Wind Farm Program FLaP*, Wind Energy, 2003.
4) Ainslie, JF. *Calculating the flowfield in the wake of wind turbines*, Journal of Wind Engineering and Industrial Aerodynamics, Elsevier BV, 1988.

# Stochastic wake meandering

- Model wake meandering in a stochastic sense[5]
  - Variance ($\sigma_m^2$) of wake centre offset ($\delta_m$):

$$\sigma_m^2 = 2\sigma_v^2 \Lambda^2 \left[\frac{t}{\Lambda} + exp(-t/\Lambda) - 1\right]$$

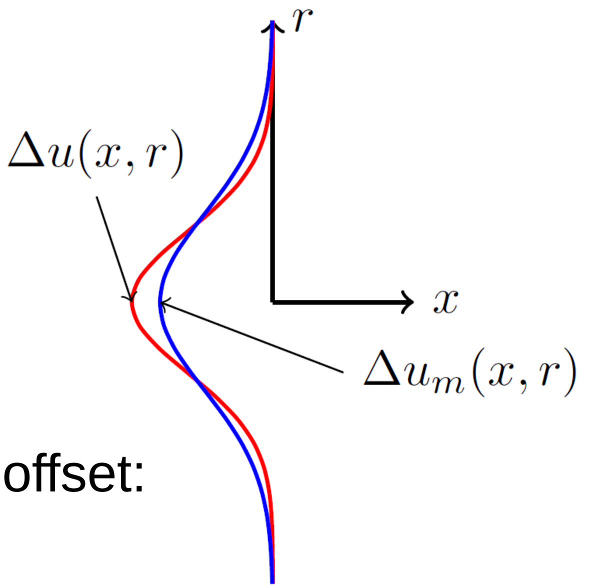
$$\sigma_v = 0.8\, I_a u_0$$

$$\Lambda = \frac{\kappa z_h}{\sigma_v}$$

  - Probability density function for offset:

$$f(\delta_m) = \frac{1}{\sqrt{2\pi}} exp\left(-\frac{\delta_m^2}{2\,\sigma_m^2}\right)$$

  - Convolution of the velocity deficit and lateral offset:




$$\Delta u_m(x,r) = \int_{-\infty}^{\infty} \Delta\, u(x,r) f(x,\delta_m) d\delta_m$$

$$= \Delta u_c(x) \left[1 + \left(\frac{\sigma_m}{w}\right)^2\right]^{-\frac{1}{2}} exp\left(-\frac{r^2}{2(w^2+\sigma_m^2)}\right)$$

5) Braunbehrens, R. & Segalini, A. *A statistical model for wake meandering behind wind turbines*. Journal of Wind Engineering and Industrial Aerodynamics, 2019.

# Multiple wake interaction

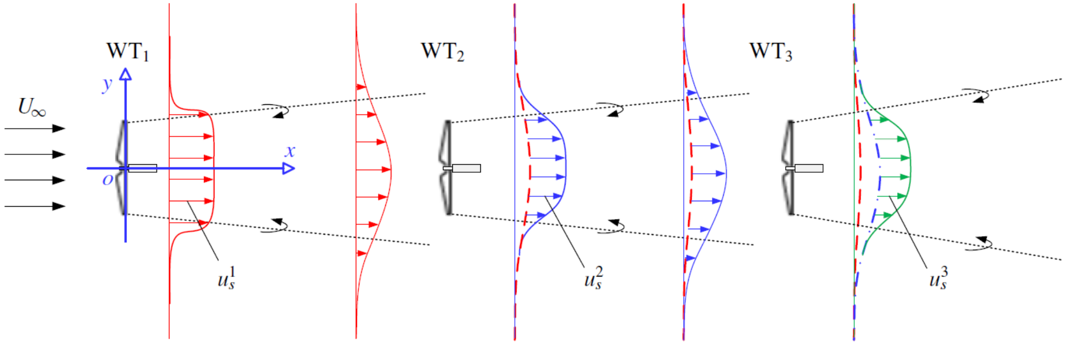



Image from [6]

- Engineering wind farm model:
  - Wind turbines interact through superposition of individual wake velocity deficits and turbulence intensities.
  - Superposition method usually derived on principles of conservation of momentum or energy in individual deficits
  - Differences in definition of reference wind speed for individual wakes and deficit summation, give rise to many different superposition methods.


6) Zong, H. & Porté-Agel, F. *A momentum-conserving wake superposition method for wind farm power prediction*, Journal of Fluid Mechanics, Cambridge University Press, 2020.

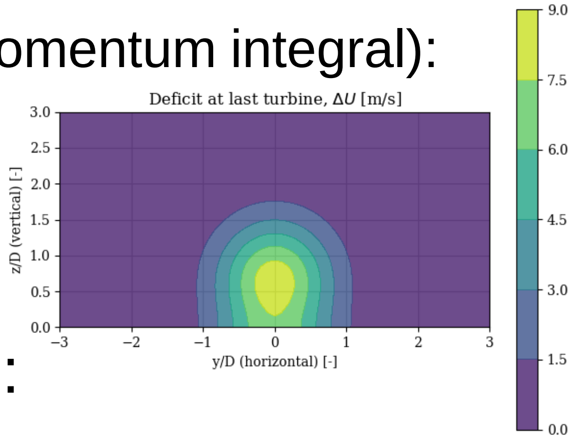
# Wake deficit summation[6]

- Momentum conservation for a single turbine:
  - Wake mean convection velocity (linearized momentum integral) :

$$\bar{u}^i(x) = \frac{\iint u^i(x,r)\,\Delta u^i(x,r)\,dy\,dz}{\iint \Delta u^i(x,r)\,dy\,dz}$$

- Momentum conservation for multiple turbines
  - Farm mean convection velocity (linearized momentum integral):

$$\bar{U}(x) = \frac{\iint U(x,y,z)\,\Delta U(x,y,z)\,dy\,dz}{\iint \Delta U(x,y,z)\,dy\,dz}$$



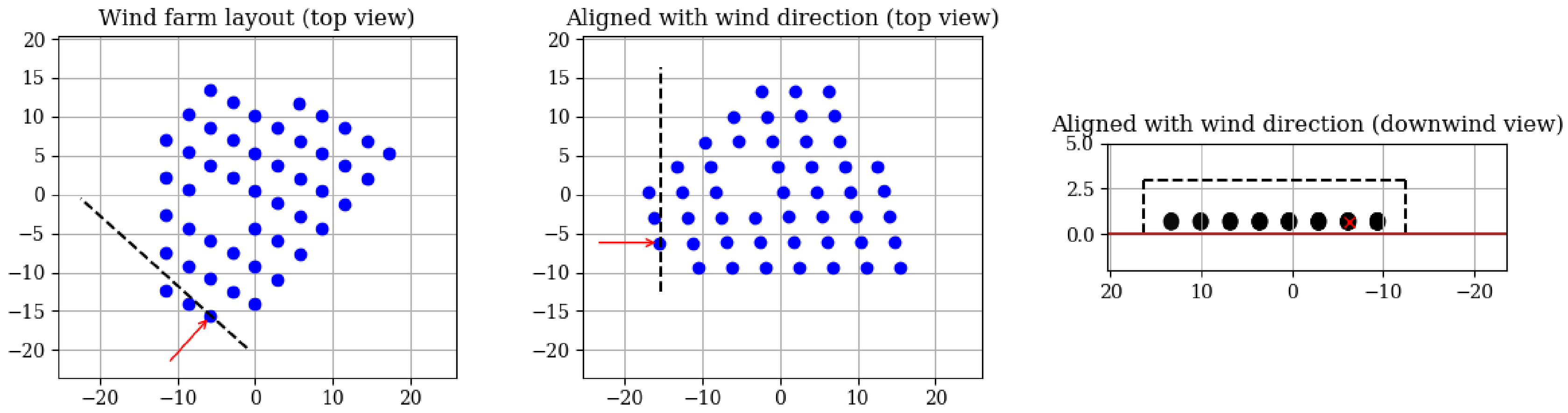

- Momentum conserving wake summation:

$$\Delta U(x,y,z) = \sum_i^N \frac{\bar{u}(x)}{\bar{U}(x)} \Delta u^i(x,y,z)$$

6) Zong, H. & Porté-Agel, F. *A momentum-conserving wake superposition method for wind farm power prediction*, Journal of Fluid Mechanics, Cambridge University Press, 2020.

# Wake deficit summation (cont.)

- Implications for wind farm model:
  - Evaluate wakes sequentially as encountered along the mean wind direction.
  - Define a plane normal to wind direction, on which to calculate mean convection velocity integrals.
  - Requires iteration (not expensive, wakes are not re-evaluated, only their summation)



# Wake added turbulence

- Derive a mean wake turbulence intensity, from the eddy viscosity model:[3]

$$\bar{I} = \frac{\varepsilon}{\kappa^2 u_0 z_h} \Rightarrow \bar{\sigma} = \frac{\varepsilon}{\kappa^2 z_h}$$

- Separate into ambient and wake added contributions

$$\bar{\sigma} = \sqrt{\sigma_a + \sigma_+} \Rightarrow \sigma_+ = \sqrt{|\bar{\sigma} - \sigma_a|}$$

- In wind farm, superimpose ambient and wake added contributions:[7]

$$\sigma = \sigma_a + \sqrt{\sum_i \sigma_{+,i}^2}$$

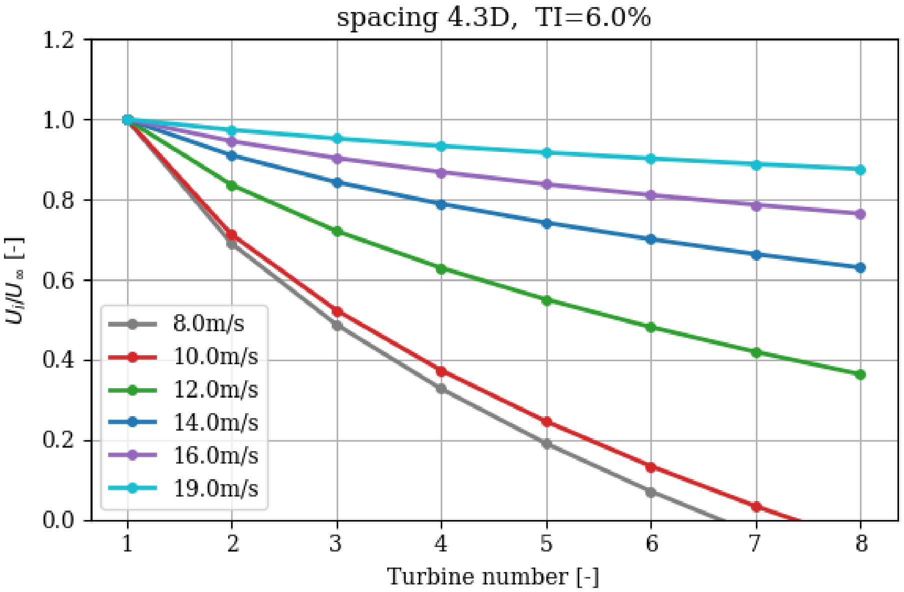
3) Lange, B. et al. *Modelling of Offshore Wind Turbine Wakes with the Wind Farm Program FLaP*, Wind Energy, 2003.
7) Wessel, A., Peinke, J., Lange, B. *Modelling Turbulence Intensities Inside Wind Farms*. Wind Energy, Springer Berlin Heidelberg, 2007.

# Results: Wake summation influence

- Row of 8 turbines: mean inflow velocity at each turbine (non-dimensionalized with undisturbed wind speed)

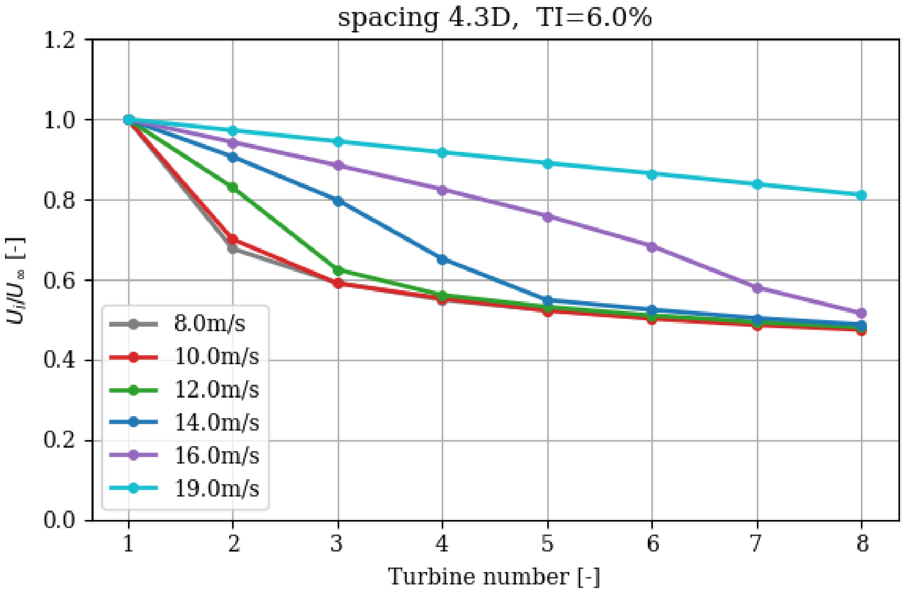
**Linear summation**

$$U = U_\infty - \sum_i (U_\infty - u_i)$$




**Mom. conserving summation**

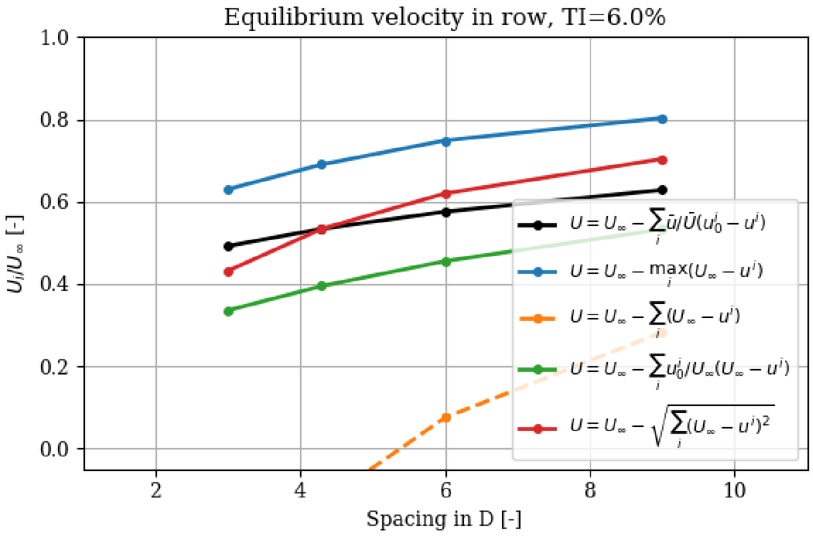
$$U = U_\infty - \sum_i \bar{u}/\bar{U}\ (u_{0,i} - u_i)$$



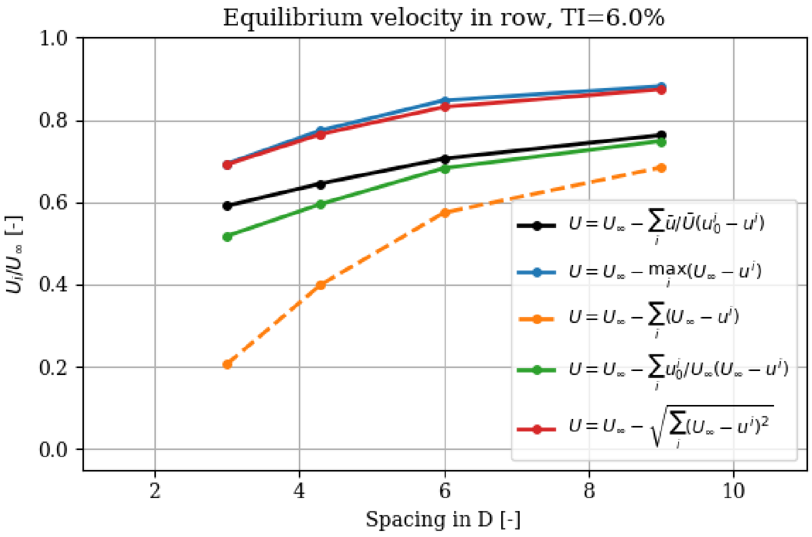
# Results: Wake summation influence

- Influence of inter-turbine spacing and summation method

### Wake meandering




### Wake meandering + Wake added turbulence



# Results: Lillgrund Row B

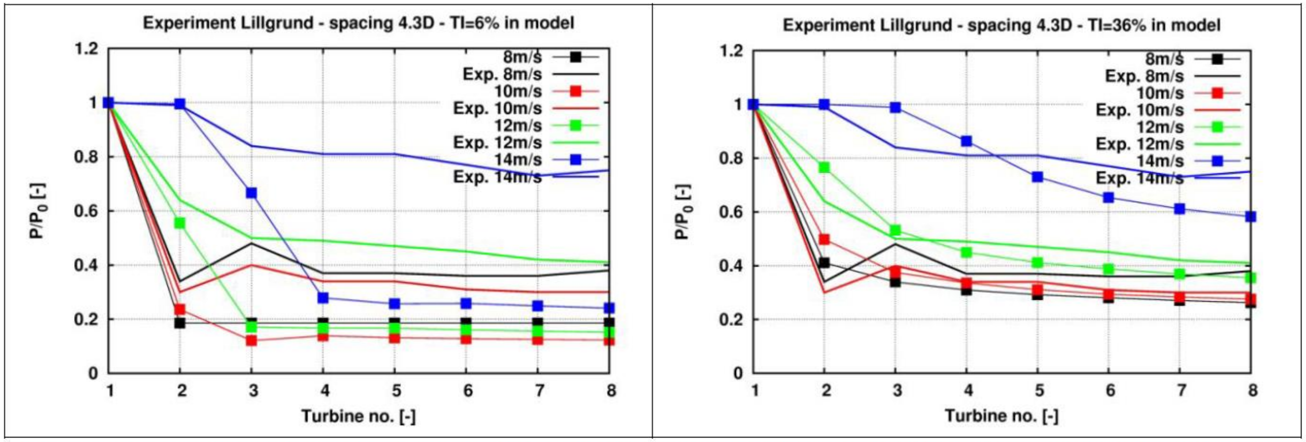
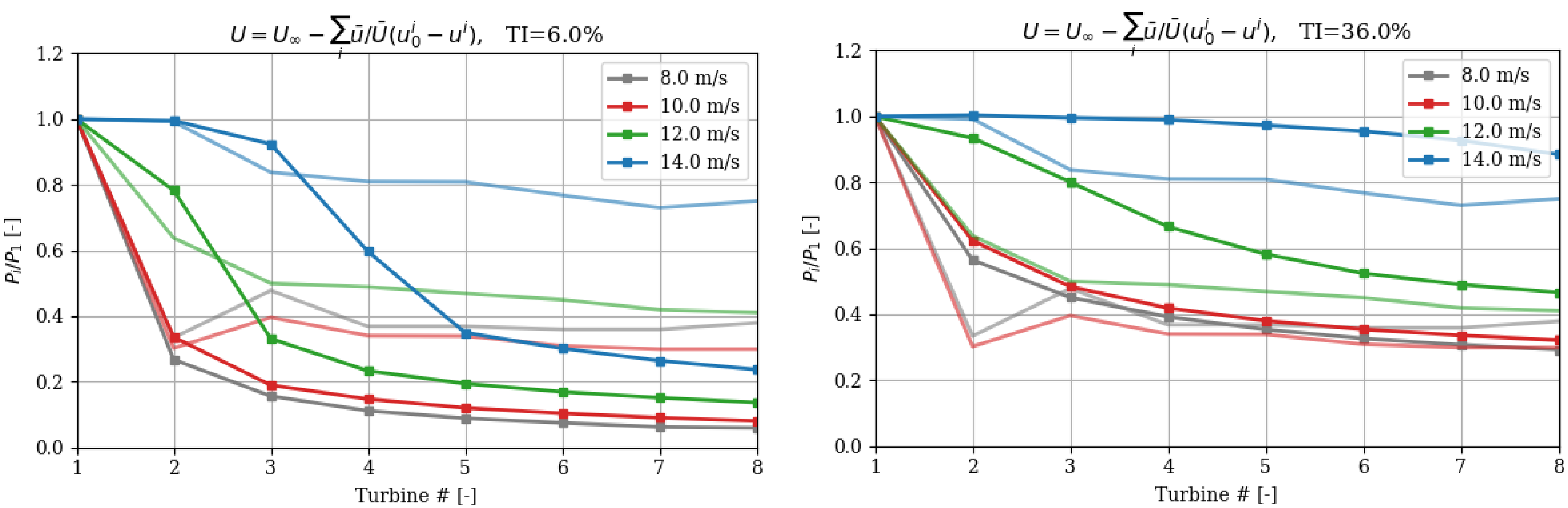



Figure 10 Comparison of model results from the Lillgrund wind farm. In the left figure the TI used in the model is 6%. To account for wake meandering the TI was increased to 36% in the simulations in the right graph.

Image from [8]

**Without** meandering
or added turbulence



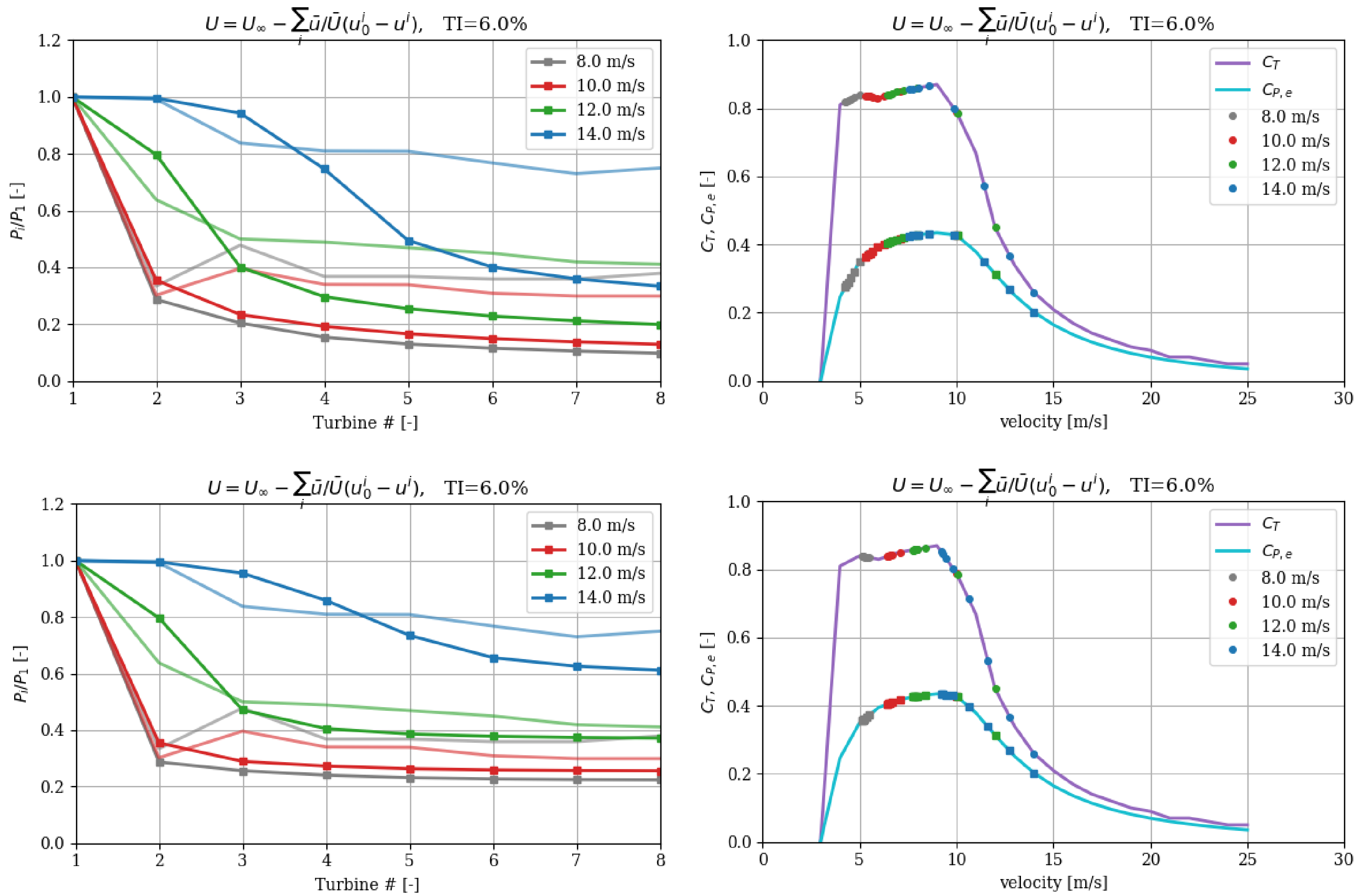


8) Madsen, H. A., et al., *Wake flow characteristics at high wind speed,* 34th Wind Energy Symposium, AIAA, 2016

# Results: Lillgrund Row B

Meandering only

Meandering + added turbulence



# Conclusions & Further work

- Presented an engineering wind farm model that simulates relevant physics as simply as possible:
  - Wake added turbulence intensity
  - Stochastic wake meandering
  - Momentum conserving velocity deficit summation
  - Potential to be useful for wind farm power production and load prediction at an early design stage

- Further work
  - Better quantify improvements / changes in velocity field
  - Quantify influence on rotor loads, compare with aeroelastic calculations
  - Continue validating sub-models, and refine as required