From wind inflow field towards turbine loads: Analysis from a stochastic POD model and LES simulations

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Outline

- Introduction
- LES simulation
- Stochastic reduced order models
- Turbulent box and nudging
- Conclusion

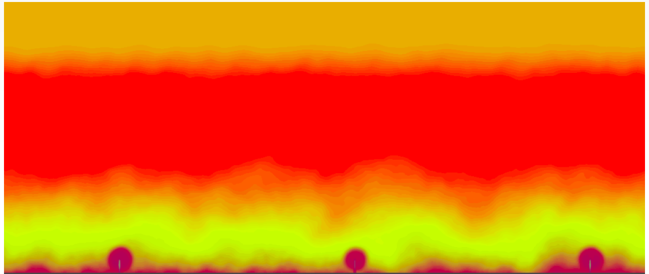
Introduction

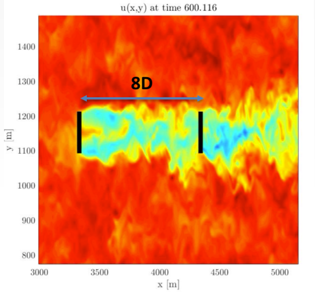
- Highly variable wakes affects the performance of the wind farm. Such effects can be studied using Lidar measurements or numerical wake models like LES.
- LES runs are computationally expensive for practical wind energy applications.
- Reduced order model can bridge between computationally expensive CFD models and capturing dominant scales and dynamics

LES simulations

- 6912×2304×1459 m with grid size of dxdydz=6 m. The grid cell is stretched in *z* direction after 800 m with the factor of 1.04, maximum cell size is capped at dz_{max}=12 m. Model is run for **neutral** atmospheric boundary layer. Two turbine configuration.

- 7670×3830×1600 m with grid size of dx=dy=dz=10 m. Model is run for **neutral** atmospheric boundary layer. Three turbine configuration and high frequency (5 Hz) outputs in few points.

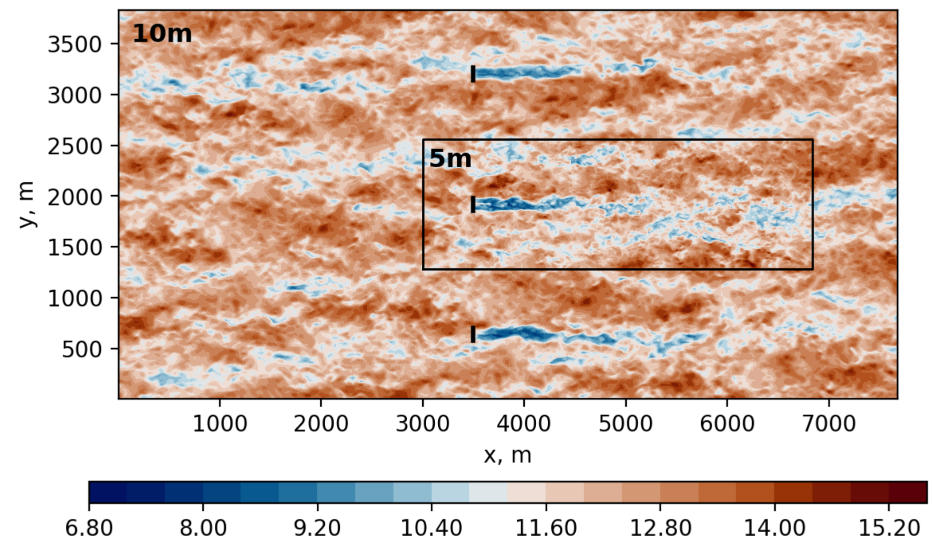




PALM LES setup: Three turbines

- Parent domain: Δ = 10 m
- Child domain: Δ = 5 m
- Three NREL 5 MW wind turbines
- No wake-wake/turbine interaction.
- Free flow: U_0 = 12.5 m/s, true neutral.

- **Simulations**:
  - No nested domain (coarse, 10m).
  - One-way nesting.
  - Two-way nesting.
  - 3 hours simulated, 2 hours of data at 5Hz



POD method

Let us write $\boldsymbol{x} = (x, y, z)$ for the position vector, $\boldsymbol{u} = (u, v, w)$ for the wind velocity vector, $\overline{\boldsymbol{u}}$ as the mean, and $\boldsymbol{u}' = \boldsymbol{u} - \overline{\boldsymbol{u}}$ as fluctuating velocity, all estimated from the LES data. The idea behind the POD is to decompose the fluctuating velocity field into a number of deterministic spatial fields, $\boldsymbol{\phi}_i(\boldsymbol{x})$, modulated by random time-dependent weighting coefficients, $a_i(t)$:

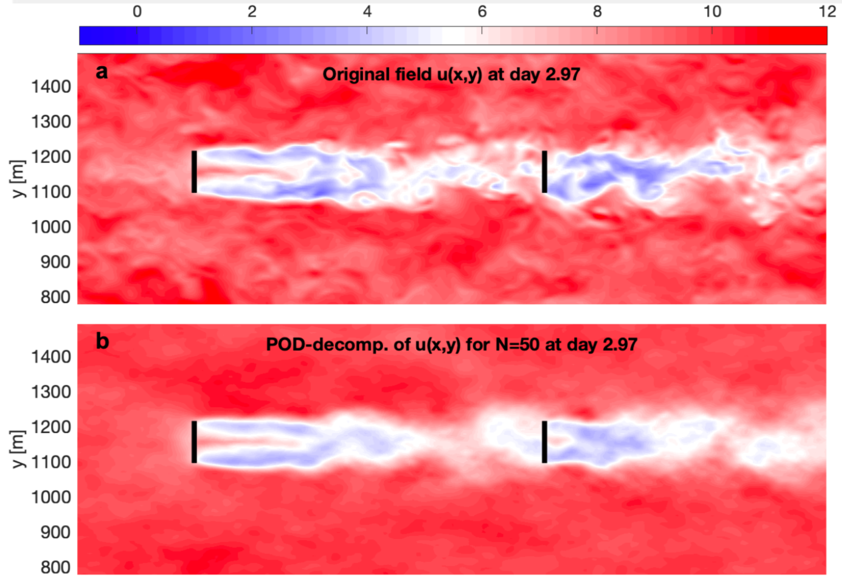
$$\boldsymbol{u}'(\boldsymbol{x}, t) = \sum_{i=1}^{N} a_i(t)\boldsymbol{\phi}_i(\boldsymbol{x}), \tag{1}$$

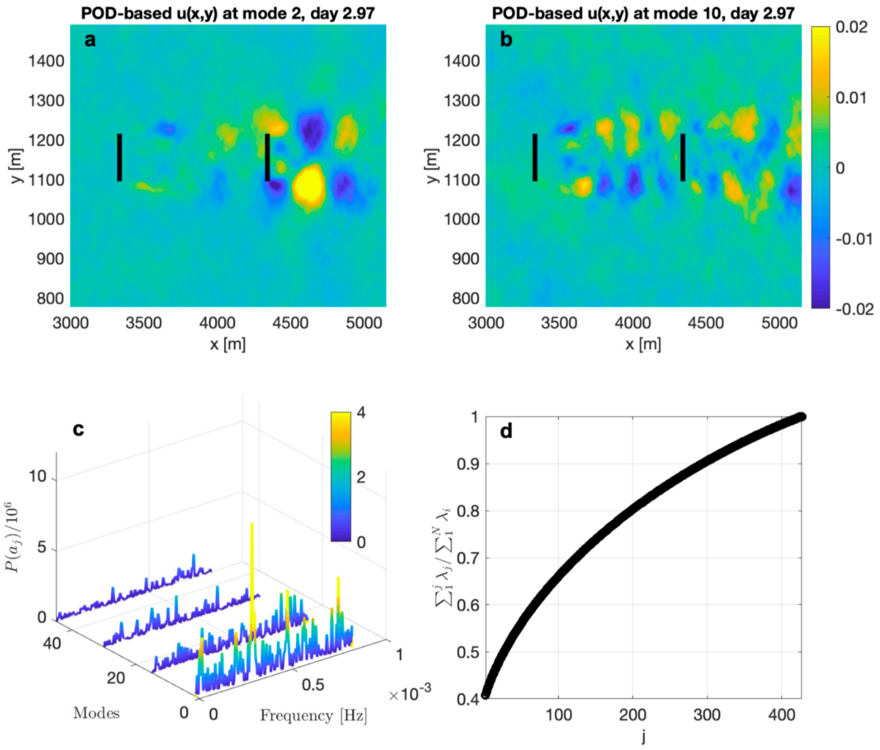
$$a_j(t) = \int\int\int_{\boldsymbol{x}} \boldsymbol{u}'(\boldsymbol{x}, t)\boldsymbol{\phi}_j(\boldsymbol{x})d\boldsymbol{x} \quad \text{and} \quad \overline{a_i(t)a_j(t)} = \lambda_j\delta_{ij}, \tag{3}$$

where the overbar denotes temporal averaging, and λ_j denotes the strength of the jth POD mode. Here, $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_N$ are real eigenvalues (calculated from SVD) so that $\overline{a_j(t)^2} = \lambda_j$ for $j = 1, \cdots, N$. In the POD, modes are ranked by their energy.

POD method

(a,b) POD spatial basis functions for mode number of 2 and 10, respectively; (c) Energy power spectra of aj for j = 1, 15, 30, 45; and (d) cumulative energy of POD modes.



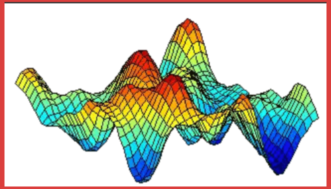


POD method and stochasticity

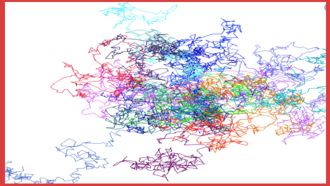
all dynamical information associated with temporal variation of wind is encapsulated in the time-dependent weighting coefficients

Stochasticity of time dependent coefficient

- Gaussian random process (GP)

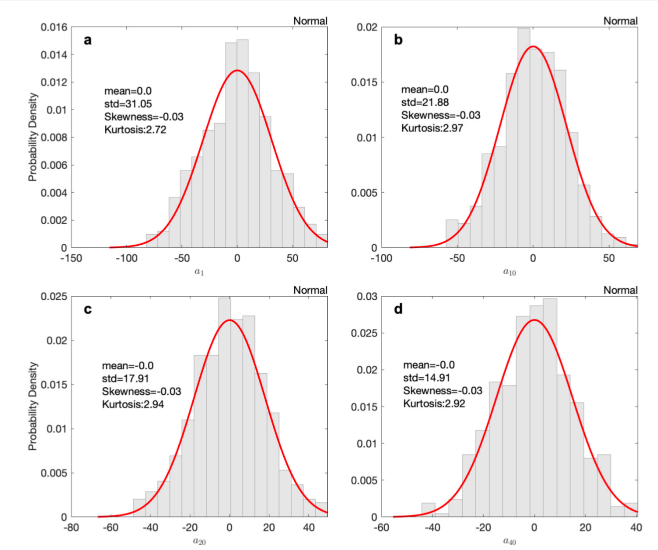


- Stochastic differential equations (SDE)



Stochastic POD: GP

We assume that the empirical probability density of *a* fits properly with a normal distribution with an auto-correlation approximated by an exponential-decaying function in time.



Assume that $\hat{a}_j$ is a Gaussian process

Corresponding to

$$a_j$$

The GP at j^{th} model can be constructed based on mean and a covariance function

$$\hat{a}(t) \sim GP(\mu_j, k_j(t,t'))$$

$$\begin{aligned} \mu_j(t) &= \Xi[\hat{a}_j(t)], \\ k_j(t,t') &= \Xi[(\hat{a}_j(t)-\mu_j)(\hat{a}_j(t')-\mu_j)]. \end{aligned}$$

Stochastic POD: GP

Using above definition, at each mode we can produce a random time series for our GP with a certain modal zero-mean error, $y_j=a_j+e_j$. Note that we aim to use the GP to eaither extrapolate or interpolate the stochastic process. Therefore, for new set of random time series y^*, we will have:

$$\begin{bmatrix} y_j \\ y_j^* \end{bmatrix} \sim N\left(0, \begin{bmatrix} k_j(t,t)+\sigma_\epsilon^2 I & k_j(t,t^*) \\ k_j(t^*,t) & k_j(t^*,t^*) \end{bmatrix}\right), \rightarrow \overline{y_j^*} = k_{*j}^T (k_j + \sigma_\epsilon^2 I)^{-1} y_j,$$

Stochastic POD: GP

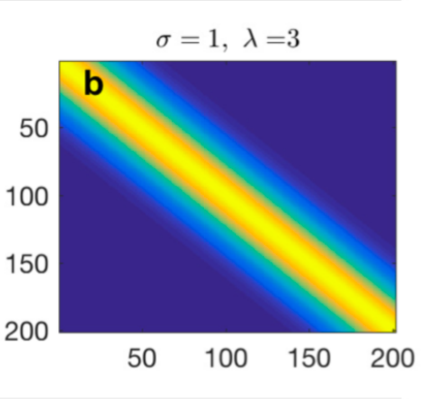
$$k(t,t')$$

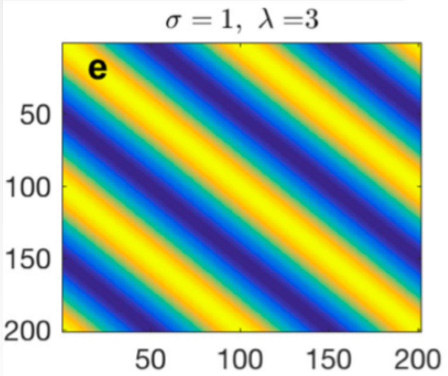
Squared Exponential (SE) covariance

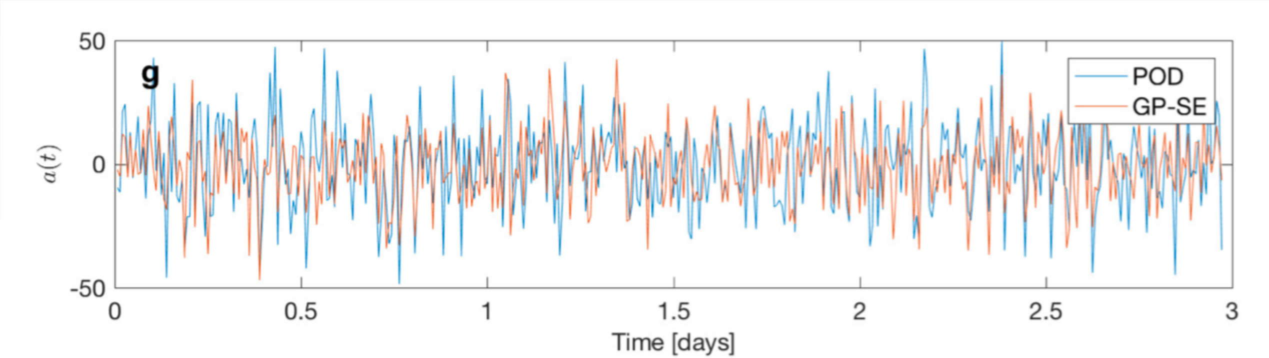
$$\sigma_{ker} \exp\left(-\frac{(t-t')^2}{\xi^2}\right)$$

periodic kernel

$$\sigma_{ker} \exp\left(-\frac{2\sin^2(|t-t'|/\chi)}{\xi^2}\right)$$







Stochastic POD: SDE

the one-dimensional stochastic differential equation for the evolution of a ̂ⱼ obeys the following general form

$$d\hat{a}_j(t) = -\alpha_j(\mu_j - \hat{a}_j(t)) \cdot dt + \overbrace{\sigma_j\sqrt{2\alpha_j} \cdot dW(t)}^{d\gamma(t)},$$

where γ is a driftless subordinator, μ_j and σ_j are the mean and the standard deviation of $\hat{a}_j(t)$ at the j^{th} POD mode, respectively. The autocorrelation is governed by an exponential-decaying function with decay rate of α_j for the jth POD mode as follows:

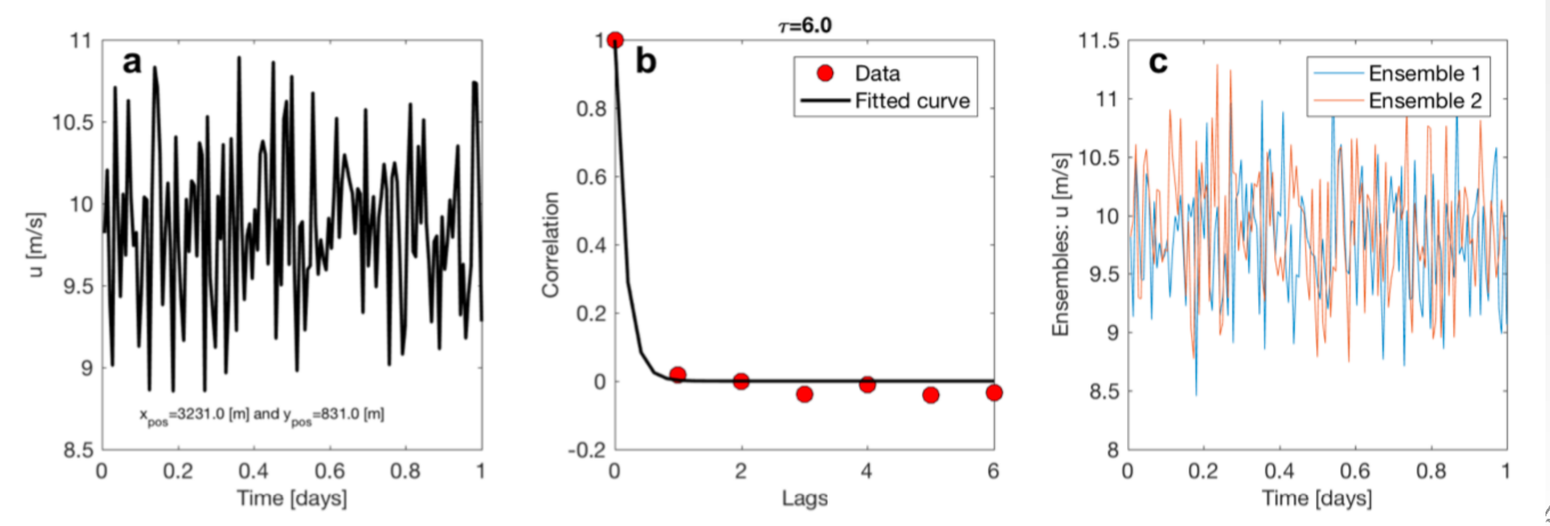
$$\rho(\tau) = \overline{\hat{a}_j(t)\hat{a}_j(t+\tau)} = e^{-\alpha_j \cdot \tau},$$

Stochastic POD: SDE

To generate time series, we use Euler scheme in Ito calculus as:

$$\hat{a}_j^{n+1} = \hat{a}_j^n + f(t_j, \hat{a}_j^n)dt + g(t_j, \hat{a}_j^n)\Delta W_j^n,$$

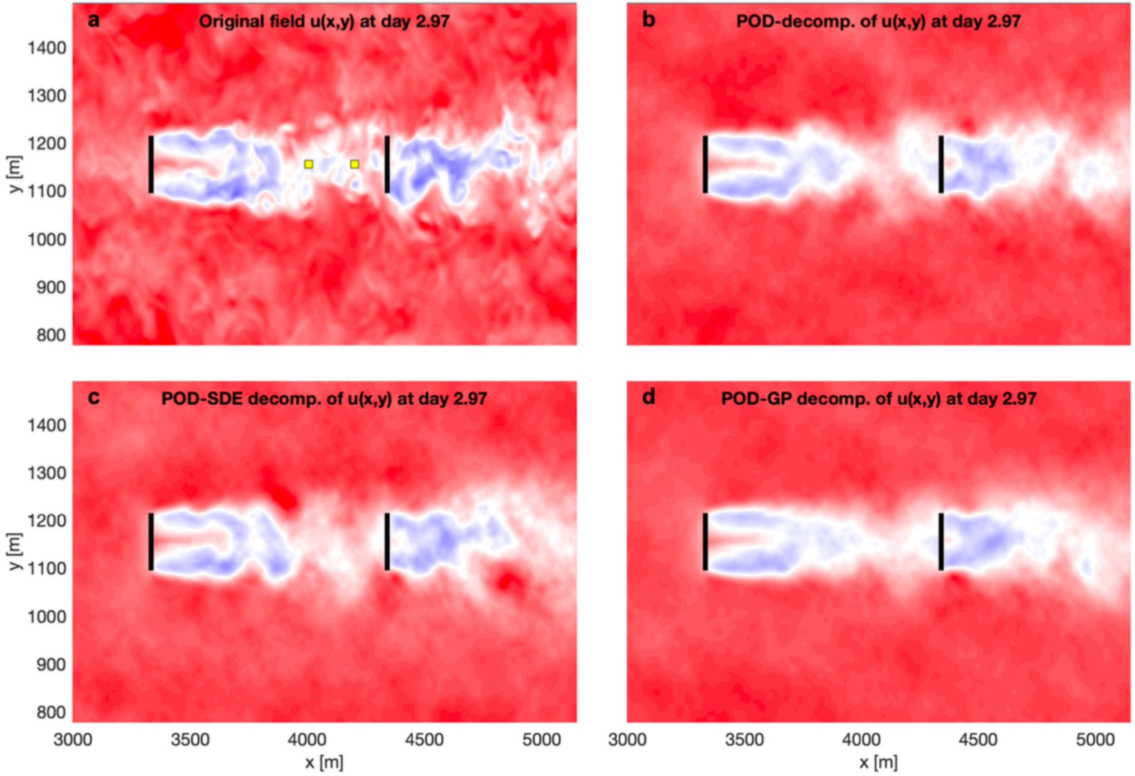
$$\Delta W_j = W_j^{n+1} - W_j^n \sim N(0, \sqrt{dt})$$



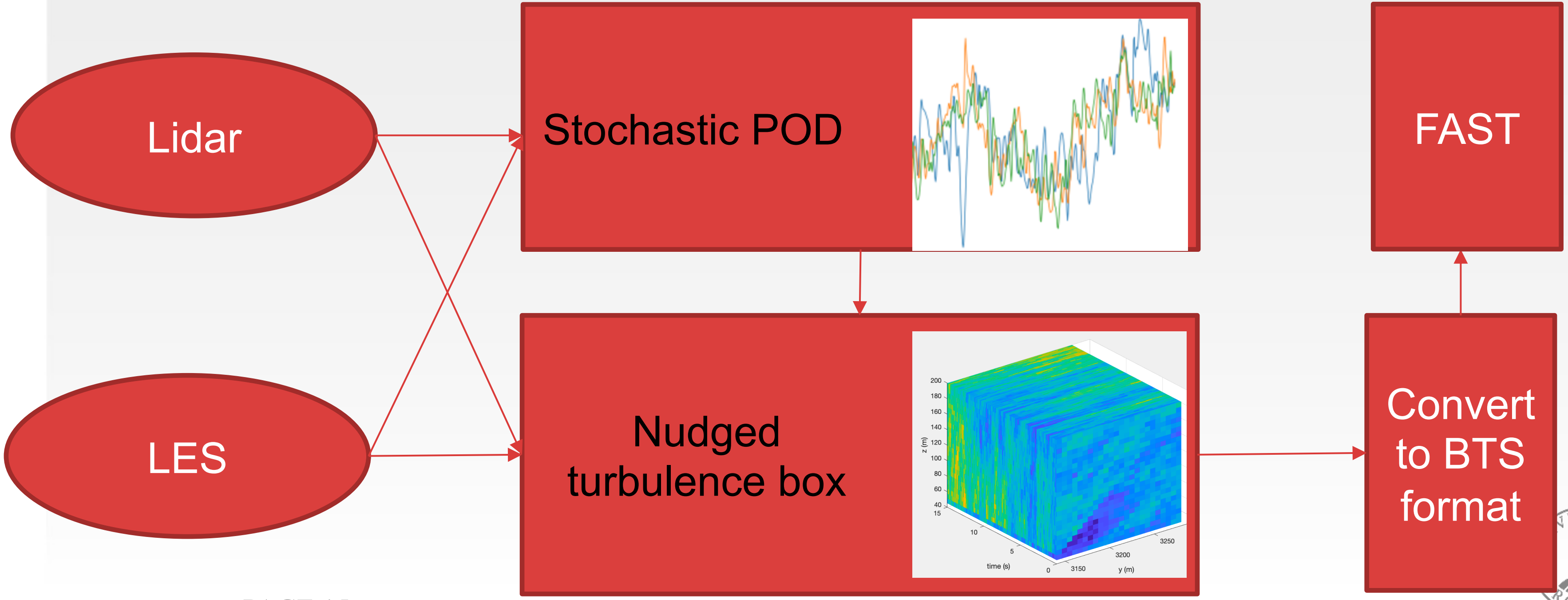
(a) Wind time series at point (x_{pos},y_{pos}) = (3231,831) m from LES; (b) autocorrelation of the data (red markers) and fitted exponential curve (black line); and (c) two SDE-based realization.

Stochastic POD: SDE versus GP

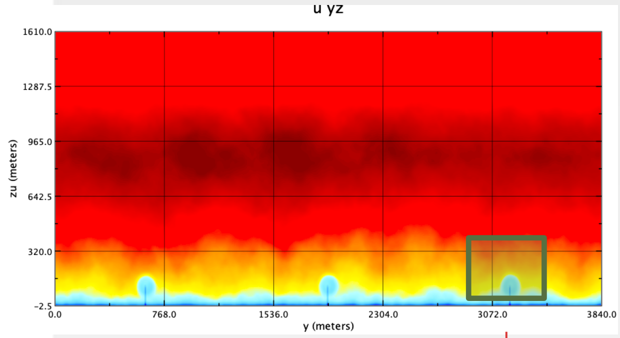
u component of wind from LES at day t = 2.97; (b) reconstructed u by the standard POD scheme; (c) reconstructed u from the SDE-based model; and (d) reconstructed u from the GP-based model.

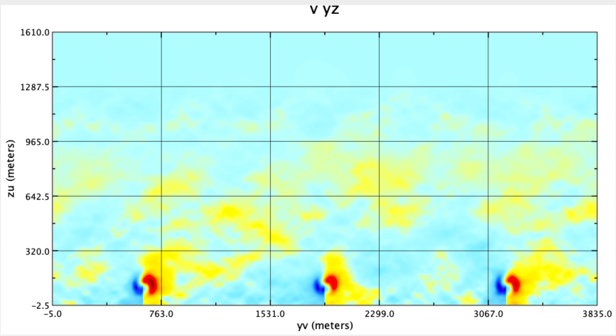


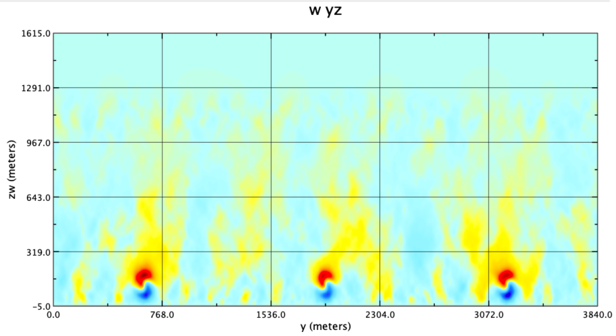
Stochastic POD and turbulent box

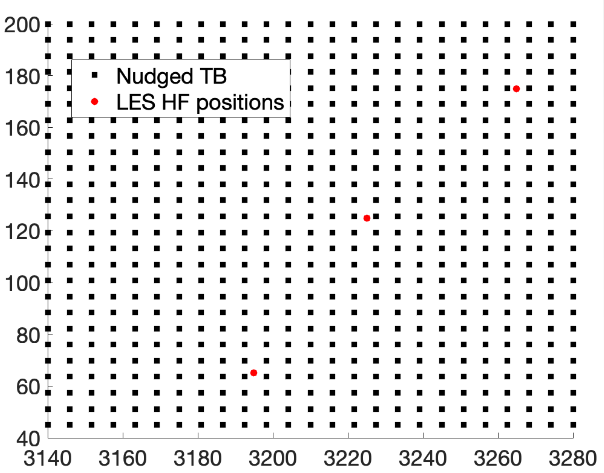


Stochastic POD and turbulent box

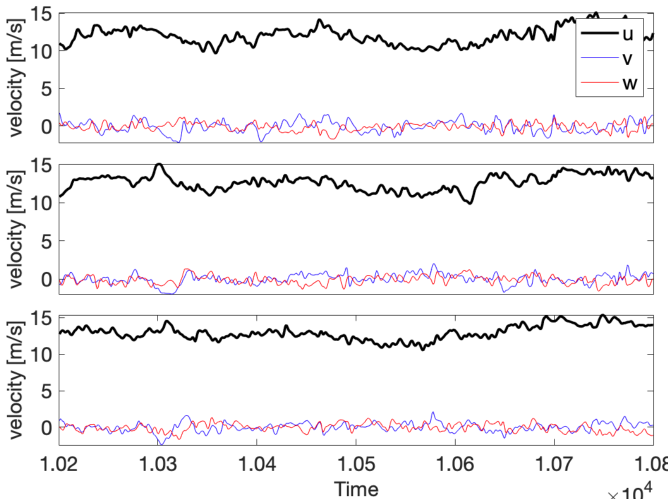








5 Hz time series from three points at rotor area of the first turbine from right.

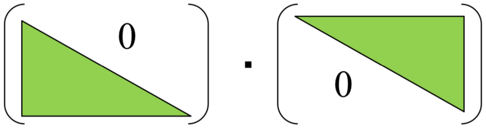


Stochastic POD and turbulent box

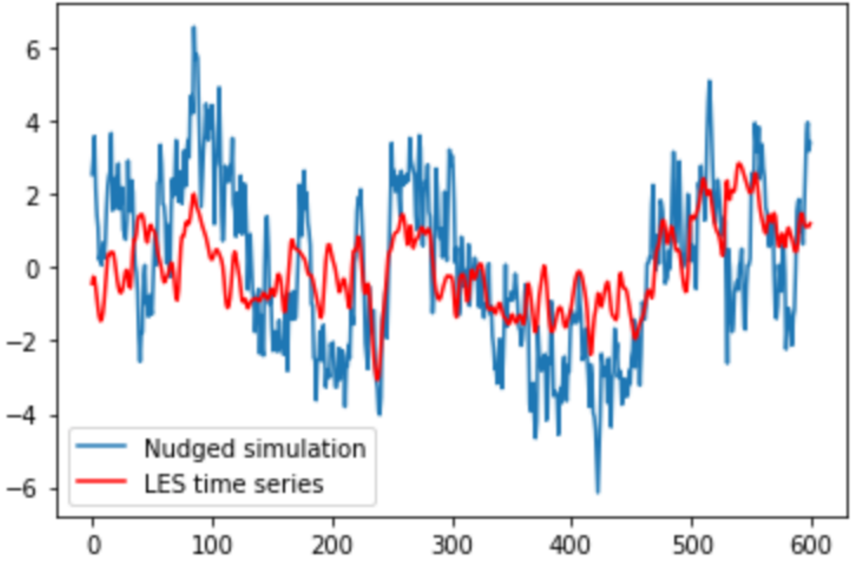
$$R_{ij} = \begin{pmatrix} \langle u'u' \rangle & \langle u'v' \rangle & \langle u'w' \rangle & \langle u'\phi' \rangle \\ \langle v'u' \rangle & \langle v'v' \rangle & \langle v'w' \rangle & \langle v'\phi' \rangle \\ \langle w'u' \rangle & \langle w'v' \rangle & \langle w'w' \rangle & \langle w'\phi' \rangle \\ \langle \phi'u' \rangle & \langle \phi'v' \rangle & \langle \phi'w' \rangle & \langle \phi'\phi' \rangle \end{pmatrix}$$

Here, before decomposition, we exert a block associated with all observation

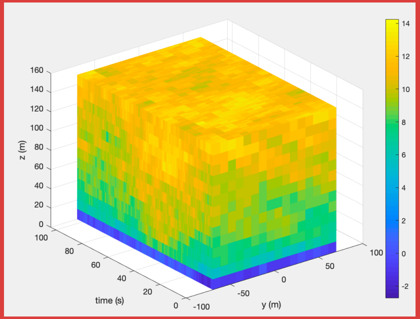
$$R_{ik} = a_{ik} \cdot a_{kj} = a \cdot a^{\mathrm{T}} =$$

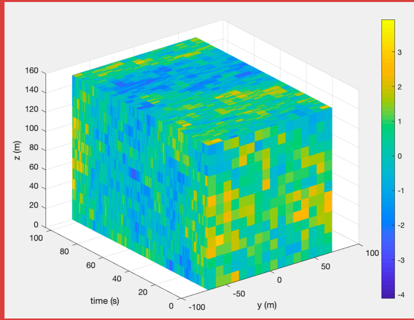


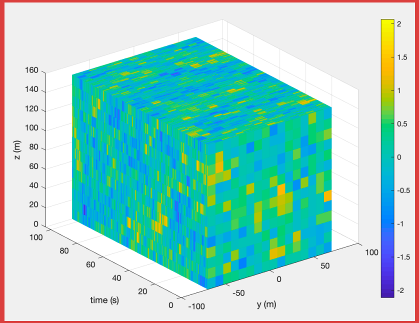
$$a_{ij} = \begin{pmatrix} \sqrt{R_{11}} & 0 & 0 & 0 \\ R_{21}/a_{11} & \sqrt{R_{22} - a_{21}^2} & 0 & 0 \\ R_{31}/a_{11} & (R_{32} - a_{21}a_{31})/a_{22} & \sqrt{R_{33} - a_{31}^2 - a_{32}^2} & 0 \\ R_{41}/a_{11} & (R_{42} - a_{21}a_{41})/a_{22} & (R_{43} - a_{31}a_{41} - a_{32}a_{42})/a_{33} & \sqrt{R_{44} - a_{41}^2 - a_{42}^2 - a_{43}^2} \end{pmatrix}$$



Stochastic POD and turbulent box







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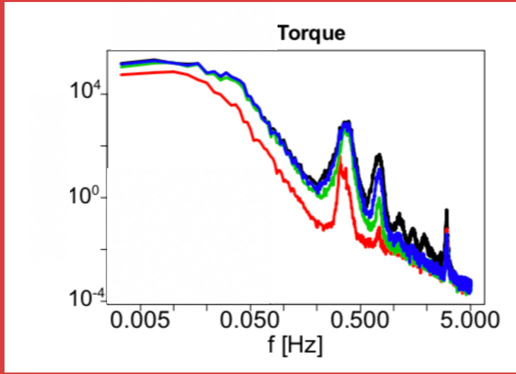
NREL FAST model



See [1] for more references

Conclusions

- POD decomposition for two sets of LES experiments were discussed.
- A turbulent box model was used to use LES results at few points to be used in the structural analysis code
- More analysis with FAST model will be given.

References

M. Bakhoday-Paskyabi, M. Krutova, F. N. Nielsen, J. Reuder, amd O. El Guernaoui, On Stochastic Reduced-Order and LES-based Models of Offshore Wind Turbine Wakes, Journal of Physics, 2020.

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