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Gradient-based optimization of a 15 MW wind turbine spar floater

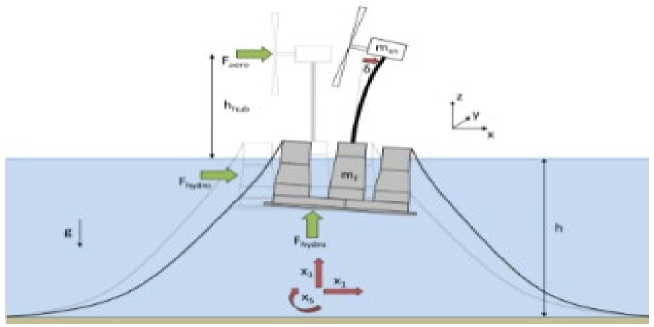
Nicolò Pollini, Antonio Pegalajar-Jurado, Suguang Dou, Henrik Bredmose, Mathias Stolpe

DTU Wind Energy

Innovation Fund Denmark

The present research is part of the FloatStep Project

Innovation Fund Denmark (2018-2022)



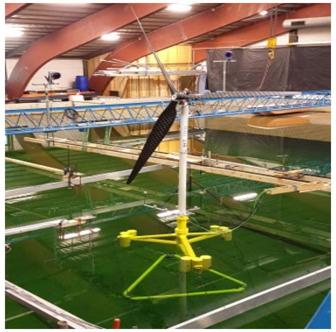


Image: Bourbon Offshore

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Introduction

The traditional aero-elastic tools for calculating wind turbine response in time domain are too expensive for optimization of Floating Offshore Wind Turbine (FOWT).

The optimization of FOWT can benefit from low-order models and fast frequency-domain analyses!

Floating offshore wind turbines are subjected to:

- The forcing from wind and waves
- The controller actions

The design of support structures for FOWT combines:

- Requirements of stability
- Dynamic response levels for nacelle acceleration
- Tower inclination
- Mooring loads
- A large set of design load cases

Objectives & approach

To combine a **frequency domain** response model with **gradient-based optimization** for **fast** conceptual and preliminary **design** of **15 MW floating wind turbine spar-buoy support structures**.

- Floater response calculated with **QuLAF** (Quick Load Analysis of Floating wind turbines [1])
- QuLAF has been extended to include the **analytical design sensitivity analysis** [2]
- Optimization problems solved using modern **gradient-based optimization**, e.g. SQP (Sequential Quadratic Programming)
- Precomputed aerodynamic loads in HAWC2 (the results shown here rely on precomputed aerodynamic loads from FAST as in [1])
- Post-processing of the optimize design to discrete design values chosen from catalogues
- First optimization of a steel spar-buoy for a 15 MW reference floating wind turbine

[1] Pegalajar-Jurado A, Borg M and Bredmose H. 2018 An efficient frequency-domain model for quick load analysis of floating offshore wind turbines. Wind Energy Science

[2] Dou S, Pegalajar-Jurado A, Wang S, Bredmose H, Stolpe M. 2020 Optimization of floating wind turbine support structures using frequency-domain analysis and analytical gradients. Journal of Physics: Conference Series

Related work

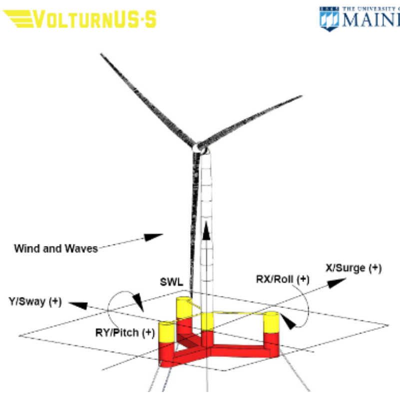
- Hegseth JM, Bachynski EE. 2019. A semi-analytical frequency domain model for efficient design evaluation of spar floating wind turbines. Marine Structures
- Dou S, Pegalajar-Jurado A, Wang S, Bredmose H, Stolpe M. 2020. Optimization of floating wind turbine support structures using frequency-domain analysis and analytical gradients. Journal of Physics: Conference Series
- Souza CES, Hegseth JM, Bachynski EE. 2020. Frequency-dependent aerodynamic damping and inertia in linearized dynamic analysis of floating wind turbines. Journal of Physics: Conference Series
- Hegseth JM, Bachynski EE, Martins JR. 2020. Integrated design optimization of spar floating wind turbines. Marine Structures
- …

15-MW offshore reference wind turbine

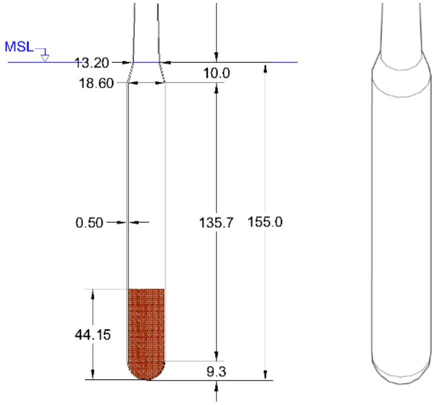
Made by NREL and DTU

Publically available as FAST and HAWC2 models at https://github.com/IEAWindTask37/IEA-15-240-RWT

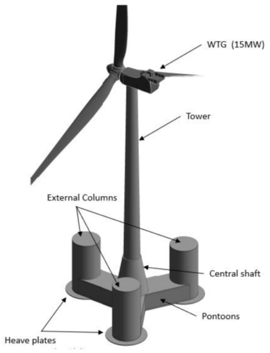
Public floater designs for the 15-MW offshore reference wind turbine



- The steel semisub from University of Maine: https://github.com/IEAWindTask37/IEA-15-240-RWT

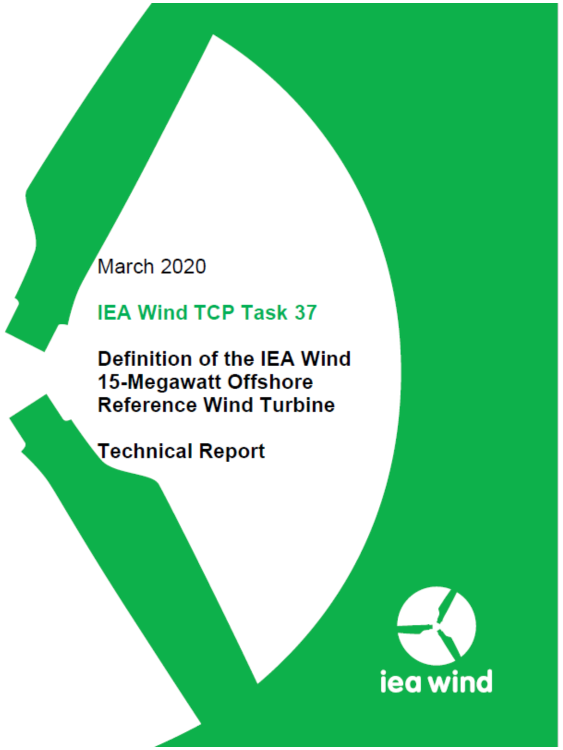


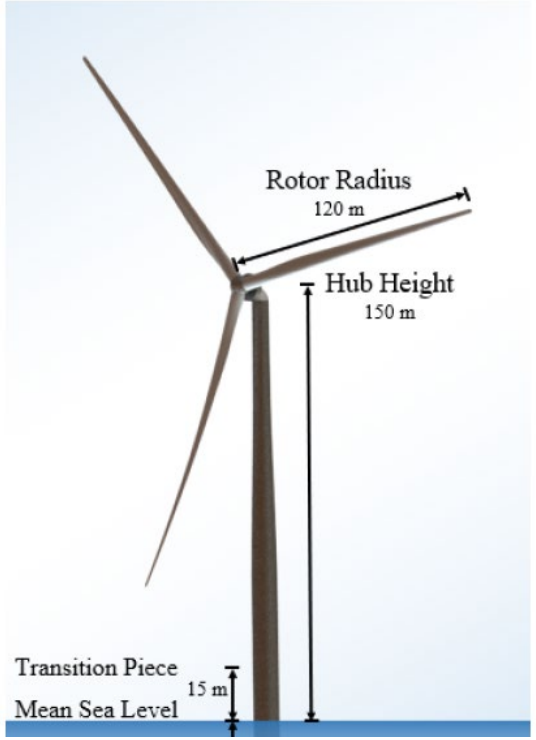
- The concrete spar from UPC: https://zenodo.org/record/4322446



- The concrete semisub developed by COBRA: https://zenodo.org/record/4322585

15-MW offshore reference wind turbine





Parameter	Units	Value
Power rating	MW	15
Turbine class	-	IEC Class 1B
Specific rating	W/m^2	332
Rotor orientation	-	Upwind
Number of blades	-	3
Control	-	Variable speed Collective pitch
Cut-in wind speed	m/s	3
Rated wind speed	m/s	10.59
Cut-out wind speed	m/s	25
Design tip-speed ratio	-	9 0
Minimum rotor speed	rpm	5.0
Maximum rotor speed	rpm	7.56
Maximum tip speed	m/s	95
Rotor diameter	m	240
Airfoil series	-	FFA-W3
Hub height	m	150
Hub diameter	m	7.94
Hub overhang	m	11.35
Rotor precone angle	deg	-4.0
Blade prebend	m	4
Blade mass	t	65

Model and documentation on GitHub:
https://github.com/IEAWindTask37/IEA-15-240-RWT

Wind turbine tower properties

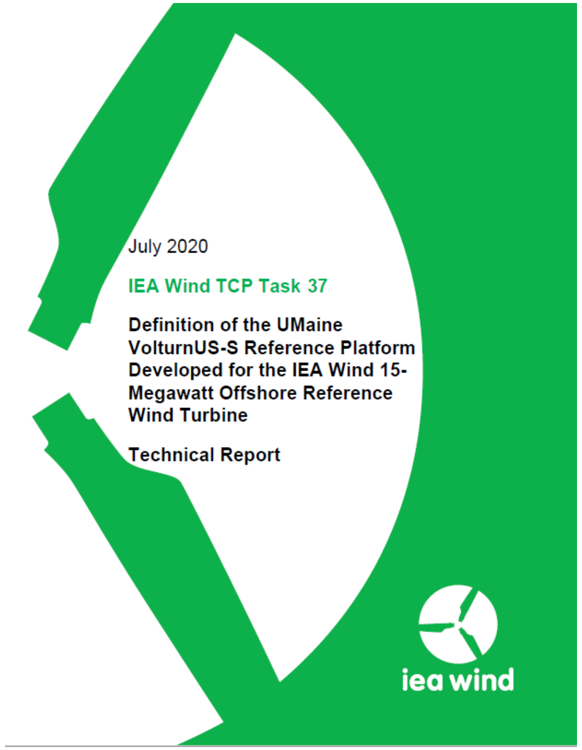


Table 8. Floating Tower Properties

Parameter	Value	Units
Mass	1,263	ton
Length	129.495	m
Base Outer Diameter	10	m
Top Outer Diameter	6.5	m
1st Fore-Aft Bending Mode	0.496	Hz
1st Side-Side Bending Mode	0.483	Hz

Table 9. Steel Material Properties for the Floating Tower

Parameter	Symbol	Value	Units
Young's Modulus	E	200e11	Pascals (Pa)
Shear Modulus	G	793e10	Pa
Density	ρ	785e3	kg/m^3

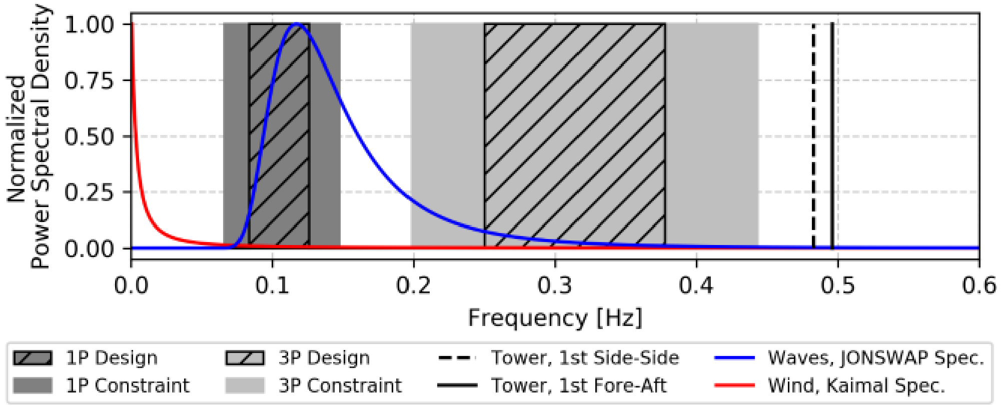


Figure 10. Tower natural frequencies relative to excitation frequencies

Model and documentation on GitHub:
https://github.com/IEAWindTask37/IEA-15-240-RWT

Governing equations

- Design variables ($\mathbf{v}$): geometric properties of the floater and the mooring system.
- The equations of motion for a floating wind turbine in QuLAF are:

$$\left(-\omega^2 \left(\mathbf{M}(\mathbf{v}) + \mathbf{A}(\mathbf{v})\right) + i\,\omega \mathbf{B}(\mathbf{v}) + \mathbf{C}(\mathbf{v})\right) \widehat{\boldsymbol{\xi}}_j(\omega) = \widehat{\mathbf{F}}^{\mathrm{h}}_j(\mathbf{v},\omega) + \widehat{\mathbf{F}}^{\mathrm{a}}_j(\omega)$$

- $\mathbf{M}$ structural mass; $\mathbf{A}$ added mass; $\mathbf{B}$ damping; $\mathbf{C}$ restoring matrix (hydrostatic and mooring); $\mathbf{F}^{\mathrm{h}}$ hydrodynamic loads (Morison's equation); $\mathbf{F}^{\mathrm{a}}$ precomputed aerodynamic loads.
- The optimization problem includes limits on eigenvalues from the eigenvalue problems

$$(\mathbf{C}(\mathbf{v}) - \lambda_i(\mathbf{M}(\mathbf{v}) + \mathbf{A}(\mathbf{v})))\phi_i = \mathbf{0}$$

- and limits on the maximum static responses

$$\mathbf{C}(\mathbf{v})\mathbf{u} = \mathbf{f}^s$$

Mooring system

- Three identical and uniformly distributed catenary lines
- Analytical derivation of the mooring stiffness matrix [1]

- *l*, *h*: horizontal and vertical mooring line projections
- *H, V:* horizontal and vertical force at fairlead, obtained from catenary equation with Newton-Raphson method
- *D, R*: vertical position and radius of the fairlead
- n: number of mooring lines

$$\boldsymbol{K}_m = \begin{bmatrix} K_{11} & 0 & 0 & 0 & K_{15} & 0 \\ 0 & K_{22} & 0 & K_{24} & 0 & 0 \\ 0 & 0 & K_{33} & 0 & 0 & 0 \\ 0 & K_{42} & 0 & K_{44} & 0 & 0 \\ K_{51} & 0 & 0 & 0 & K_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & K_{66} \end{bmatrix}$$

(only surge, heave, pitch)

$$K_{11} = \frac{1}{2} n \left[K_{11}^{p} + \frac{H}{l} \right],$$

$$K_{15} = -n \left[-\frac{R}{2} K_{12}^{p} + \frac{D}{2} K_{11}^{p} + \frac{\mathrm{DH}}{l} \right],$$

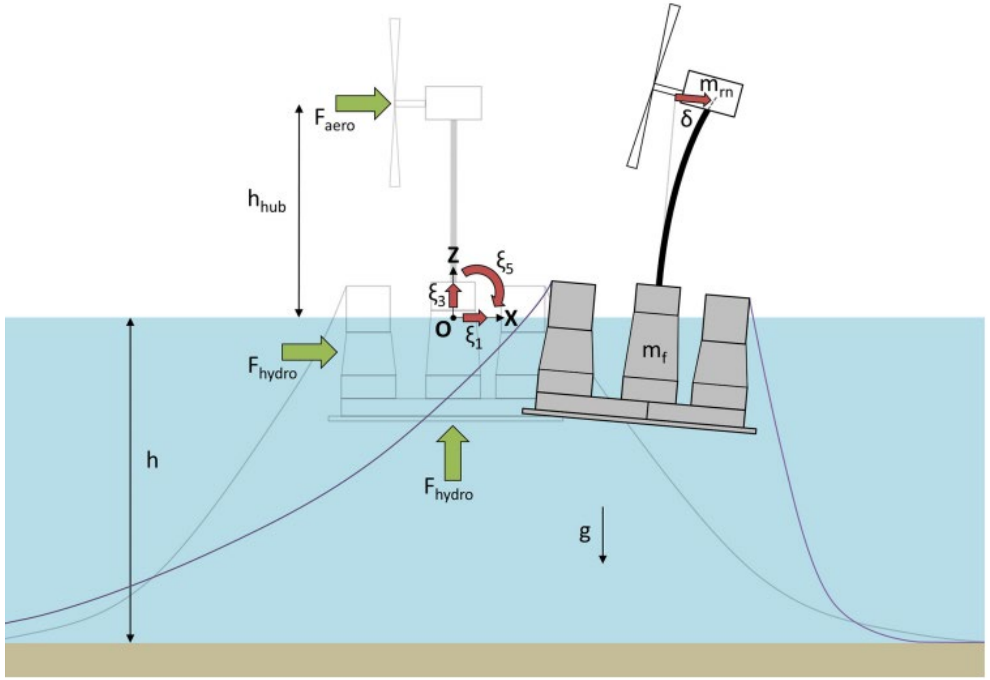
$$K_{22} = K_{11}, \quad K_{24} = -K_{15}, \quad K_{33} = n\, K_{22}^{p}, \quad K_{42} = K_{24}$$

$$K_{44} = n \left[-\mathrm{DR}\, K_{12}^{p} + \frac{D^2}{2} K_{11}^{p} + \frac{R^2}{2} K_{22}^{p} + \mathrm{DV} + \frac{\mathrm{HR}}{2} + \frac{D^2 H}{2l} \right], \quad K_{51} = K_{15}$$

$$K_{55} = K_{44}, \quad K_{66} = n \left[\frac{H\, R^2}{l} + \mathrm{HR} \right] \qquad (26)$$

[1] Al-Solihat MK, Nahon M. 2016 Stiffness of slack and taut moorings. Ships and Offshore Structures

Optimization problem formulation



Pegalajar-Jurado et al. *Wind Energy Science,* 2018

We consider floating wind turbine systems with spar-buoy floaters and with four degrees of freedom:

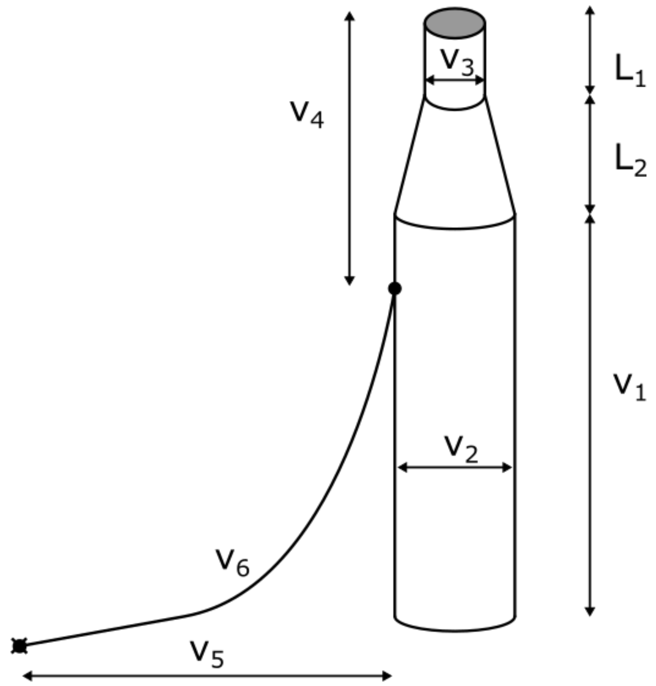
- Surge
- Heave
- Pitch
- Tower top fore-aft (15 MW WT, IEA Wind Task 37)

Moreover:

- Hydro loads: computed internally in the optimization code
- Aero loads: precomputed with rigid-structure rotor computations in FAST

$$
\begin{aligned}
\underset{\mathbf{x}}{\text{minimize}} \quad & f(\mathbf{x}) & \\
\text{subject to} \quad & g_i(\mathbf{x}) \leq 0 & i = 1, \ldots, m \\
& \mathbf{A}\mathbf{x} \leq \mathbf{b} & \\
& lb_j \leq x_j \leq ub_j & j = 1, \ldots, n
\end{aligned}
$$

Optimization problem formulation



$$
\begin{aligned}
\underset{\mathbf{v}\in\mathcal{V}}{\text{minimize}} \quad & c(\mathbf{v}) && \text{(cost)}\\
\text{subject to} \quad & \lambda_i^{min} \leq \lambda_i(\mathbf{v}) \leq \lambda_i^{max} && \text{(frequencies)}\\
& \theta^{dyn}(\mathbf{v}) \leq \theta_{max}^{dyn} && \text{(dynamic pitch)}\\
& a^{dyn}(\mathbf{v}) \leq a_{max}^{dyn} && \text{(tower top acceleration)}\\
& \theta^{stat}(\mathbf{v}) \leq \theta_{max}^{stat} && \text{(static pitch)}\\
& u^{stat}(\mathbf{v}) \leq u_{max}^{stat} && \text{(static surge)}\\
& lb_j \leq v_j \leq ub_j && \text{(bounds)}
\end{aligned}
$$

$$
\mathcal{V} = \left\{
\begin{aligned}
& \mathbf{v} \mid z_b(\mathbf{v}) \geq z_{cm}(\mathbf{v}), && \text{(buoyancy center)}\\
& l_m^{min} \leq l_m(\mathbf{v}) \leq l_m^{max}, && \text{(mooring linen length)}\\
& V(\mathbf{v}) \leq \eta w(\mathbf{v}), && \text{(vertical force at the fairlead)}\\
& D_{min} \leq D_{head} \leq D_{spar} && \text{(spar body/head diameters)}
\end{aligned}
\right\}
$$

$$
\mathbf{v} = \begin{bmatrix}
v_1 & \text{draft of spar}\\
v_2 & \text{diameter of spar body}\\
v_3 & \text{diameter of spar head}\\
v_4 & \text{depth of fairlead}\\
v_5 & \text{anchor radius}\\
v_6 & \text{mooring line length}
\end{bmatrix}
$$

Problem formulation similar to:

Dou S, Pegalajar-Jurado A, Wang S, Bredmose H, Stolpe M. 2020 Optimization of floating wind turbine support structures using frequency-domain analysis and analytical gradients. Journal of Physics: Conference Series

Design sensitivity analysis

The optimization problem is solved with a Sequential Quadratic Programming (SQP) algorithm
→ The **gradients** of the optimization problem functions are needed!

- Equations of motion:

$$\frac{\partial \widehat{\boldsymbol{\xi}}_j(\omega)}{\partial v_k} = \left(-\omega^2(\mathbf{M}+\mathbf{A})+i\omega\mathbf{B}+\mathbf{C}\right)^{-1}\left(\frac{\partial \widehat{\mathbf{F}}^{\mathrm{h}}_j(\omega)}{\partial v_k} + \left(\omega^2\frac{\partial(\mathbf{M}+\mathbf{A})}{\partial v_k} - i\omega\frac{\partial \mathbf{B}}{\partial v_k} - \frac{\partial \mathbf{C}}{\partial v_k}\right)\widehat{\boldsymbol{\xi}}_j(\omega)\right)$$

- Eigenvalue problem:

$$\frac{\partial \lambda_i(\mathbf{v})}{\partial v_k} = \phi_i^T\left(\frac{\partial \mathbf{C}(\mathbf{v})}{\partial v_k} - \lambda_i\frac{\partial(\mathbf{M}(\mathbf{v})+\mathbf{A}(\mathbf{v}))}{\partial v_k}\right)\phi_i$$

- Static response (both for pitch and surge):

$$\frac{\partial \mathbf{u}(\mathbf{v})}{\partial v_k} = \frac{\partial \mathbf{C}^{-1}(\mathbf{v})}{\partial v_k}\mathbf{f}^s = -\mathbf{C}^{-1}(\mathbf{v})\frac{\partial \mathbf{C}(\mathbf{v})}{\partial v_k}\mathbf{C}^{-1}(\mathbf{v})\mathbf{f}^s = -\mathbf{C}^{-1}(\mathbf{v})\frac{\partial \mathbf{C}(\mathbf{v})}{\partial v_k}\mathbf{u}(\mathbf{v})$$

Post-processing of the design

1) From continuous SQP solution we pick the closest entries from "catalogues". We keep some of the linear constraints from previous optimization. With some manipulations, this is formulated as a MILP problem.

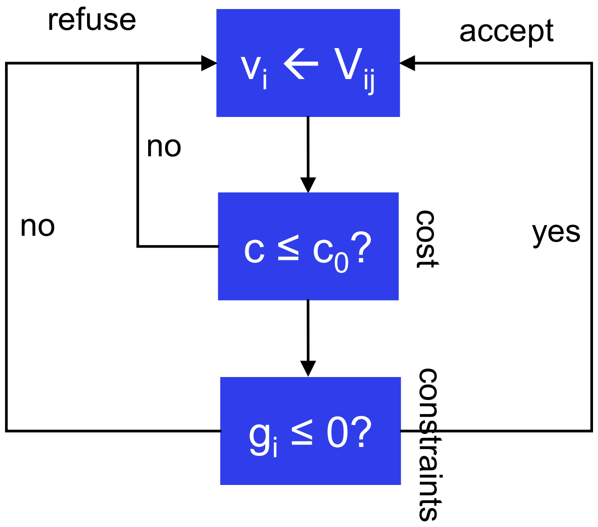
$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & \|\mathbf{v} - \mathbf{v}^*\|_1 \\ \text{subject to} \quad & \mathbf{A}\mathbf{v} \leq \mathbf{b} \\ & \mathbf{v} = \mathbf{V}\mathbf{x} \\ & \mathbf{x} \in \mathcal{X} \end{aligned}$$

$$\mathcal{X} = \left\{ \mathbf{x} \mid x_{ij} = \{0, 1\},\ v_i = \sum_j V_{ij}\, x_{ij},\ \sum_j x_{ij} = 1 \right\}$$

"Catalogue" of i-th variable v_i:

$$V_{ij} = [lb_i : \Delta v_i : ub_i]$$

2) From the discrete MILP solution we explore the closest entries from "catalogues". We look for designs that reduce the cost, and fulfill the constraints. It is an heuristic search, inspired by the Relaxation Induced Neighborhood Search (RINS) [1]



[1] Danna E, Rothberg E, Le Pape C. 2005 Exploring relaxation induced neighborhoods to improve MIP solutions. Mathematical Programming

Environmental conditions

Table 1. Environmental conditions with wind speed (WS) in m/s, the significant wave height H_s in m, and the peak period T_p in s

EC	1	2	3	4	5	6	7	8	9	10	11	12
WS	5.00	7.00	9.00	10.59	11.00	13.00	15.00	17.00	19.00	21.00	23.00	25.00
H_s	1.00	1.36	1.72	2.00	2.09	2.52	2.95	3.38	3.82	4.25	4.68	5.11
T_p	5.00	5.36	5.72	6.00	6.09	6.50	6.92	7.33	7.75	8.17	8.58	9.00

Design requirements

Table 2. A list of constraints in the optimization problem of the spar-buoy floater

Type	Number	Description	Values
Stability	1	buoyancy center higher than mass center	0 (min)
Static response	2	maximum pitch angle (θ_{max}^{stat}) maximum surge (u_{max}^{stat})	8 ° 50 m
System periods	5	minimum surge period (T_1^{min}) min/max heave/pitch periods ($T_i^{min}, T_i^{max}, i = 2, 3$)	80 s $25, 40\,s$
Mooring line	3	min/max mooring line length v_6 maximum percentage of suspended line (η)	l_m^{min}, l_m^{max} 75%
Spar head diameter	2	minimum value $v_2\, 2\, h_{taper}\, \tan(\alpha_{max})$ maximum $v_2 - \epsilon$	$\alpha_{max} = 30°$ 0.1
Dynamic response	24	maximum pitch angle (θ_{max}^{dyn}) maximum nacelle acceleration (a_{max}^{dyn})	10 ° 0.2 g

Numerical results

Gradient-based optimization with Sequential Quadratic Programming

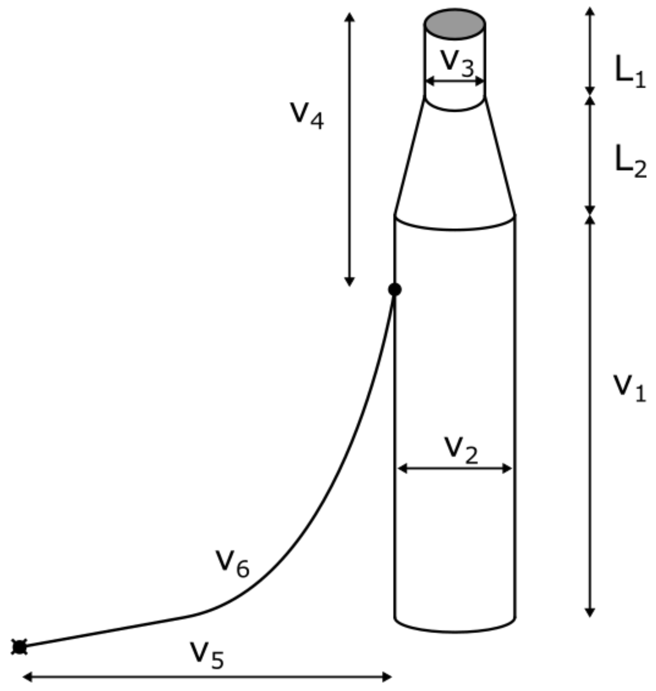


Table 3. The limits, initial and optimized values of the design variables

Variables	Min	Max	Initial	Optimized 13 constraints	Optimized 37 constraints
Draft v_1	60	180	130	97.004	104.600
Spar diameter v_2	5	20	18	20.000	18.864
Spar head diameter v_3	5	20	10	10.762	9.626
Fairlead position v_4	-160	-12	-50	-12.000	-12.000
Anchor radius v_5	600	2000	650	728.176	745.240
Mooring line length v_6	600	2100	750	825.976	841.404

Numerical results

Post-processing of the results obtained without dynamic constraints

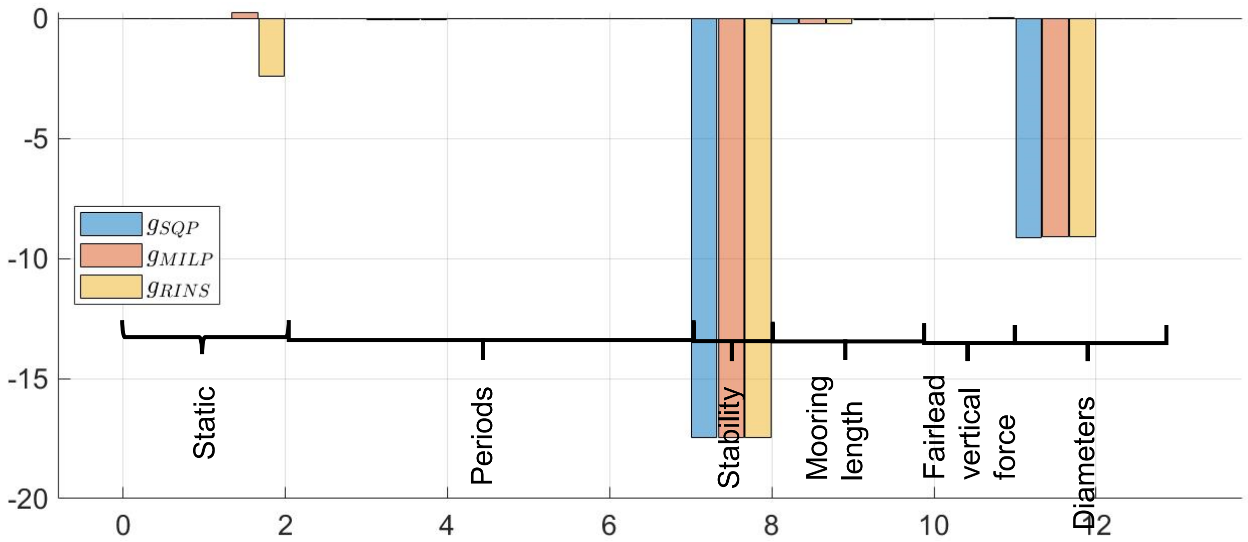
Catalogue:

	v_1	v_2	v_3	v_4	v_5	v_6
Minimum v_{min}	60	5	5	-160	600	600
Step Δv	1.0	0.1	0.1	0.1	1.0	1.0
Maximum v_{max}	180	20	20	-12	2000	2100

Optimized designs (time for optimization: 1 min 2 s):

Optimization step	v_1	v_2	v_3	v_4	v_5	v_6	Cost (10^6 \$)
Initial	130.000	18.000	10.000	-50.000	650.000	750.000	9.8285
SQP	97.004	20.000	10.762	-12.000	728.176	825.976	8.4600
MILP	97.000	20.000	10.800	-12.000	728.000	826.000	8.4629
RINS	97.000	20.000	10.800	-12.000	728.000	824.000	8.4627

Values of the constraints ($g_i \leq 0$):



Numerical results

Post-processing of the results obtained with dynamic constraints

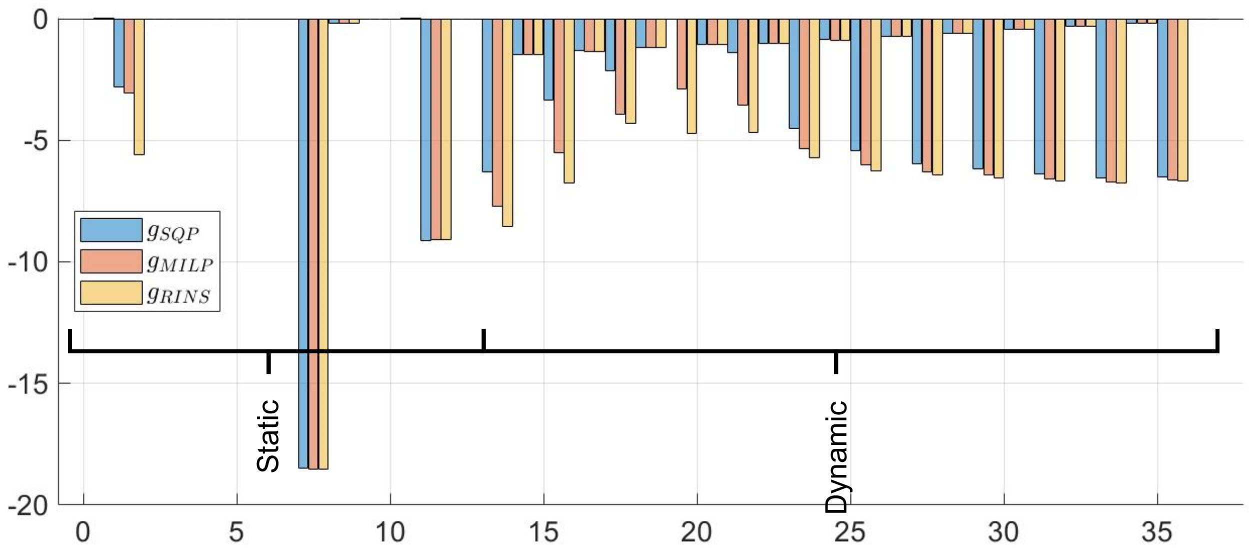
Catalogue:

	v_1	v_2	v_3	v_4	v_5	v_6
Minimum v_{min}	60	5	5	-160	600	600
Step Δv	1.0	0.1	0.1	0.1	1.0	1.0
Maximum v_{max}	180	20	20	-12	2000	2100

Optimized designs (time for optimization: 15 min 29 s):

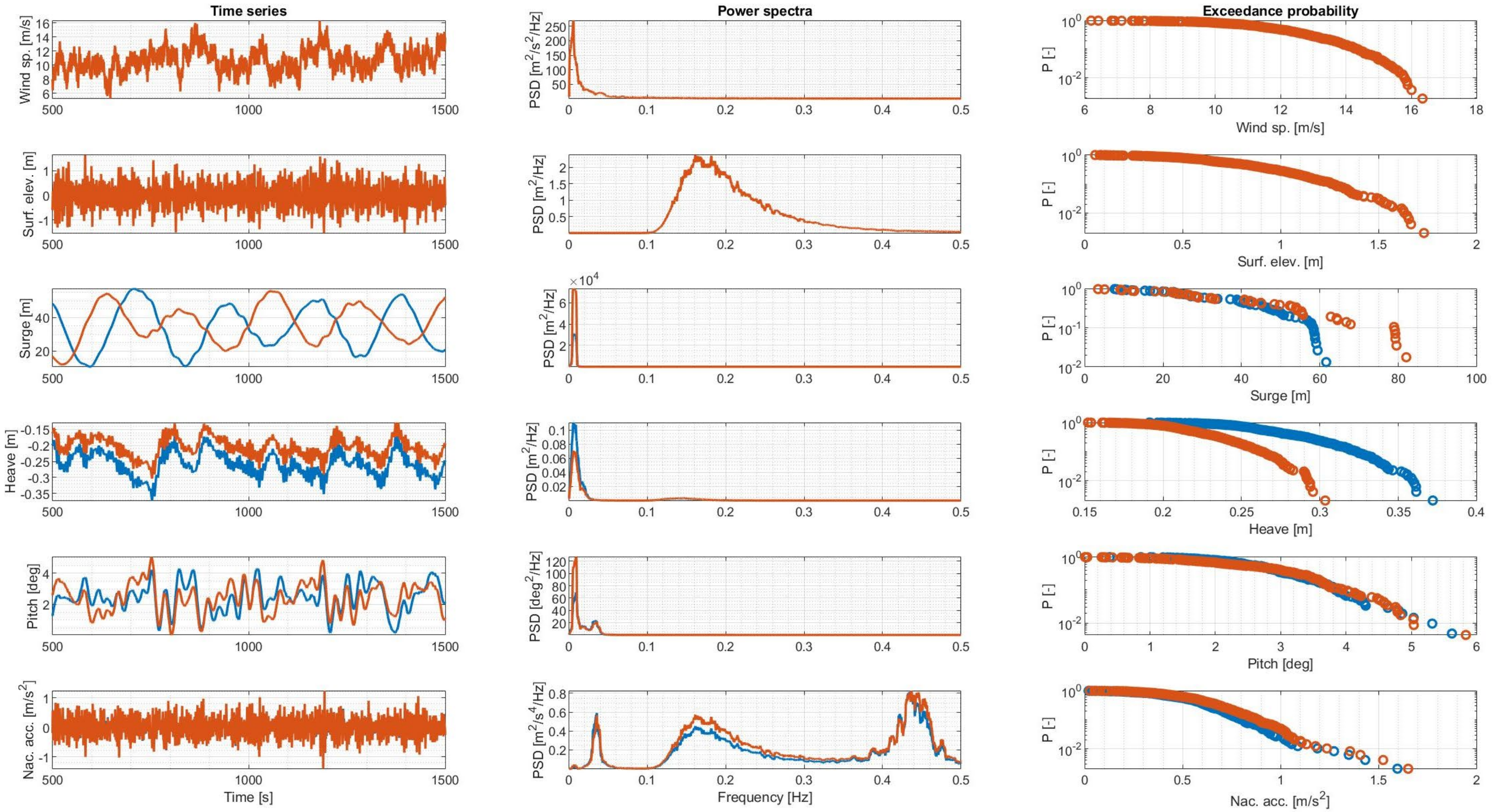
Optimization step	v_1	v_2	v_3	v_4	v_5	v_6	Cost (10^6 \$)
Initial	130.000	18.000	10.000	-50.000	650.000	750.000	9.8285
SQP	104.600	18.864	9.626	-12.000	745.240	841.404	8.4690
MILP	105.000	18.800	9.600	-12.000	745.000	841.000	8.4689
RINS	105.000	18.800	9.600	-12.000	745.000	839.000	8.4686

Values of the constraints ($g_i \leq 0$):



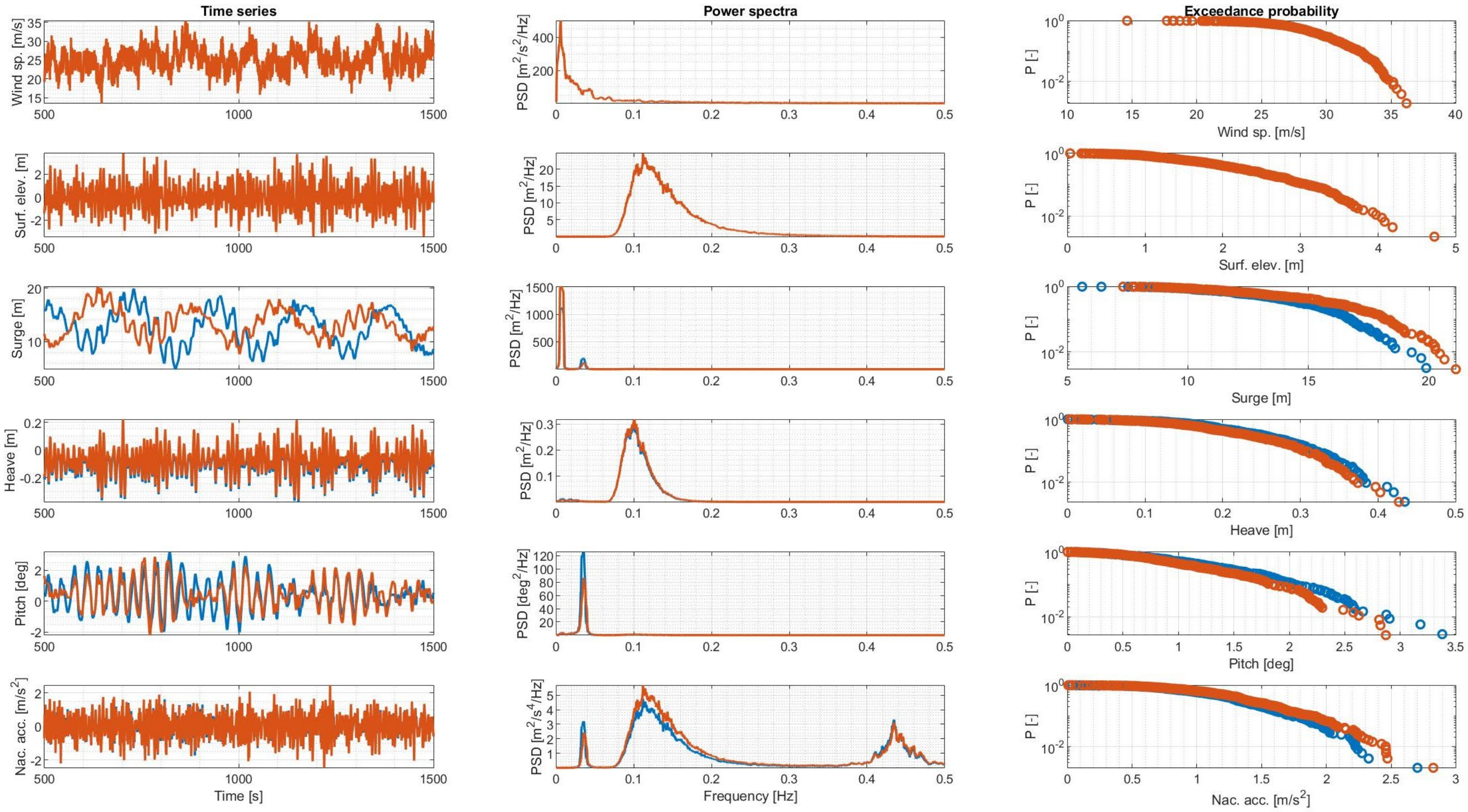
Numerical results

Dynamic responses at rated wsp. Optimized designs with (in blue) and without (in red) inclusion of the dynamic constraints.



Numerical results

Dynamic responses at cut-out wsp. Optimized designs with (in blue) and without (in red) inclusion of the dynamic constraints.



Conclusions

- An optimization framework for floating wind turbine support structures has been presented
- The framework builds on fast frequency domain analyses and analytical design sensitivity analyses
- Optimization approach applied to the design of the first steel spar-buoy floater for the 15 MW reference wind turbine developed within IEA Wind Task 37 (available on GitHub)
- A post-processing procedure of the optimized design has been proposed
- The post-processing relies on Mixed Integer Linear Programming, and a local search heuristic
- The optimization approach provides optimized conceptual designs within minutes on a standard PC

Acknowledgment

This work was carried out as part of the FloatStep project. The research leading to these results has received funding from Innovation Fund Denmark (IFD) under grant no. 8055-00075B.

Innovation Fund Denmark