On the Stochastic Reduced-Order and LES-based Models of Offshore Wind Turbine Wake

Mostafa Bakhoday-Paskyabi

Mostafa.Bakhoday-Paskyabi@uib.no

Maria Krutova, Finn-Gunnar Nielsen, Joachim Reuder, and Omar El Guernaoui

UNIVERSITY OF BERGEN

Bergen Offshore Wind Centre

Geophysical Institute, University of Bergen

Outline

- Motivation/Background
- LES modelling for 2 turbines configuration
- POD/Galerkin ROMs modelling
- Numerical results
- Future/Follow-up works

Motivation

- We are interested in wake modelling of offshore wind turbines.

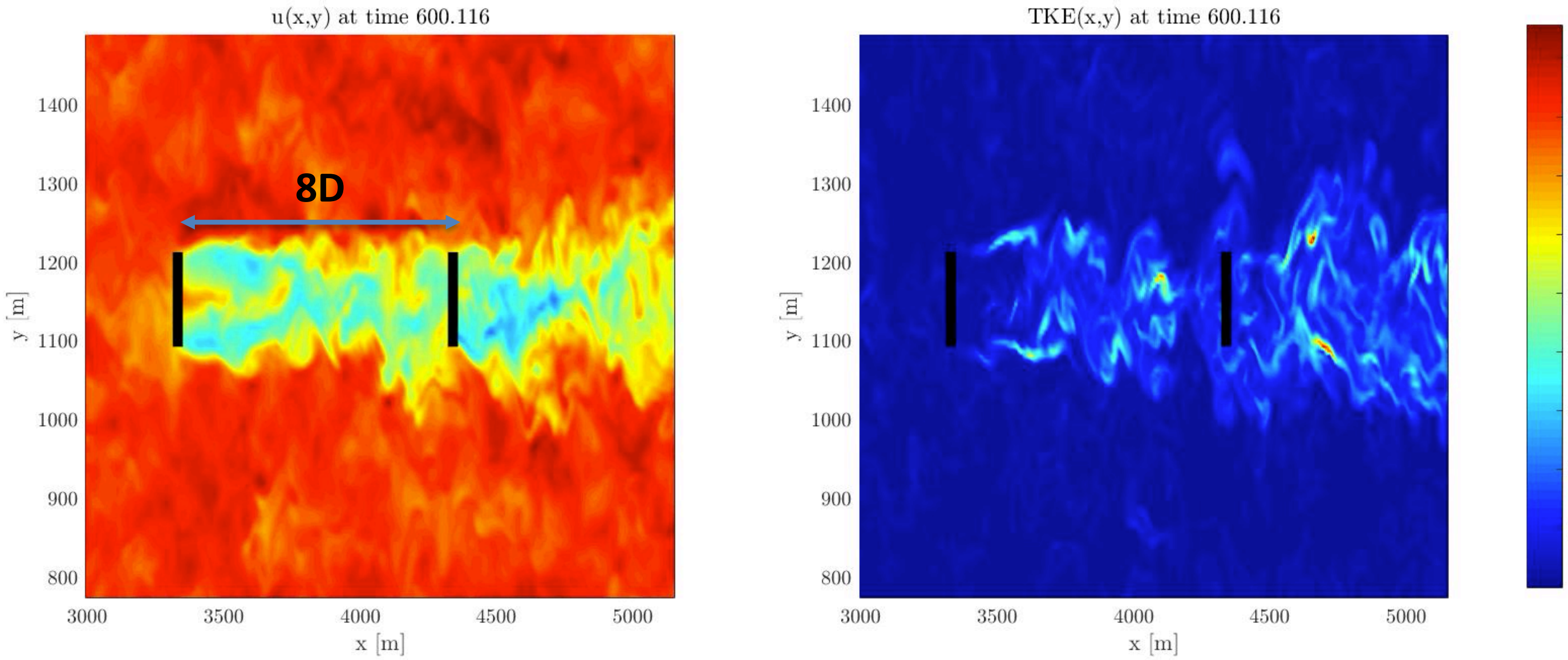
- Of primary interest is short- and long-term predictive simulations based on reduced order models.

- Secondary interest: ROMs application in short-term control of wind farm.

LES modelling for 2 turbines configuration

6912⨉2304⨉1459 m with grid size of dxdydz=6 m. The grid cell is stretched in *z* direction after 800 m with the factor of 1.04, maximum cell size is capped at dz_{max}=12 m.

Model is run for **neutral** atmospheric boundary layer.



Proper Orthogonal Decomposition

Data-driven ROMs are promising for:

- predictive methodologies and flow control applications due to the simplified definition of turbulence dynamics, speed of calculation, and portability to control methods

$$\mathbf{u}(\mathbf{x},t) = \sum_{i=1}^{N} a_i(t)\Phi^{(i)}(\mathbf{x}). \qquad a_i(t) = \int_D \mathbf{u}(\mathbf{x},t)\Phi^{(i)}(\mathbf{x})d\mathbf{x}. \qquad \langle a_i(t)a_j(t)\rangle_t = \lambda_i\delta_{ij} \,,$$

$$A = \begin{bmatrix} \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix}_{n_t \times m},$$

For the LES data, we formulate a snapshot matrix

where $m = 3n_x \times n_y \times n_z$.

n_x, n_y, n_z are the number of grid points in the streamwise, spanwise, and vertical directions, respectively

Find more references in [1]

Proper Orthogonal Decomposition

$$A = \begin{bmatrix} \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix}_{n_t \times m},$$

$$A = U\Sigma^{-1}V^T$$

Correlation function

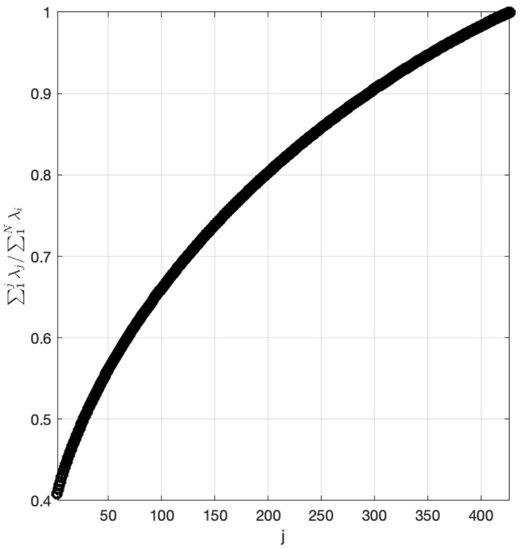
$$C = AA^T = U\Sigma^2 U^T$$

Obtain the POD modes in V:

$$\Sigma^{-1}U^T A$$

$$a_i(t) = \int_D \mathbf{u}(\mathbf{x}, t)\Phi^{(i)}(\mathbf{x})d\mathbf{x}.$$

Find more references in [1]

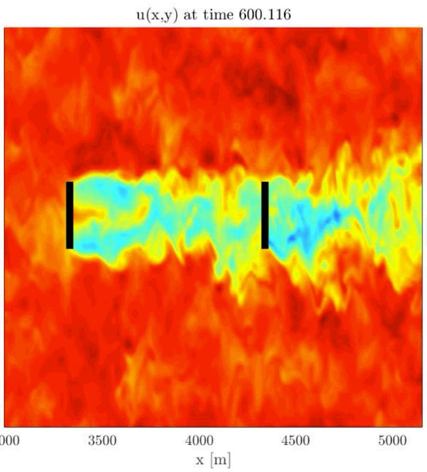


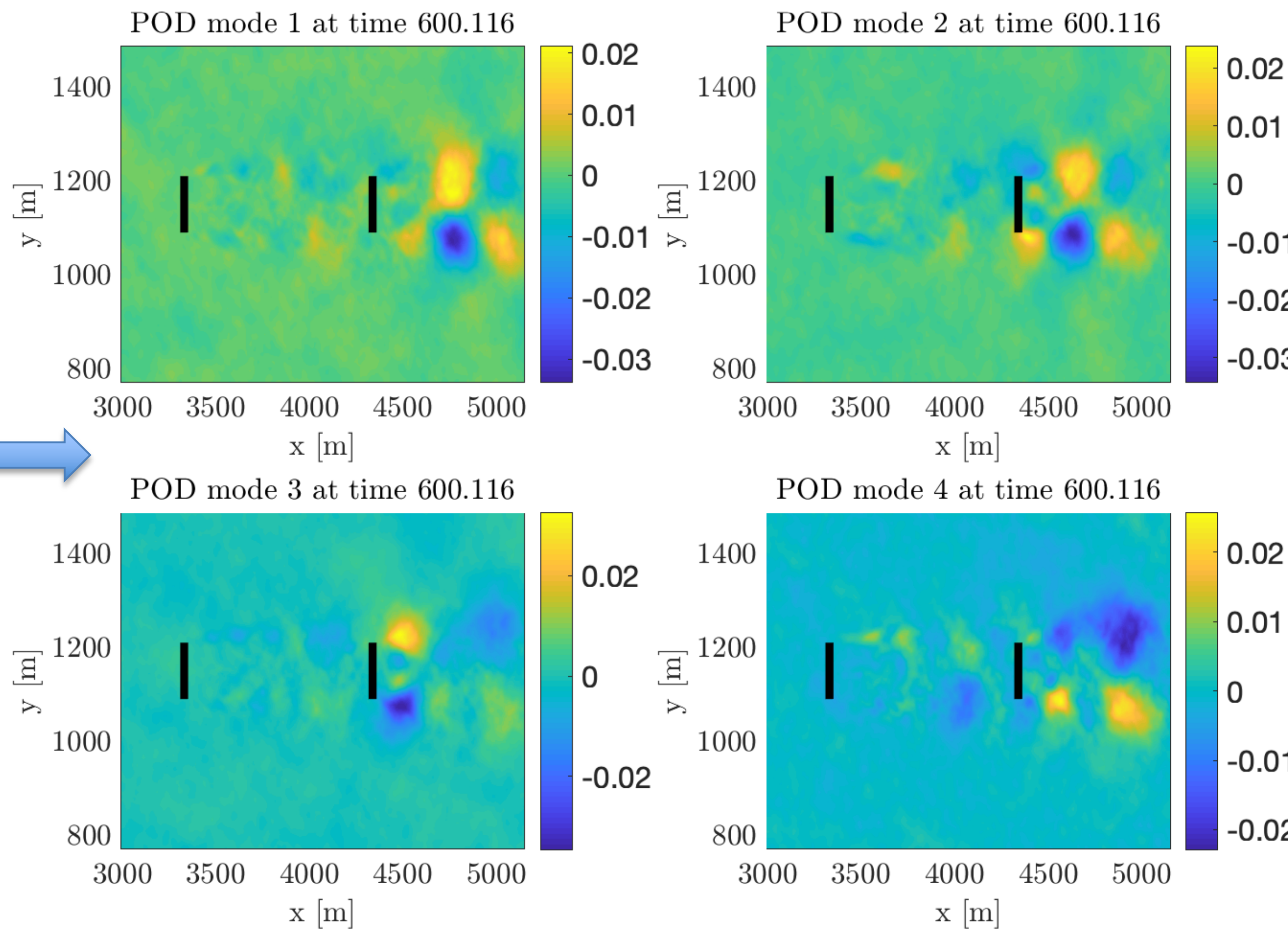
Distribution of energy in the proper orthogonal decomposition mode basis according to eigenvalues λ_n

Eigne values of Σ^2 represents kinetic energy corresponding to each POD mode.

Proper Orthogonal Decomposition

First 4 POD modes





Proper Orthogonal Decomposition

We show second POD mode
For u and v components
Of wind at hub-height.

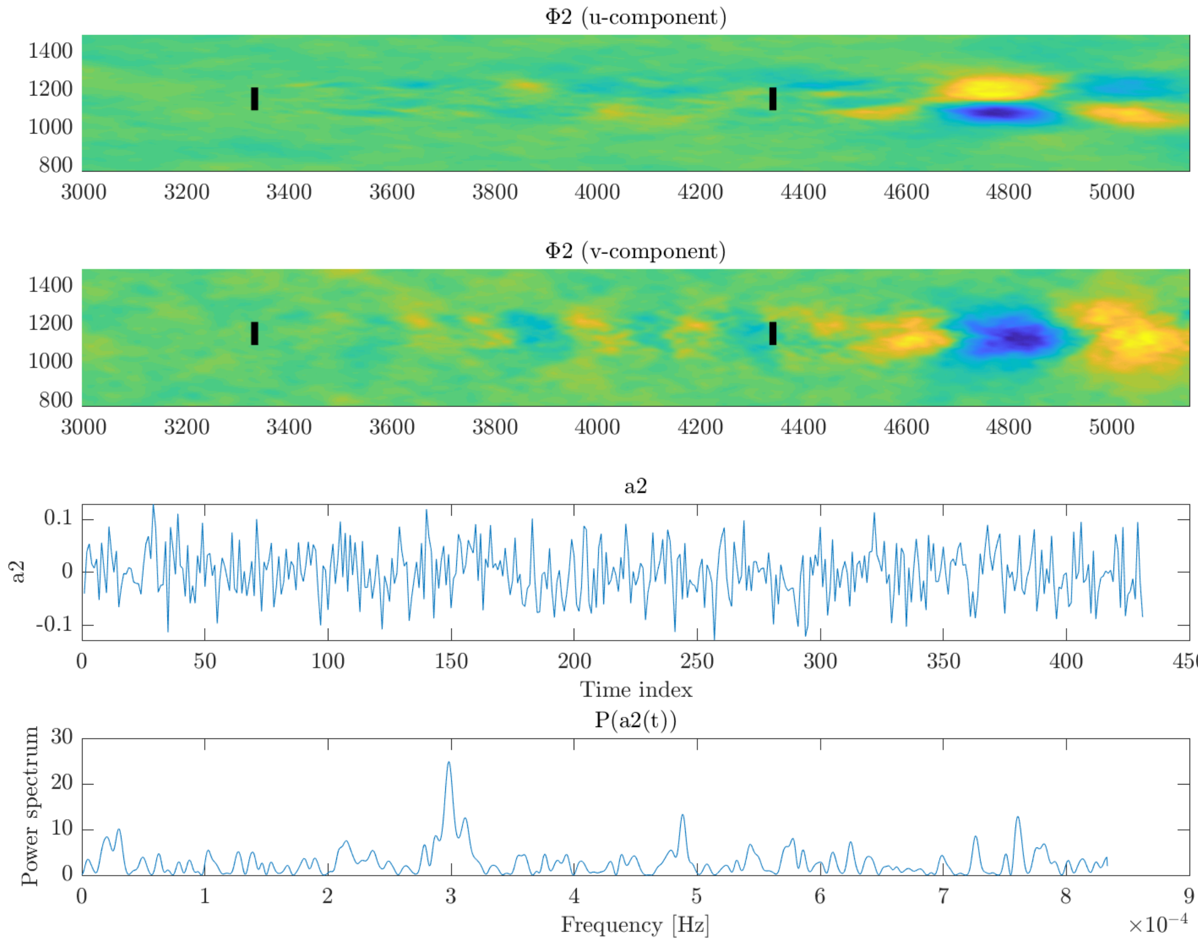
u*=(u,v)*

$$\mathbf{u}(\mathbf{x},t) = \sum_{i=1}^{N} a_i(t)\Phi^{(i)}(\mathbf{x}).$$

Timeseries of time-dependent weight coefficients

$$a_i(t) = \int_D \mathbf{u}(\mathbf{x},t)\Phi^{(i)}(\mathbf{x})d\mathbf{x}.$$

Power spectrum of $a_2(t)$



Note that no modelling of the temporal dynamics is involved in description of the field.

Results: Compare techniques

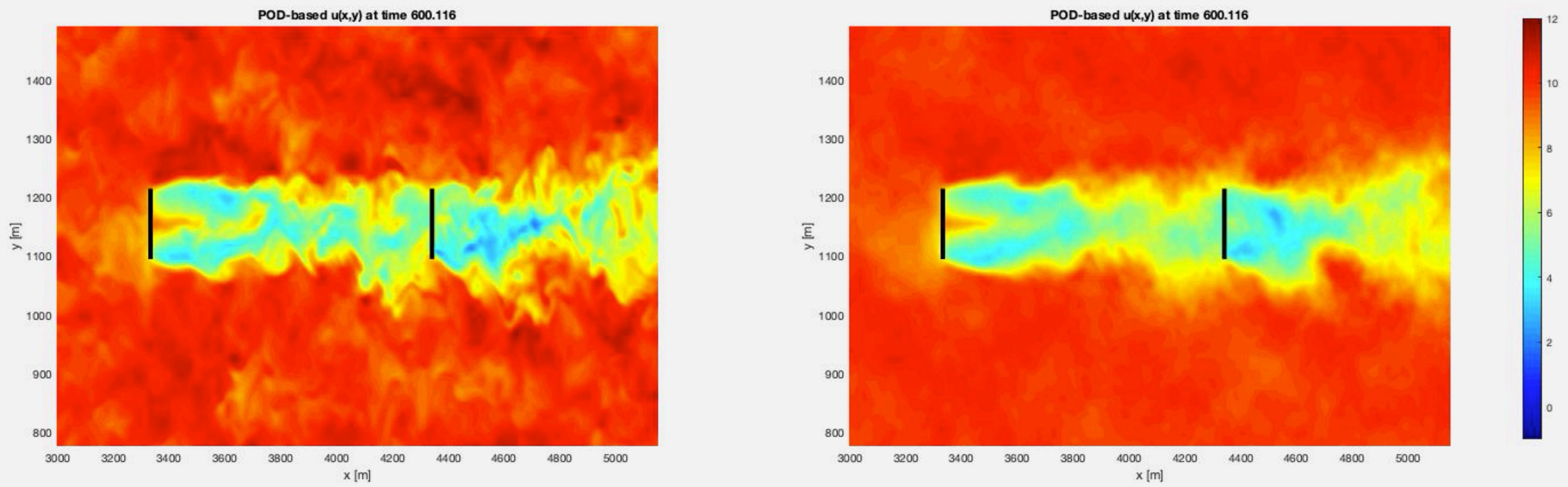
$$\mathbf{u}(\mathbf{x},t) = \sum_{i=1}^{N} a_i(t)\Phi^{(i)}(\mathbf{x}).$$

$$a_i(t) = \int_D \mathbf{u}(\mathbf{x},t)\Phi^{(i)}(\mathbf{x})d\mathbf{x}.$$

Original u-component versus the one reconstructed from the standard POD analysis

We are using N=50 modes for POD analysis.

How can we account for small scale dynamics?



Note that no modelling of the temporal dynamics is involved in description of the field.

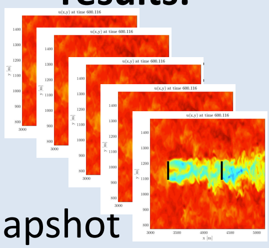
POD-Galerkin method to model reduction

turbine

$$\frac{\partial \tilde{\mathbf{u}}}{\partial \mathbf{x}} + (\tilde{\mathbf{u}} \cdot \nabla)\tilde{\mathbf{u}} + \nabla \cdot \tau = \mathbf{f},$$

$$\nabla \cdot \tilde{\mathbf{u}} = 0$$

High fidelity LES results:



Snapshot

Step 1

Modal decomposition (POD):

$$\boldsymbol{u}(\boldsymbol{x}, t) \approx \sum_{j=1}^{N} a_j(t)\, \boldsymbol{\phi}_j(\boldsymbol{x})$$

Step 2

Galerkin projection of fluid PDEs:

$$\left(\boldsymbol{\phi}_j, \dot{\boldsymbol{u}} + \boldsymbol{F}(\boldsymbol{u})\right) = 0$$

"Small" ROM ODE system:

$$\dot{a}_k = f(a_1, \dots, a_N)$$

- ***Snapshot matrix***: $\boldsymbol{X} = (\boldsymbol{x}^1, \dots, \boldsymbol{x}^K) \in \mathbb{R}^{NxK}$
- ***SVD:*** $\boldsymbol{X} = \boldsymbol{U\Sigma V}^T$
- ***Truncation:*** $\boldsymbol{\Phi}_N = (\boldsymbol{\phi}_1, \dots, \boldsymbol{\phi}_N) = \boldsymbol{U}(:, 1{:}\,\boldsymbol{N})$

K: # of snapshots
N: # of dofs in ROM
F: includes linear, nonlinear, turbulence, and turbine effects

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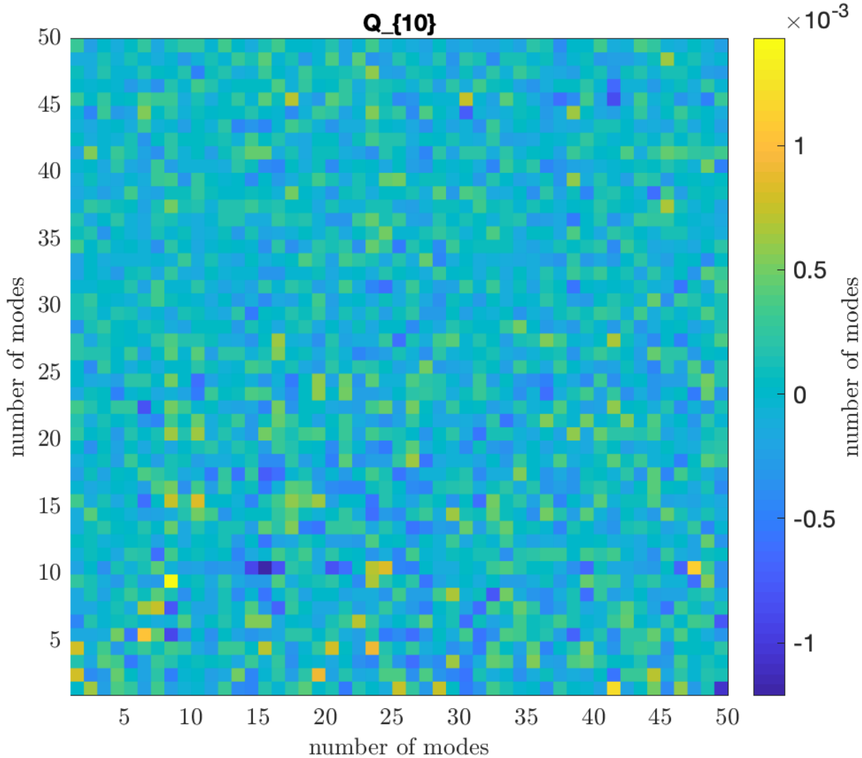
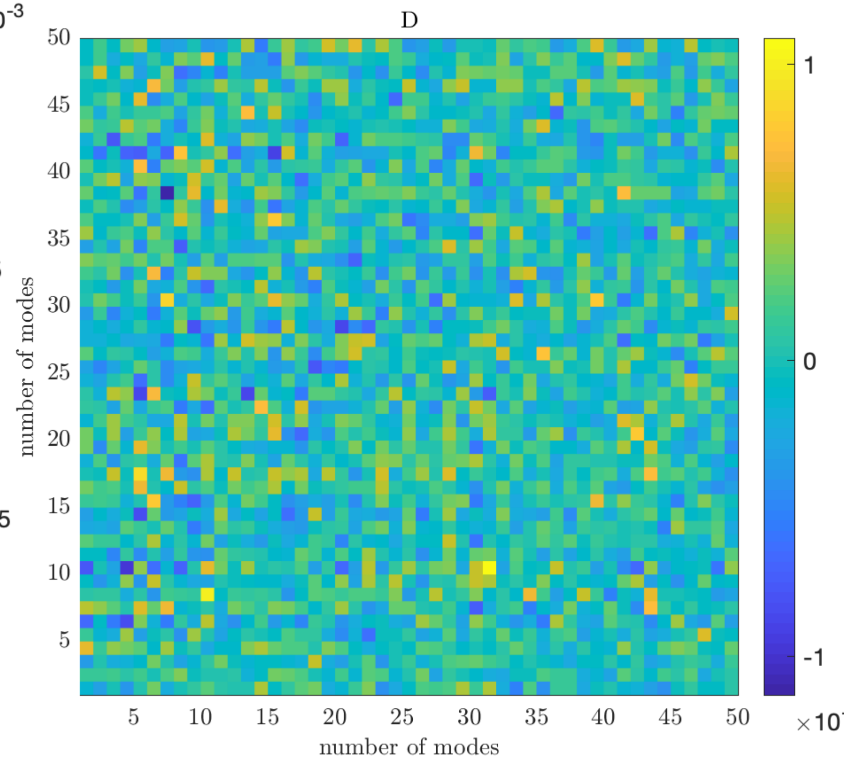
POD-Galerkin method to model reduction

D_i, *L_{ij}*, *Q_{ijk}*, and *C_{ijkl}*, imply constant, linear, quadratic, and cubic mode interactions, respectively

$$\frac{da_i}{dt} = D_i + \sum_{j=1}^{N_r} L_{ij} a_j + \sum_{j,k=1}^{N_r} Q_{ijk} a_j a_k + \sum_{j,k,l=1}^{N_r} C_{ijkl} a_j a_k a_l.$$





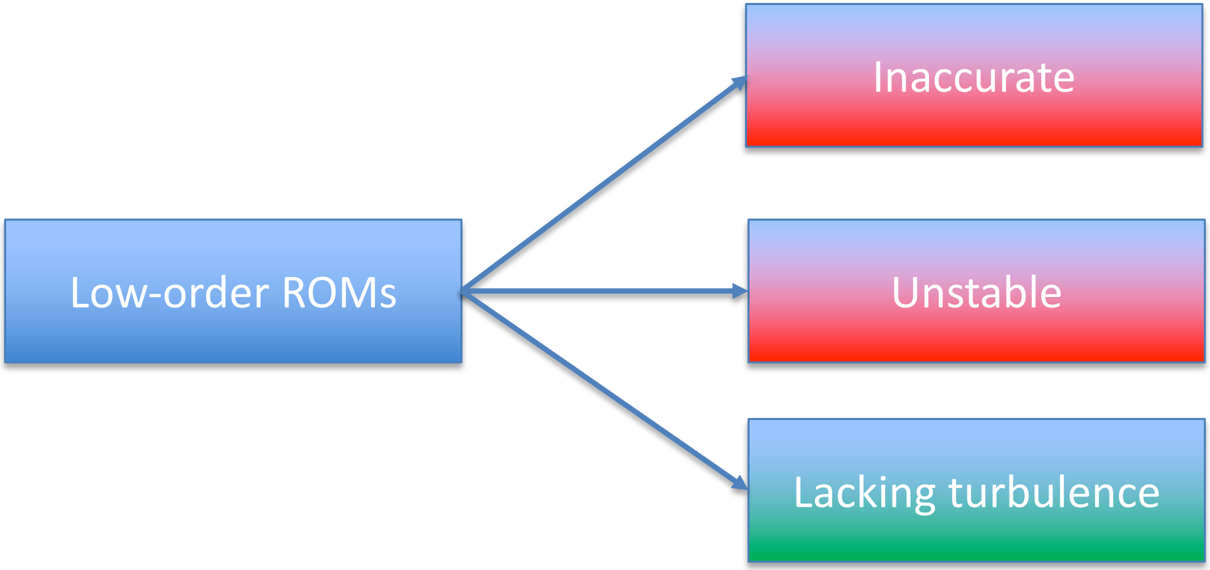
Here we account for the the non-linear coupling of different scales.

Find more references in [1,2,3

Mode truncation instability

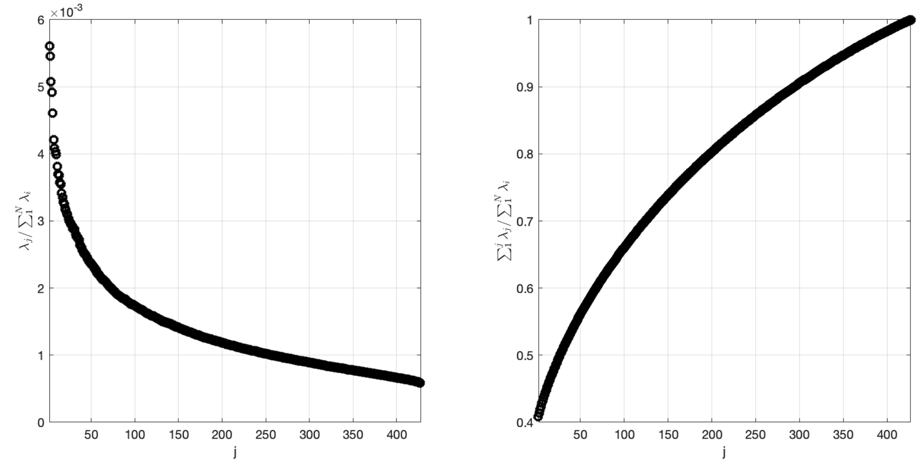
Projection-based POD necessitates ***truncation.***

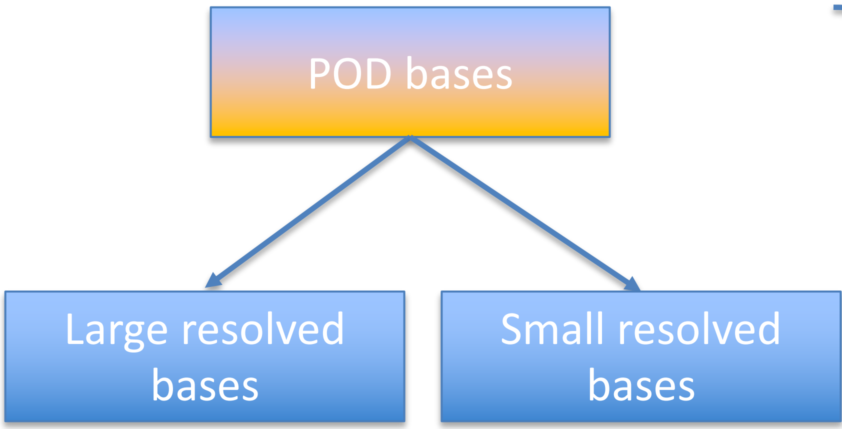
- POD can properly capture the ***large scales of motions (energy-containing eddies)*** of the flow (i.e., modes with large POD eigenvalues).
- Small POD eigenvalues are key for the corresponding ***dynamical equations***.
- Higher-order modes are associated with energy ***dissipation and small scale turbulence***



POD Closure Models: Overview

- Mixing Length (ML)
- Smagorinsky (S)
- Variational Multi-Scale (VMS)
- Dynamic Subgrid (DS)





$$\mathbf{X}^r = \mathbf{X}^r_L \oplus \mathbf{X}^r_S, \quad \text{where}$$
$$\mathbf{X}^r_L := \operatorname{span}\{\varphi_1, \varphi_2, \ldots, \varphi_{r_L}\} \quad \text{and}$$
$$\mathbf{X}^r_S := \operatorname{span}\{\varphi_{r_L+1}, \varphi_{r_L+2}, \ldots, \varphi_r\}.$$

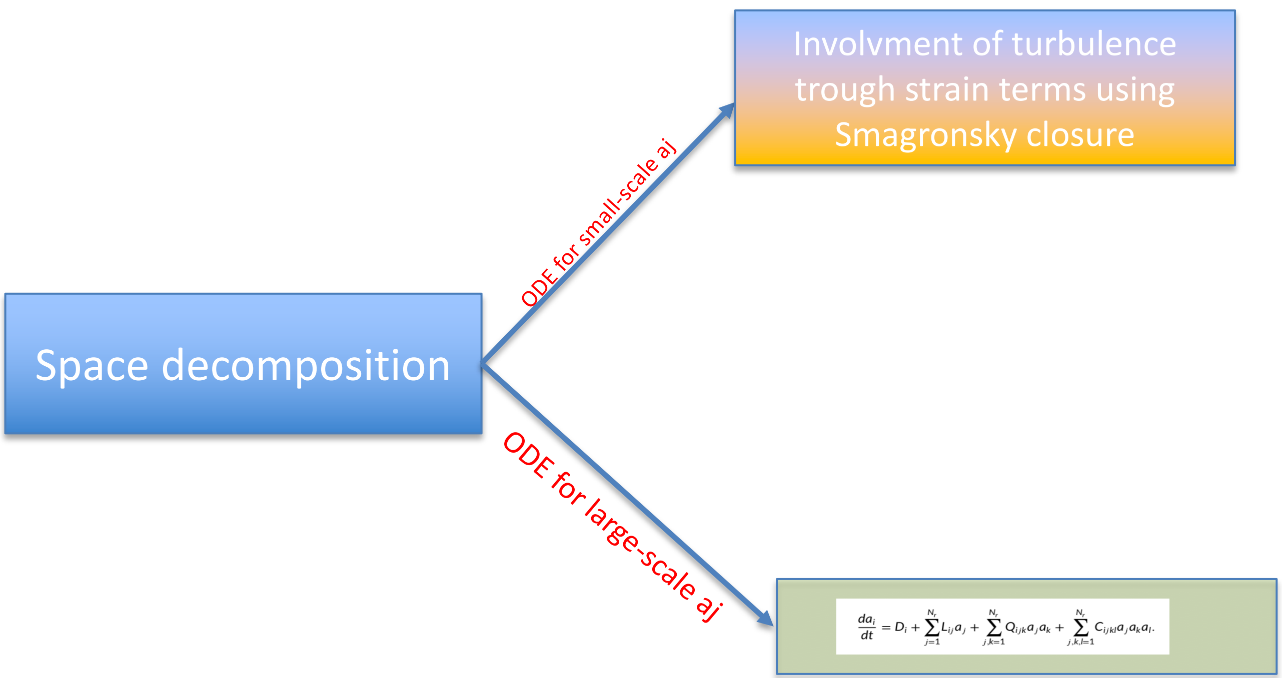
$$\boldsymbol{u}(\boldsymbol{x},t) \approx \boldsymbol{u}_L + \boldsymbol{u}_H$$

$$\boldsymbol{u}_L(\boldsymbol{x},t) \approx \boldsymbol{U} + \sum_{j=1}^{r_L} a_j(t)\,\boldsymbol{\phi}_j(\boldsymbol{x})$$

$$\boldsymbol{u}_H(\boldsymbol{x},t) \approx \boldsymbol{U} + \sum_{j=r_L+1}^{r} a_j(t)\,\boldsymbol{\phi}_j(\boldsymbol{x})$$

POD Closure Models: Overview

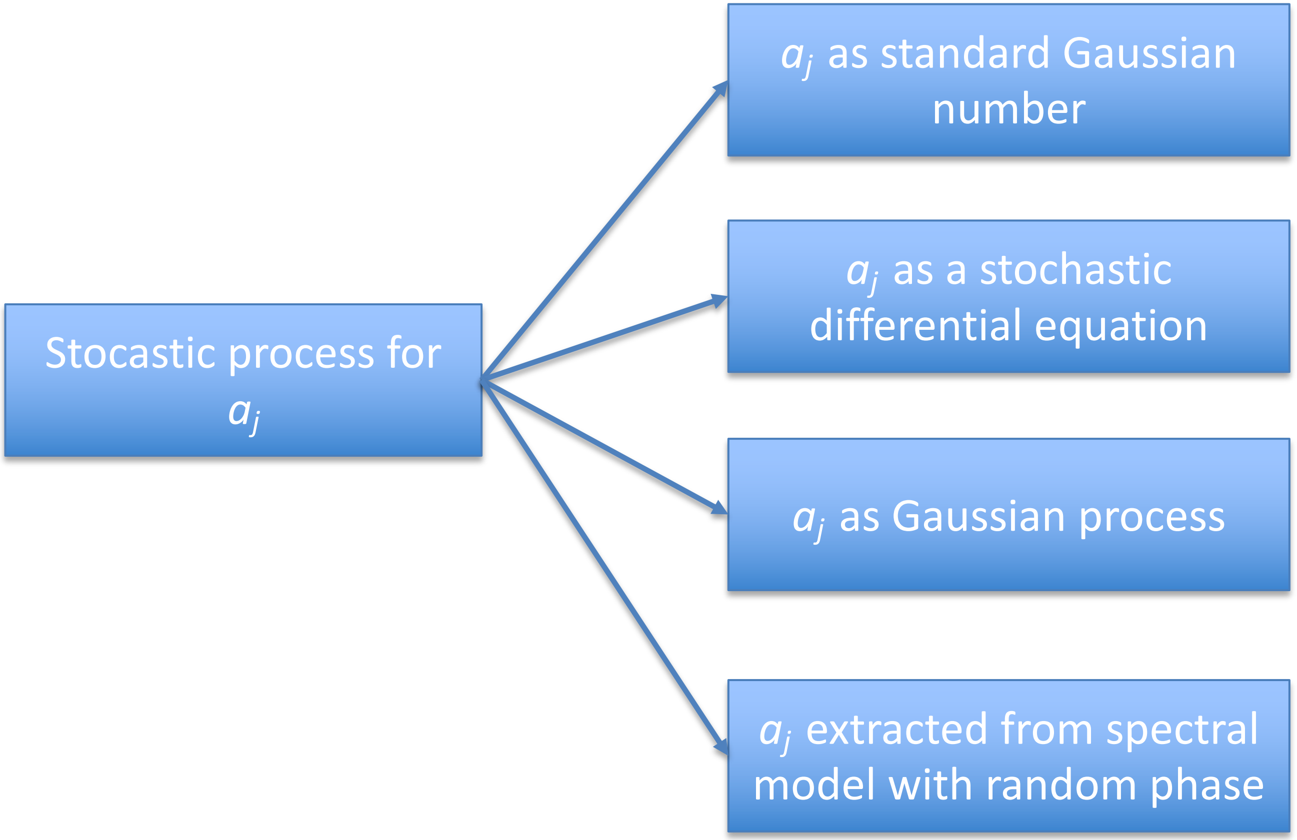
Applying previous slide’s decomposition leads to two sets of Ordinary Differential Equations (ODEs). The one related to the small scales of motion accounts for turbulence, For example through the Smagronsky representation.



Stochastic POD

Can we describe *N time-dependent* weighting coefficients ($a_j(t)$) as a stochastic system?

By assuming, a_j are statistically independent, we are able to consider them as stochastic process.



Stochastic POD

Can we describe *N time-dependent* weighting coefficients ($a_j(t)$) as a stochastic system?

By assuming, a_j are statistically independent, we are able to consider them as stochastic process.

$$da_j(t) = f(a_j(t), t) \cdot dt + g(a_j(t), t) \cdot dW(t),$$

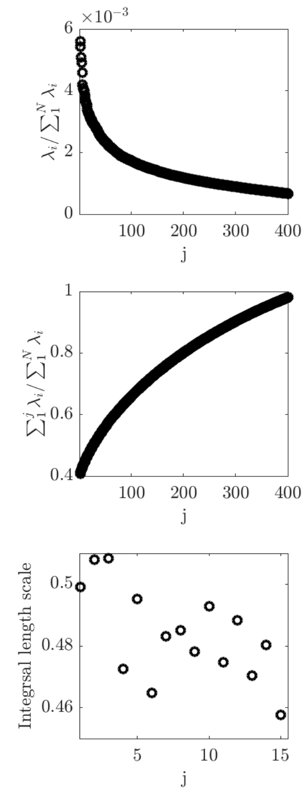
W denotes Brownien motion

$$da_j(t) = -\alpha_j(\mu_j - a_j(t)) \cdot dt + \overbrace{\sigma_j \sqrt{2\alpha_j} \cdot dW(t)}^{d\gamma(t)},$$

μ_j and σ_j are mean and standard deviation of $a_j(t)$

autocorrelation is governed by an exponential-decaying function with decay rate of α as follows

$$\rho(\tau) = \overline{a_j(t)a_j(t+\tau)} = e^{-\alpha \cdot \tau},$$



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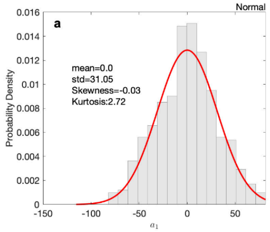
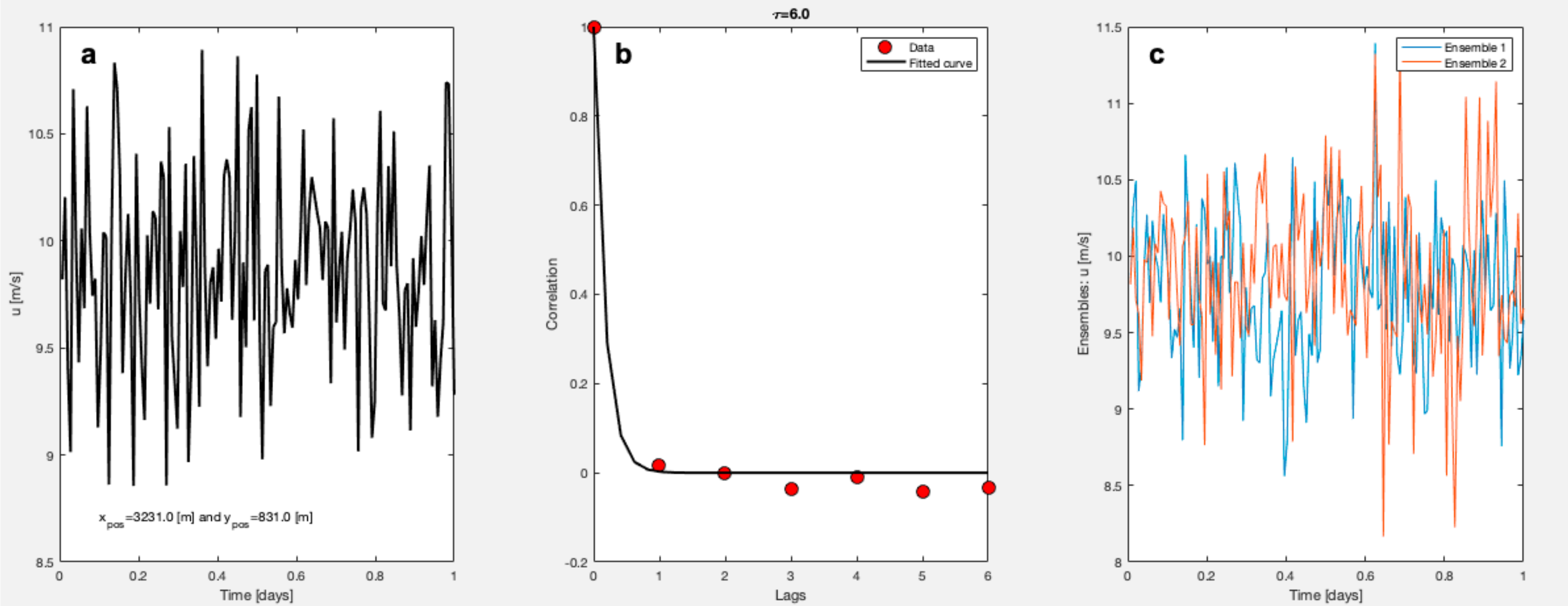
Stochastic POD: Brownian motion & a_j autocorrelation

a_j are normally distributed



Two stochastic trajectories



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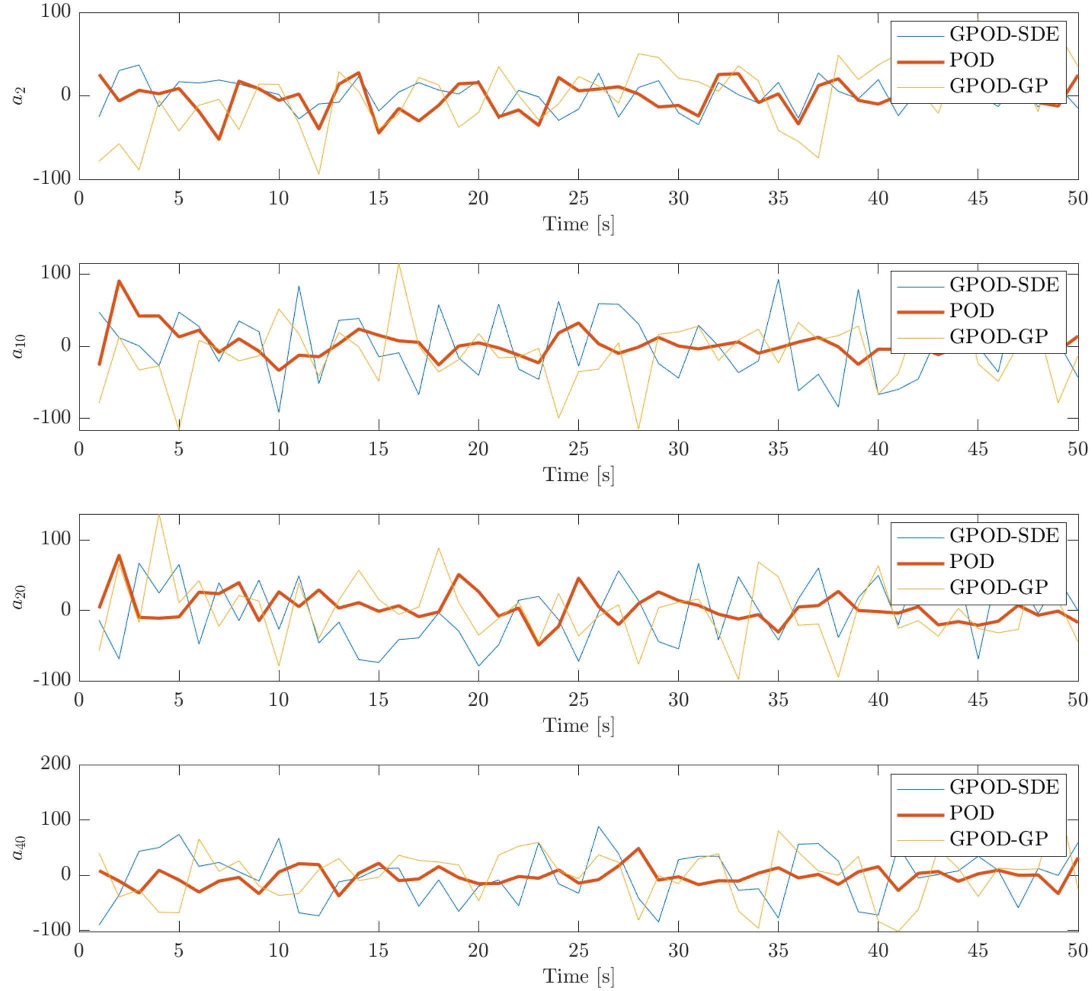
Stochastic POD

Comparisons between three different
Values of a based on:

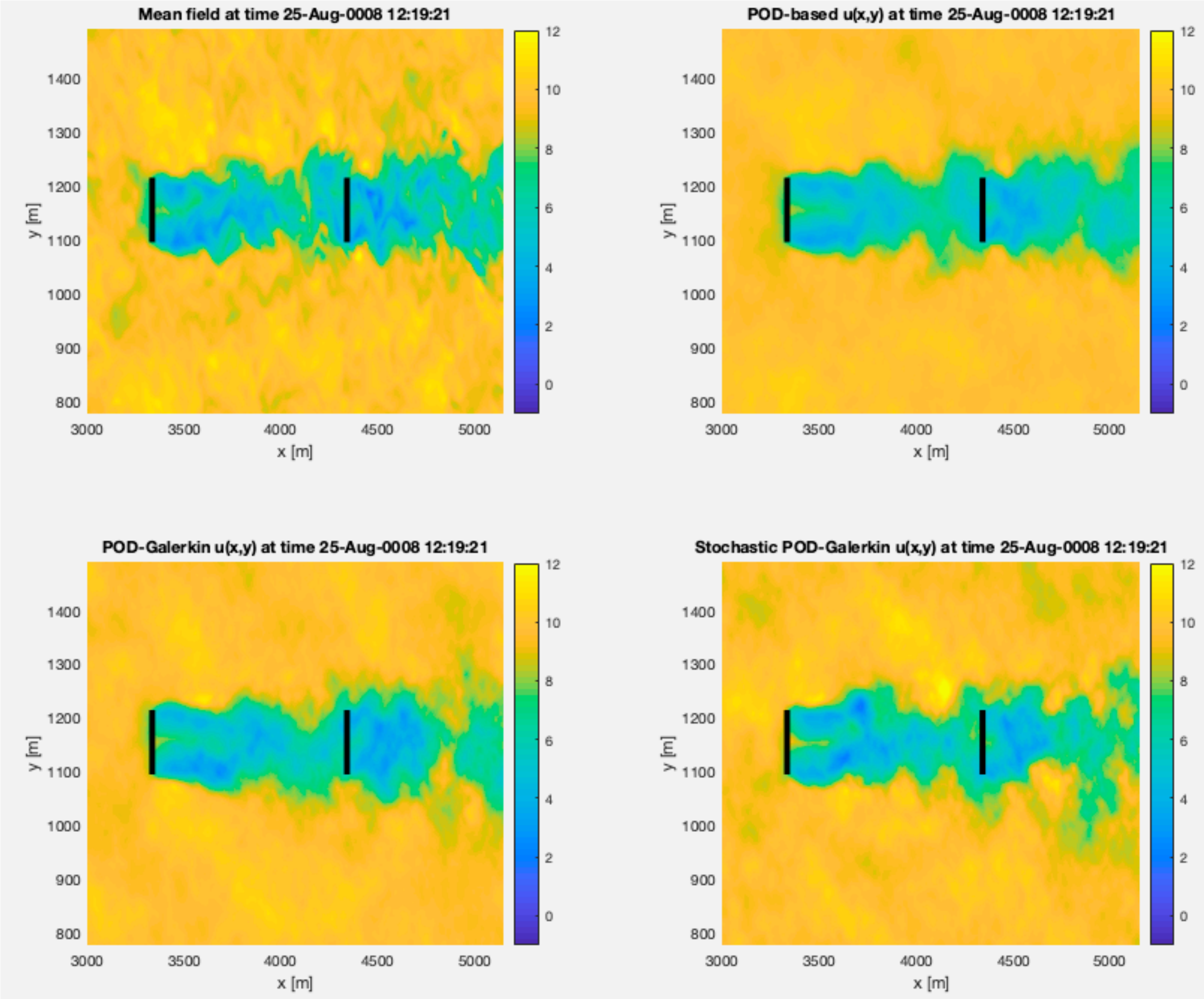
(1) POD eignvalues
(2) Gaussian random process
(3) SDE

Note that for case 2 & 3, we Use GPOD.



Results: Compare techniques

Flow field reconstruction
Based on different stochastic techniques



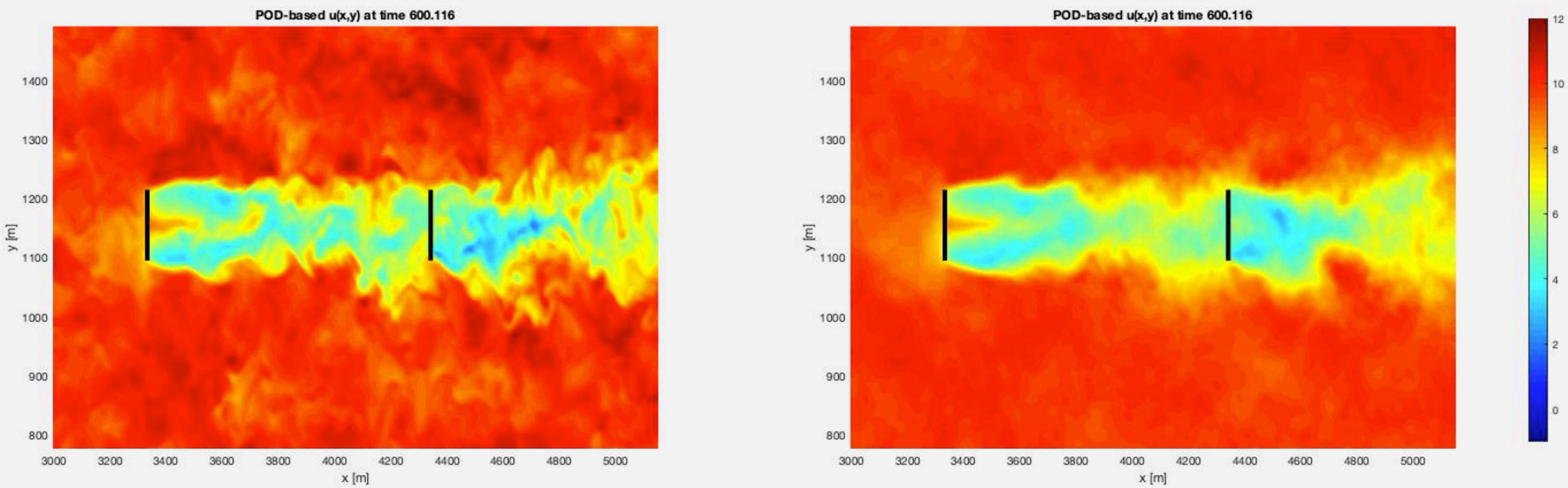
Results: Compare techniques

$$\mathbf{u}(\mathbf{x}, t) = \sum_{i=1}^{N} a_i(t)\Phi^{(i)}(\mathbf{x}).$$

$$a_i(t) = \int_D \mathbf{u}(\mathbf{x}, t)\Phi^{(i)}(\mathbf{x})d\mathbf{x}.$$

Original *u*-component versus the one reconstructed from the standard POD analysis

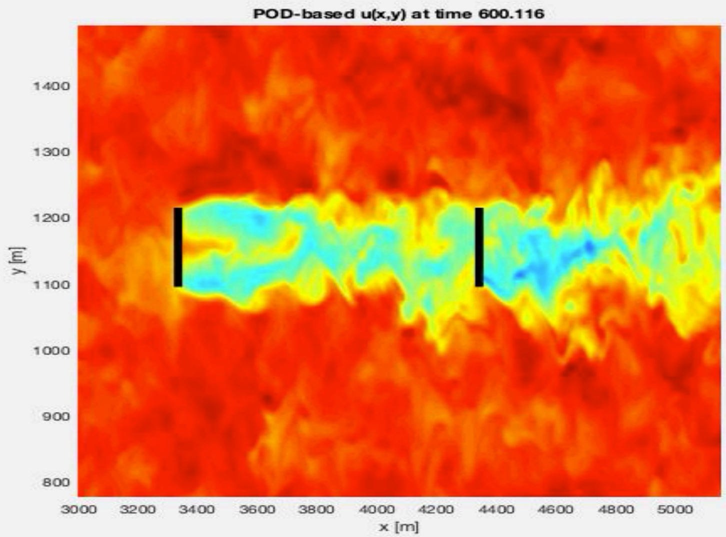
Small scale features have been filtered out in ambient and wake flow.

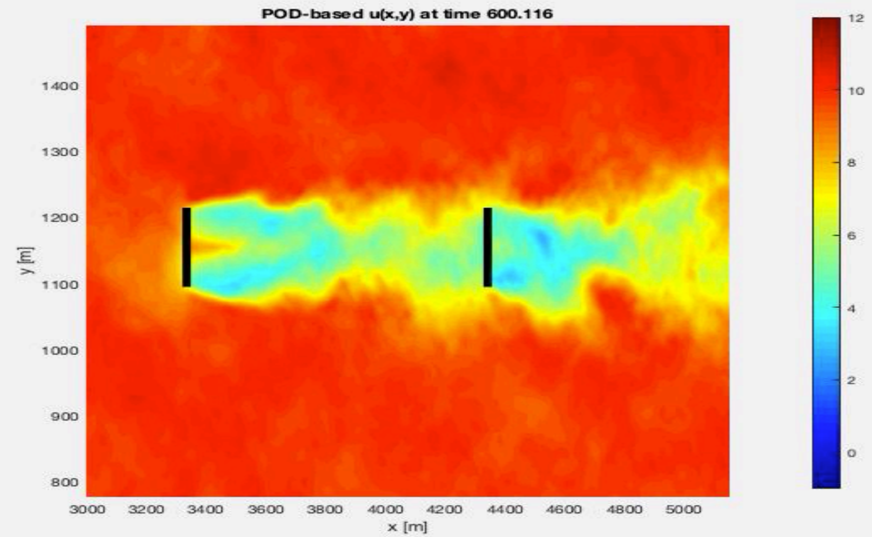


Results: Compare techniques

$$\frac{da_i}{dt} = D_i + \sum_{j=1}^{N_r} L_{ij} a_j + \sum_{j,k=1}^{N_r} Q_{ijk} a_j a_k + \sum_{j,k,l=1}^{N_r} C_{ijkl} a_j a_k a_l.$$

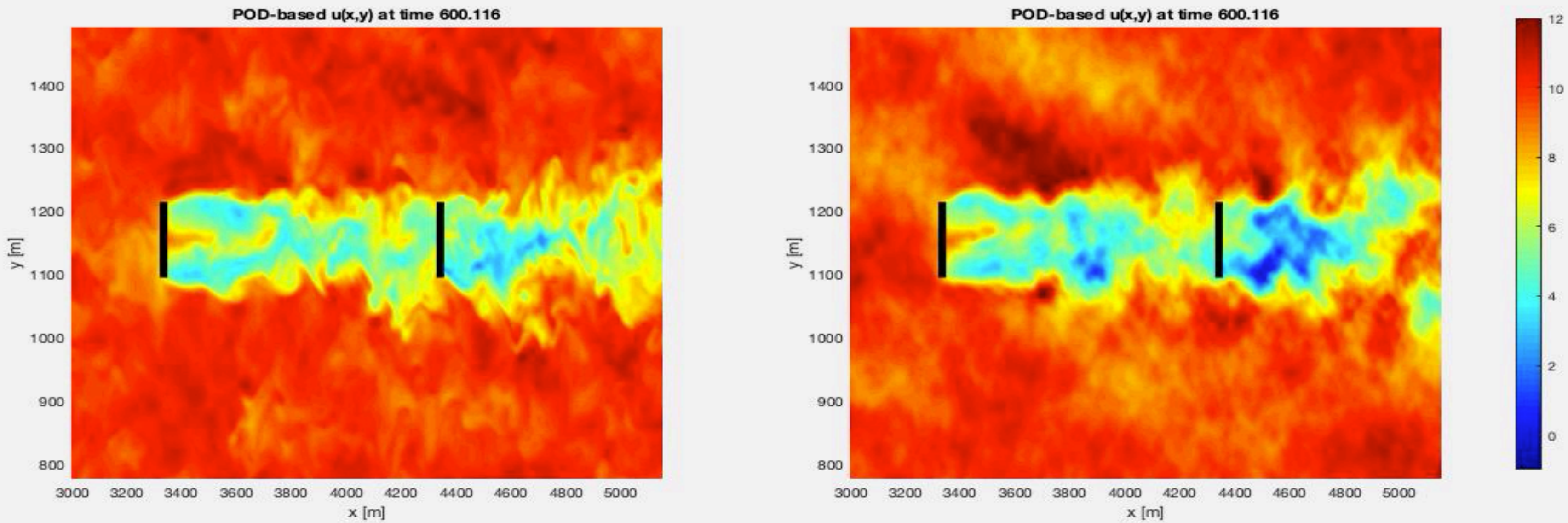
We compare the original flow field with the one reconstructed by the use of POD Galerkin (without POD closure).





Results: Compare techniques

We compare the original flow field with the one reconstructed by the use of POD Galerkin+stochastatic process (without POD closure).



Conclusion & future works

- *Tentative results suggest that considering the effects of stochastic forcing can improve the accuracy of the POD model.*
- *POD-based ROM needs further stability control.*
- Development POD closure techniques.
- Coupling the model with NREL FAST to study the load characteritics under the influence of stochastic forcing and varying atmospheric stability condition.
- Higher order statistics using POD-based approach (apropriate for turbulence study).
- Lidar-based POD-Galerkin to study coherent structures.
- POD-based short-term flow foarcast (e.g. machine-learning).

References

[1] M. Bakhoday-Paskyabi et al., On the Stochastic Reduced-Order and LES-based Model s of Offshore Wind Turbine Wake, DeepWind paper, 2020.

[2] P. Holms, J. L. Lumely, and G. Berkooz, Turbulence, coherent structures, dynamical systems and symmetry, Cambridge university press, 1998.

[3] C. Rowely, Model reduction for fluids, using balanced proper orthogonal decomposition, International Journal of Bifurcation and Chaos, vol. 15, 2005.

[4] D. Bastine et al, Stochastic wake modelling based on POD analysis, MDPI, Vol. 11, 2018.

Thanks