

# Evaluation of Gaussian wake models

LES simulation requires large amount of time and computational resources to resolve a wind turbine wake. Analytical wake models are faster but approximate. This overview compares four Gaussian models based on their ability to resolve a wake from single wind turbine.

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## Gaussian models

### Known values

- $r, D$  — wind turbine radius and diameter
- $C_T$  — thrust coefficient
- $I_a$  — ambient turbulence

$\Delta \bar{U} = 1 - \frac{U}{U_\infty}$  — wake velocity deficit normalized by free flow velocity at hub height  $U_\infty$

### Bastankhah & Porté-Agel (BPA) [1]

Early Gaussian model. Here, the version that considers ambient turbulence  $I_a$  is regarded. The model does not resolve wake field for  $x/D < 2$ .

$$\Delta \bar{U} = F_{BPA1}(\sigma, C_T, D) \exp(F_{BPA2}(\sigma, D, y, z))$$

where  $\sigma = \sigma(k^*, x, \varepsilon)$ ,  $k^*$  is parametrized as  $k^* = A + BI_a$ .

Parameters:  $A, B$

### Jensen-Gaussian [2]

Top hat distribution in original Jensen model is replaced with Gaussian.

$$\Delta \bar{U} = F_{JG1}(\bar{U}^*) \exp(F_{JG2}(r, \sigma))$$

where  $\bar{U}^* = \bar{U}^*(k, x, a)$

Parameter:  $k$

### Ishihara [3]

Full-wake model with the

$$\Delta \bar{U} = F_{I1}(a, b, c, x, D) \exp(F_{I2}(\sigma, D, y, z))$$

where  $a, b, c$  expressed as  $a, C_T^{a2} I_a^{a3}$ . Coefficients for each variable may be fitted.

### Double Gaussian [4]

Full wake model which captures double wake shape in the near wake region.

$$\Delta \bar{U} = c \cdot F(\sigma) \frac{1}{2} \{ \exp[f(r + r_0, \sigma)] + \exp[f(r - r_0, \sigma)] \}$$

where  $\sigma = (k^*, x^{1/3}, \varepsilon)$

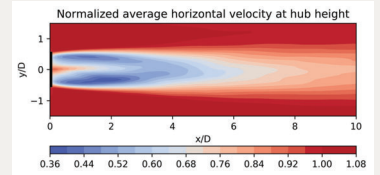
Parameters:  $k^*, \varepsilon, c$

## Methodology

A single NREL 5MW wind turbine is simulated with PALM LES code for free flow  $U_0=10$  and  $15 \text{ m/s}$  at hub height, neutral and stable conditions. The models are fitted to wake velocity profiles using least squares.

The following normalized velocity data sets are used:

- $xy$  profiles at  $x/D=1..10$
- rotor axis line
- $xy$  hub height plane (control fit)



## Factors to compare

- Number of parameters  
Less parameters – easier to fit model, more parameters – more flexibility

• Tunability  
Whether the original parameters can be tweaked for a better fit.

- Turbulence intensity  
If included explicitly, the model can account different stability conditions.

- Wake approximation  
Evaluated from root mean square error (RMSE)

- Wake shape  
Axisymmetric or 2D wake, whether the near wake can be resolved as good as far wake

## Summary

Spreading wake shape at  $U_0=10 \text{ m/s}$  is challenging for the models, all except for Double Gaussian show high RMS error in near wake.

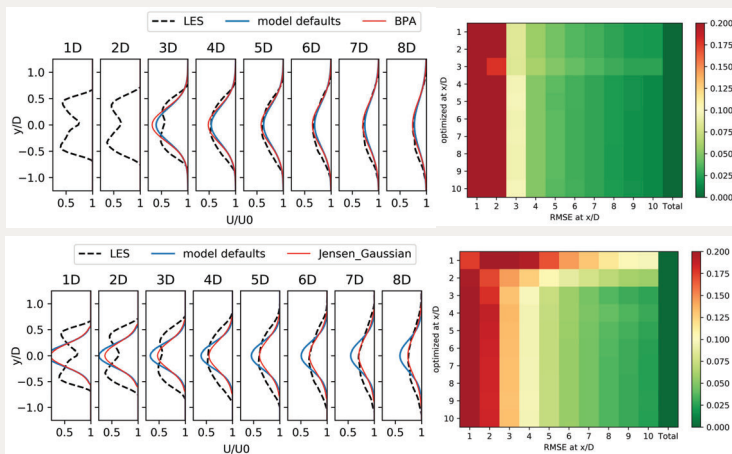
The wake at  $U_0=15 \text{ m/s}$  is narrower and more uniform. All models provide good approximation of the full wake at high speed.

However,  $U_0=10 \text{ m/s}$  is a typical wind speed for operating wind turbines. The approximation at  $U_0=10 \text{ m/s}$  is more important. Unless a good resolution of near wake is required, all models can be used to approximate the far wake.

Double Gaussian model concept is in development and may go under revisions. Ishihara model is not supposed to be tunable, the recommended parameters also provide good approximation especially for the far wake.

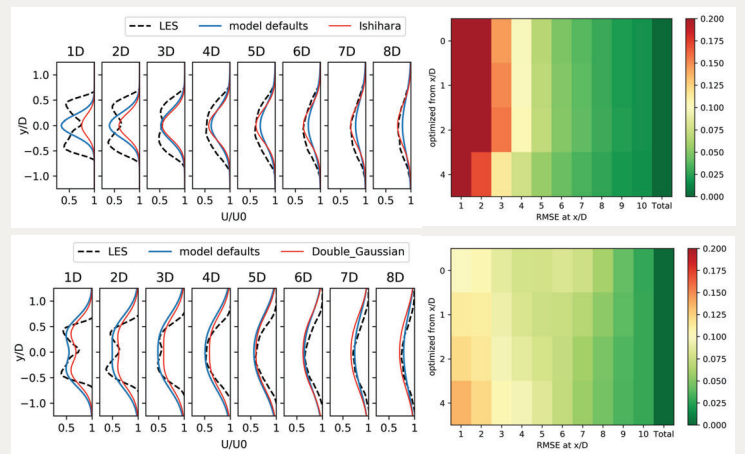
## Optimization for xy-wake profile (plots for $x/D=8$ )

BPA and Jensen-Gaussian models perform best. The returned parameters are similar regardless which of the profiles  $4 < x/D < 10$  was used for fitting.



## Optimization for rotor axis line

Full wake Ishihara and Double Gaussian models perform best, the data must be available for the full line. The fit is close to the one obtained from 2D plane.



Model	$I_a$	Full wake	# of par.	Tunable	2D wake
BPA	+	-	2	+	+/-
Jensen-Gaussian	-/+	-	1	+	-
Ishihara	+	+	9	-	+
Double Gaussian	-	+	3	+	+

## REFERENCES

- [1] A. Niyafar and F. Porté-Agel, "A new analytical model for wind farm power prediction," J. Phys. Conf. Ser., vol. 625, no. 1, 2015.
- [2] X. Gao, H. Yang, and L. Lu, "Optimization of wind turbine layout position in a wind farm using a newly-developed two-dimensional wake model," Appl. Energy, vol. 174, pp. 192–200, Jul. 2016.
- [3] T. Ishihara and G.-W. W. Qian, "A new Gaussian-based analytical wake model for wind turbines considering ambient turbulence intensities and thrust coefficient effects," J. Wind Eng. Ind. Aerodyn., vol. 177, no. May, pp. 275–292, Jun. 2018.
- [4] A. Keane, P. E. O. Aguirre, H. Ferchland, P. Clive, and D. Gallacher, "An analytical model for a full wind turbine wake," J. Phys. Conf. Ser., vol. 753, no. 3, pp. 32–39, Sep. 2016.

