Fast divergence-conforming reduced basis methods for stationary and transient flow problems

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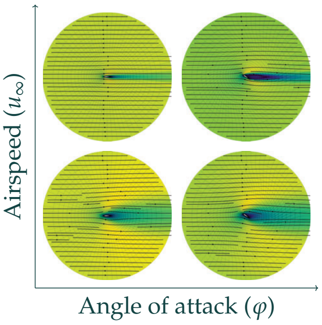


Problem: Repeated solutions of parametrized problems (left) can be extremely demanding, each query involving up to 10^6–10^9 degrees of freedom and hours or days of computational time.
Solution: Reduced Order Modelling (ROM) via Reduced Basis Methods (RBM) offers solutions with dramatic speedups and respectable accuracy.

OFFLINE

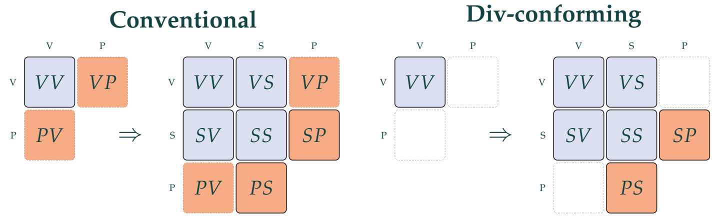
- Parametrized PDE
- High-fidelity discretization
  $$A_h(\boldsymbol{\mu})u_h(\boldsymbol{\mu}) = f_h(\boldsymbol{\mu})$$
  $$A_h(\boldsymbol{\mu}) = \sum_q \xi_q(\boldsymbol{\mu}) A_h^q$$
  $$f_h(\boldsymbol{\mu}) = \sum_q \chi_q(\boldsymbol{\mu}) f_h^q$$
- Snapshot solutions
- Projection matrix $\boldsymbol{V}$
- $$A_N^q = \boldsymbol{V}^\intercal A_h^q \boldsymbol{V}$$
  $$f_N^q = \boldsymbol{V}^\intercal f_h^q$$

ONLINE

- Parameter vector $\boldsymbol{\mu}$
- RB system assembly
  $$A_N = \sum_q \xi_q(\boldsymbol{\mu}) A_N^q$$
  $$f_N = \sum_q \chi_q(\boldsymbol{\mu}) f_N^q$$
- RB system solution
  $$A_N(\boldsymbol{\mu})u_N(\boldsymbol{\mu}) = f_N(\boldsymbol{\mu})$$
- Post-processing
  Recovery
  Visualization
  Quantities of interest

Stationary: Navier-Stokes flow around a NACA0015 airfoil with chord length of 1 m, parametrized by inflow velocity $u_\infty \in [1\,\mathrm{m/s}, 20\,\mathrm{m/s}]$ and angle of attack $\varphi \in [-35°, 35°]$. Snapshots were evaluated on the 15×15 Gauss points on the parameter domain and reduced models created with $N = 10, 20, \ldots, 50$ DoFs.

Transient: Navier-Stokes flow around a cylinder with diameter 1 m, inflow velocity 1 m/s and Re = 100. This system has two *stages*: a transient stage influenced by the initial velocity field and a stable, perpetual vortex shedding stage. Snapshots were evaluated *only* in the vortex shedding stage, and reduced models created with $N = 5, 10, 15, 20$ DoFs.



Div-conforming RBMs are faster: The reduced system matrix (size $2N$) will usually have a rank-deficient velocity-pressure block (denoted VP). Enriching the velocity space with so-called *supremizers* (denoted S) ensures a full-rank system matrix with size $3N$. A div-conforming method instead produces a fully divergence-free basis, so the VP block vanishes. This yields a block-triangular system, solvable as two size-N systems instead of one size-$3N$ system.

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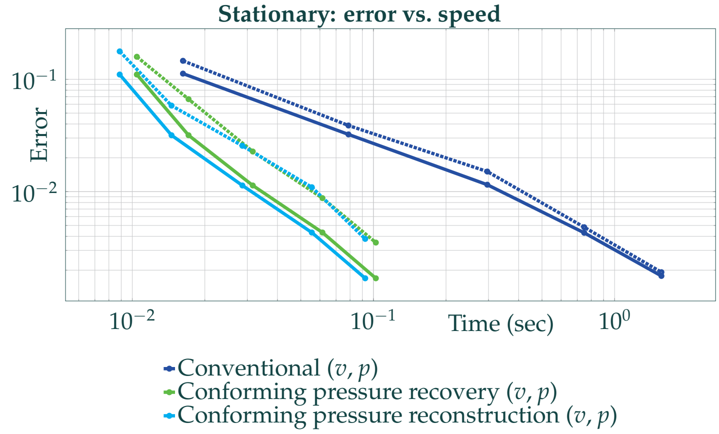
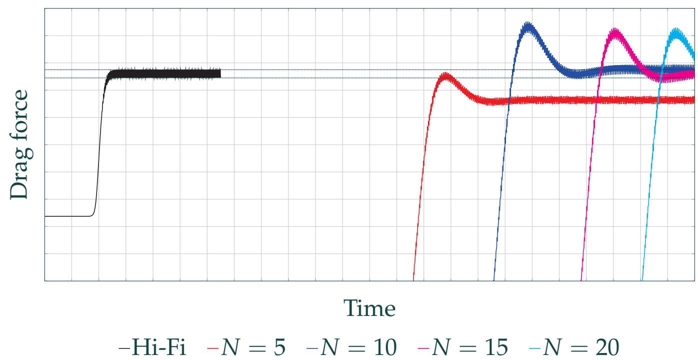
Mean solver time usage

	Hi-Fi	$N = 10$	$N = 20$	$N = 30$	$N = 40$	$N = 50$
Conventional	104 s	29 ms	126 ms	503 ms	1.02 s	2.51 s
Conforming	165 s	21 ms	54 ms	104 ms	183 ms	284 ms

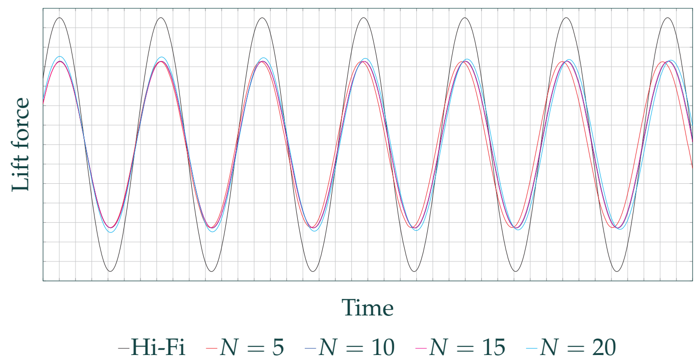
Stationary: error vs. speed



Transient: drag force vs. time



Transient: lift force vs. time



Discussion

- RBMs are able to deliver results within two to three orders of magnitude at dramatic speedups.
- Div-conforming RBMs can deliver higher speeds (one order of magnitude in present examples) by exploiting specific properties of velocity basis functions.
- RBMs based only on final stage (vortex shedding) snapshots can still step through the transient stage without permanent loss of accuracy (e.g. blowing up or crashing).