Verification of Floating Offshore Wind Linearization Functionality in OpenFAST

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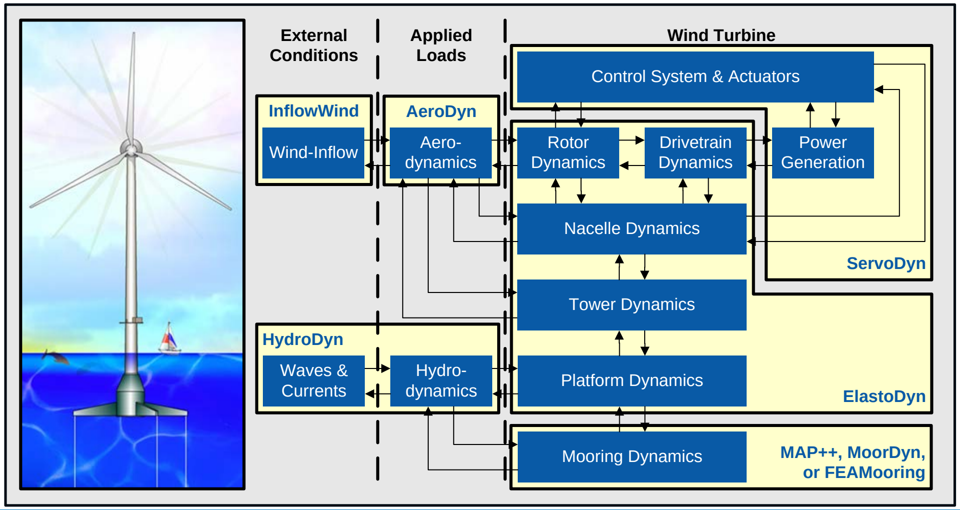
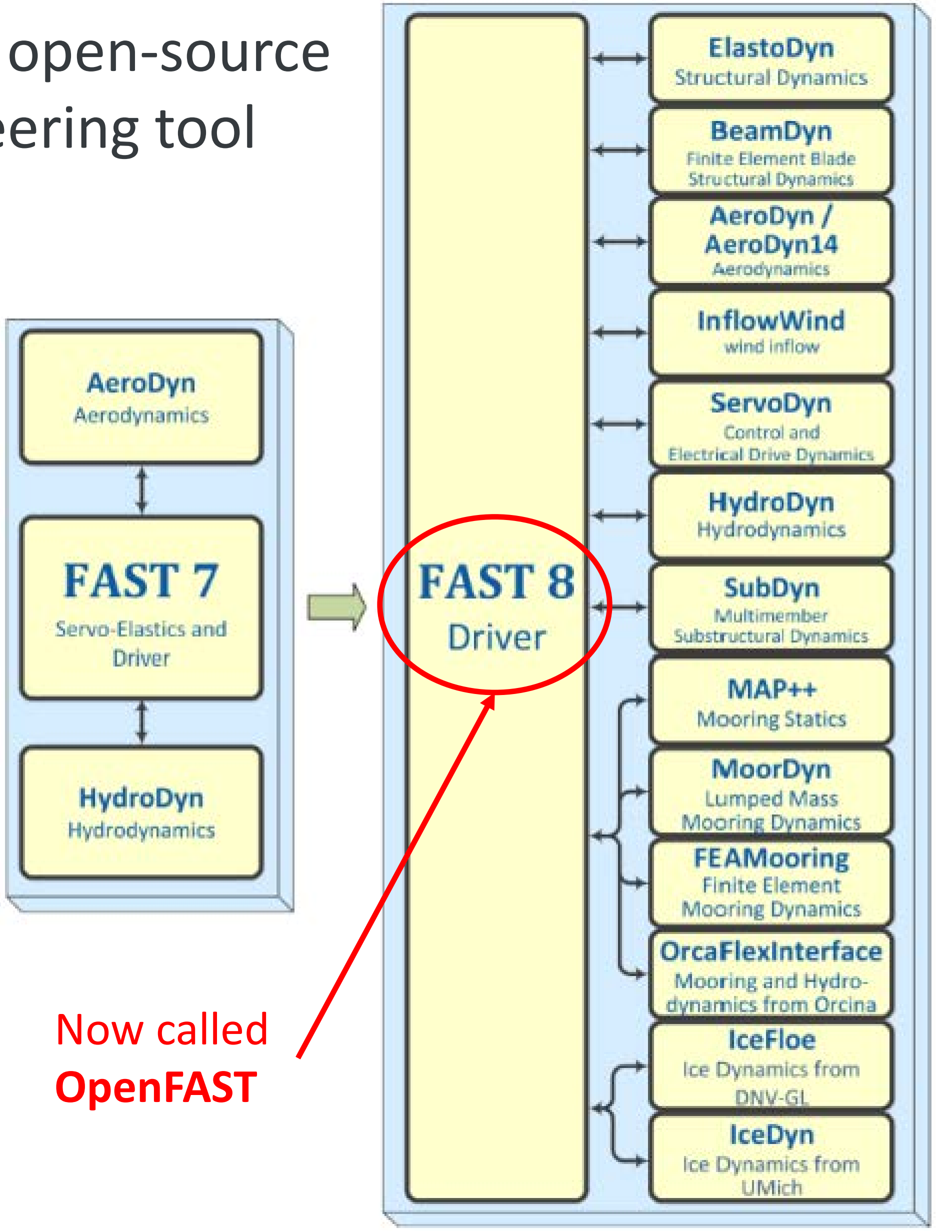
Introduction: The OpenFAST Multi-Physics Engineering Tool

- **OpenFAST** is DOE/NREL's premier open-source wind turbine multi-physics engineering tool
- **FAST** has undergone a major restructuring, w/ a new modularization framework (v8)
- Framework originally designed w/ intent of enabling full-system linearization, but functionality is being implemented in stages

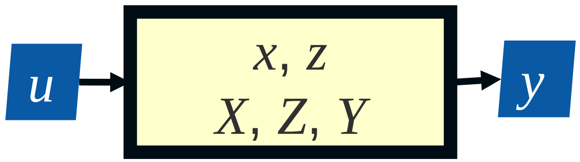




Background: Why Linearize?

- **OpenFAST** primary used for nonlinear time-domain standards-based load analysis (ultimate & fatigue)
- Linearization is about *understanding*:
  - o Useful for eigenanalysis, controls design, stability analysis, gradients for optimization, & development of reduced-order models
- Prior focus:
  - o Structuring source code to enable linearization
  - o Developing general approach to linearizing mesh-mapping w/n module-to-module coupling relationships, inc. rotations
  - o Linearizing core (but not all) features of **InflowWind**, **ServoDyn**, **ElastoDyn**, **BeamDyn**, & **AeroDyn** modules & their coupling
  - o Verifying implementation
- Recent work (presented @ IOWTC 2018):
  - o Linearizing **HydroDyn**, & **MAP++**, & coupling
  - o State-space implementation of wave-excitation & wave-radiation loads
- This work – Verifying implementation for FOWT

Module



$$\dot{x} = X\left(x,z,u,t\right)$$

$$0 = Z\left(x,z,u,t\right) \quad with \left|\frac{\partial Z}{\partial z}\right| \neq 0$$

$$y = Y\left(x,z,u,t\right)$$

$$u = u\big|_{op} + \Delta u \quad etc.$$

$$\Delta\dot{x} = A\Delta x + B\Delta u$$

$$\Delta y = C\Delta x + D\Delta u$$

with

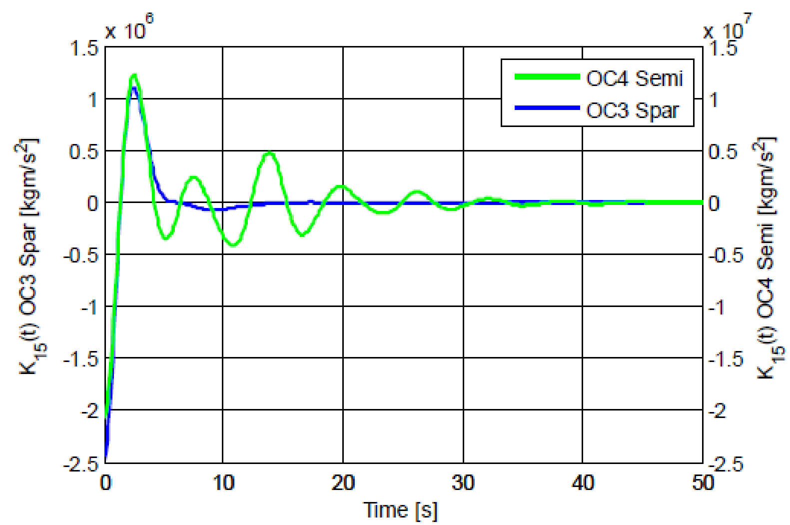
$$A = \left[\frac{\partial X}{\partial x} - \frac{\partial X}{\partial z}\left[\frac{\partial Z}{\partial z}\right]^{-1}\frac{\partial Z}{\partial x}\right]\Bigg|_{op} \quad etc.$$

Background: State-Space-Based Wave Radiation

- Wave-radiation "memory effect" accounted for in **HydroDyn** by direct time-domain (numerical) convolution
- Linear state-space (SS) approximation:
  - SS matrices derived from **SS_Fitting** pre-processor using 4 system-ID approaches

$$\dot{q}_{Ptfm} \rightarrow \boxed{F_{Rdtn} = -\int_0^t K_{Rdtn}(t-\tau)\,\dot{q}_{Ptfm}(\tau)\,d\tau} \rightarrow F_{Rdtn}$$

$$\dot{q}_{Ptfm} \rightarrow \boxed{\begin{aligned} \dot{x}_{Rdtn} &= A_{Rdtn}x_{Rdtn} + B_{Rdtn}\dot{q}_{Ptfm} \\ F_{Rdtn} &\cong C_{Rdtn}x_{Rdtn} \end{aligned}} \rightarrow F_{Rdtn}$$



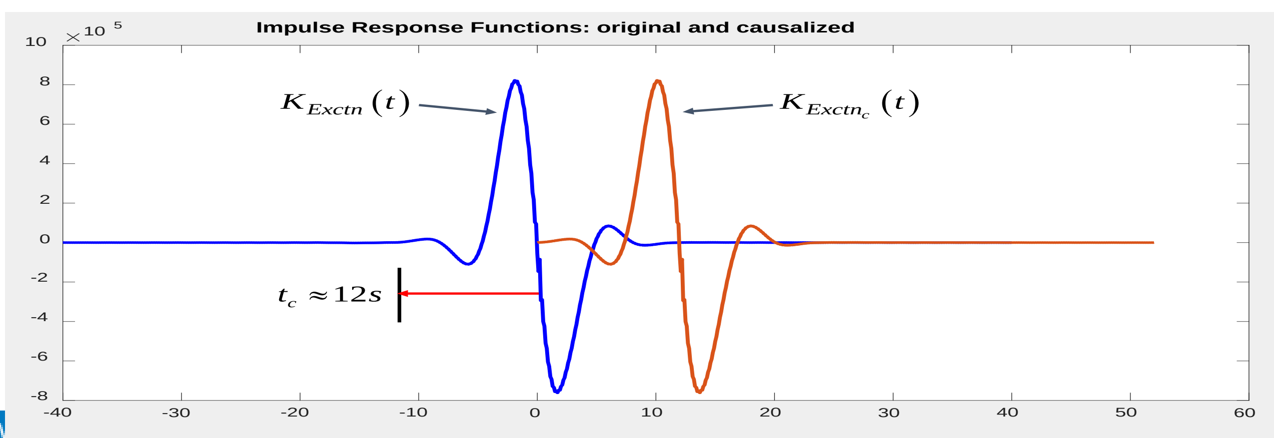
Background: State-Space-Based Wave Excitation

- First-order wave-excitation loads accounted for in **HydroDyn** by inverse Fourier transform
- Linear SS approximation:
  - SS matrices derived from extension to **SS_Fitting** pre-processor using system-ID approach
  - Requires prediction of wave elevation time t_c into future to address noncausality i.e. $\zeta_c(t) = \zeta(t + t_c)$

$\zeta \rightarrow$ $$F_{Extn} = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega, \beta)\zeta(\omega) e^{j\omega t} d\omega$$ $\rightarrow F_{Exctn}$

$\zeta \rightarrow$ $$F_{Extn} = \int_{-\infty}^{\infty} K_{Extn}(t - \tau)\zeta(\tau) d\tau$$ $\rightarrow F_{Exctn}$

$\zeta_c \rightarrow$ $$\dot{x}_{Extn} = A_{Extn} x_{Extn} + B_{Extn} \zeta_c$$ $$F_{Extn} \cong C_{Extn} x_{Extn}$$ $\rightarrow F_{Exctn}$



Background: Final Matrix Assembly

InflowWind (IfW)

$$\Delta y^{(IfW)} = D^{(IfW)} \Delta u^{(IfW)}$$

ServoDyn (SrvD)

$$\Delta y^{(SrvD)} = D^{(SrvD)} \Delta u^{(SrvD)}$$

AeroDyn (AD)

$$\Delta y^{(AD)} = D^{(AD)} \Delta u^{(AD)}$$

MAP++ (MAP)

$$\Delta y^{(MAP)} = D^{(MAP)} \Delta u^{(MAP)}$$

ElastoDyn (ED)

$$\Delta \dot{x}^{(ED)} = A^{(ED)} \Delta x^{(ED)} + B^{(ED)} \Delta u^{(ED)}$$
$$\Delta y^{(ED)} = C^{(ED)} \Delta x^{(ED)} + D^{(ED)} \Delta u^{(ED)}$$

BeamDyn (BD)

$$\Delta \dot{x}^{(BD)} = A^{(BD)} \Delta x^{(BD)} + B^{(BD)} \Delta u^{(BD)}$$
$$\Delta y^{(BD)} = C^{(BD)} \Delta x^{(BD)} + D^{(BD)} \Delta u^{(BD)}$$

HydroDyn (HD)

$$\Delta \dot{x}^{(HD)} = A^{(HD)} \Delta x^{(HD)} + B^{(HD)} \Delta u^{(HD)}$$
$$\Delta y^{(HD)} = C^{(HD)} \Delta x^{(HD)} + D^{(HD)} \Delta u^{(HD)}$$

Glue Code

$$0 = \left.\frac{\partial U}{\partial \tilde{u}}\right|_{op} \Delta u + \left.\frac{\partial U}{\partial y}\right|_{op} \Delta y$$

Full System

$$\Delta \dot{x} = A \Delta x + B \Delta u^{+}$$
$$\Delta y = C \Delta x + D \Delta u^{+}$$

$$\Delta x = \begin{Bmatrix} \Delta x^{(ED)} \\ \Delta x^{(BD)} \\ \Delta x^{(HD)} \end{Bmatrix} \quad \Delta u^{+} = \begin{Bmatrix} \Delta u^{(IfW)} \\ \Delta u^{(SrvD)} \\ \Delta u^{(ED)} \\ \Delta u^{(BD)} \\ \Delta u^{(AD)} \\ \Delta u^{(HD)} \\ \Delta u^{(MAP)} \end{Bmatrix}^{+} \quad \Delta y = \begin{Bmatrix} \Delta y^{(IfW)} \\ \Delta y^{(SrvD)} \\ \Delta y^{(ED)} \\ \Delta y^{(BD)} \\ \Delta y^{(AD)} \\ \Delta y^{(HD)} \\ \Delta y^{(MAP)} \end{Bmatrix}$$

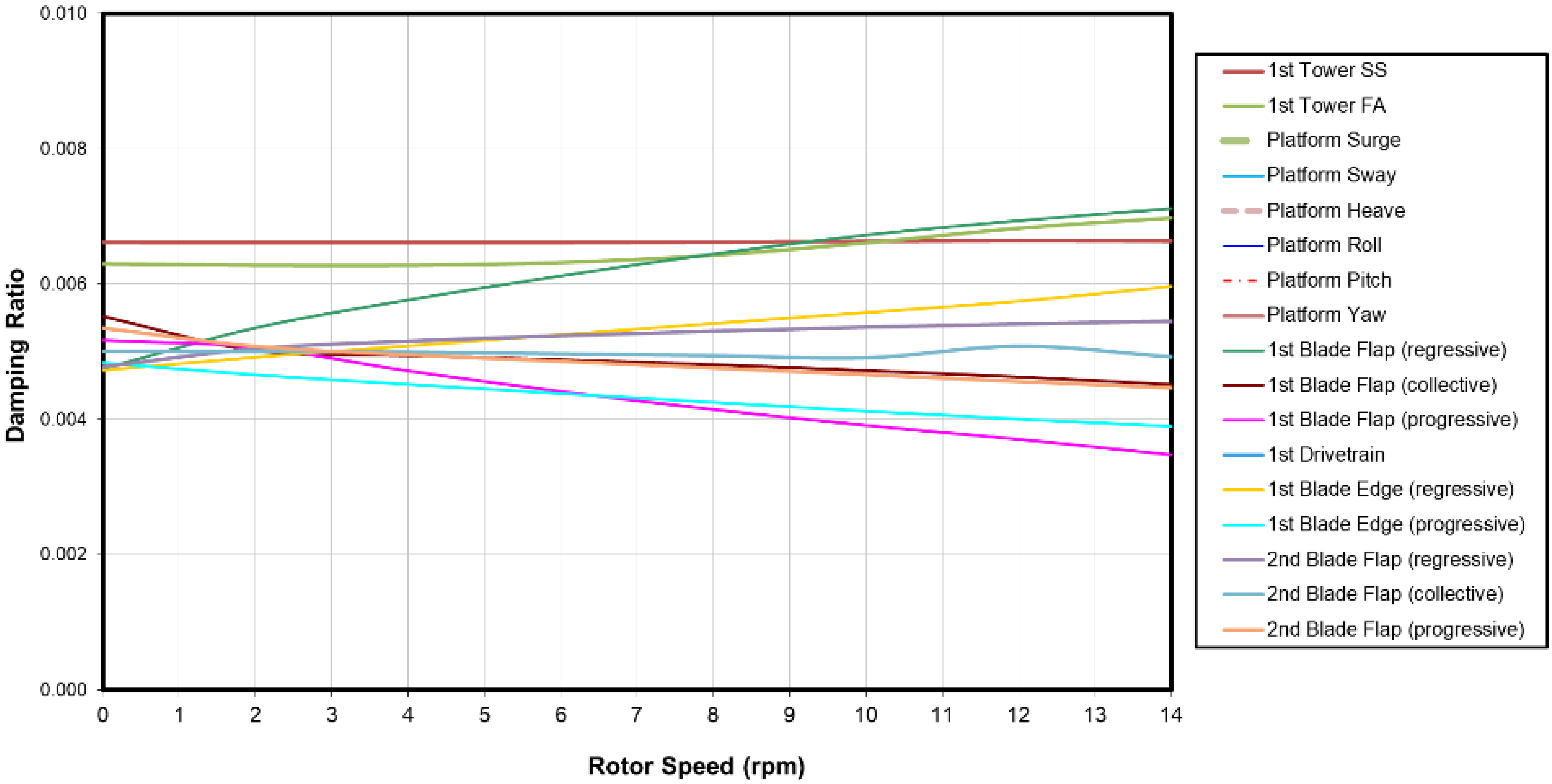
with

$$A = \begin{bmatrix} A^{(ED)} & 0 & 0 \\ 0 & A^{(BD)} & 0 \\ 0 & 0 & A^{(HD)} \end{bmatrix} - \begin{bmatrix} 0 & 0 & B^{(ED)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & B^{(BD)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & B^{(HD)} & 0 \end{bmatrix} \left[G|_{op} \right]^{-1} \left.\frac{\partial U}{\partial y}\right|_{op} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ C^{(ED)} & 0 & 0 \\ 0 & C^{(BD)} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & C^{(HD)} \\ 0 & 0 & 0 \end{bmatrix}$$

etc.

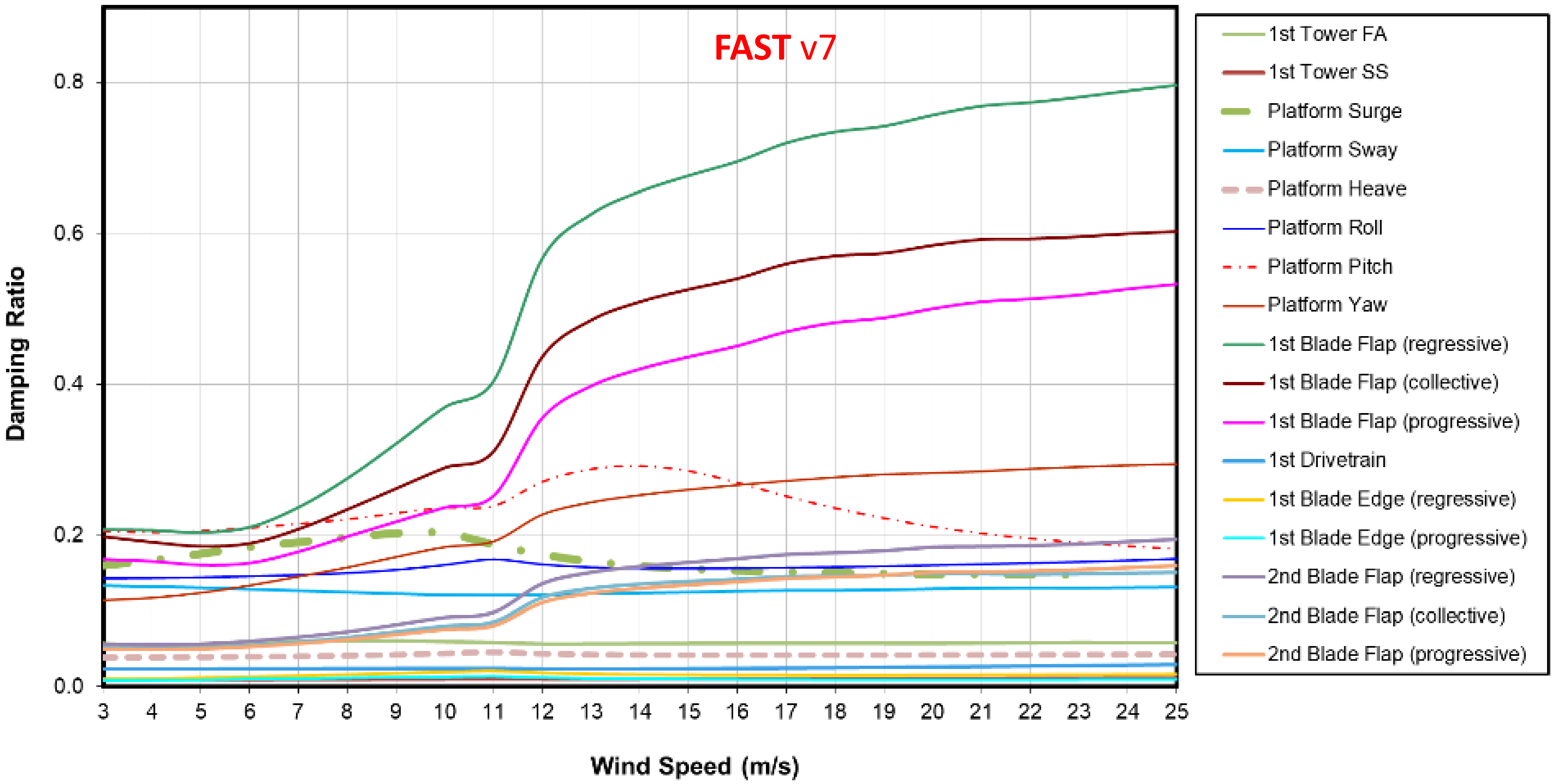
- D-matrices (included in G) impact all matrices of coupled system, highlighting important role of direct feedthrough
- While $A^{(ED)}$ contains mass, stiffness, & damping of **ElastoDyn** structural model only, full-system A contains mass, stiffness, & damping associated w/ full-system coupled aero-hydro-servo-elastics, including FOWT hydrostatics, radiation damping, drag, added mass, & mooring restoring

Results: Campbell Diagram of NREL 5-MW Turbine Atop OC3-Hywind Spar



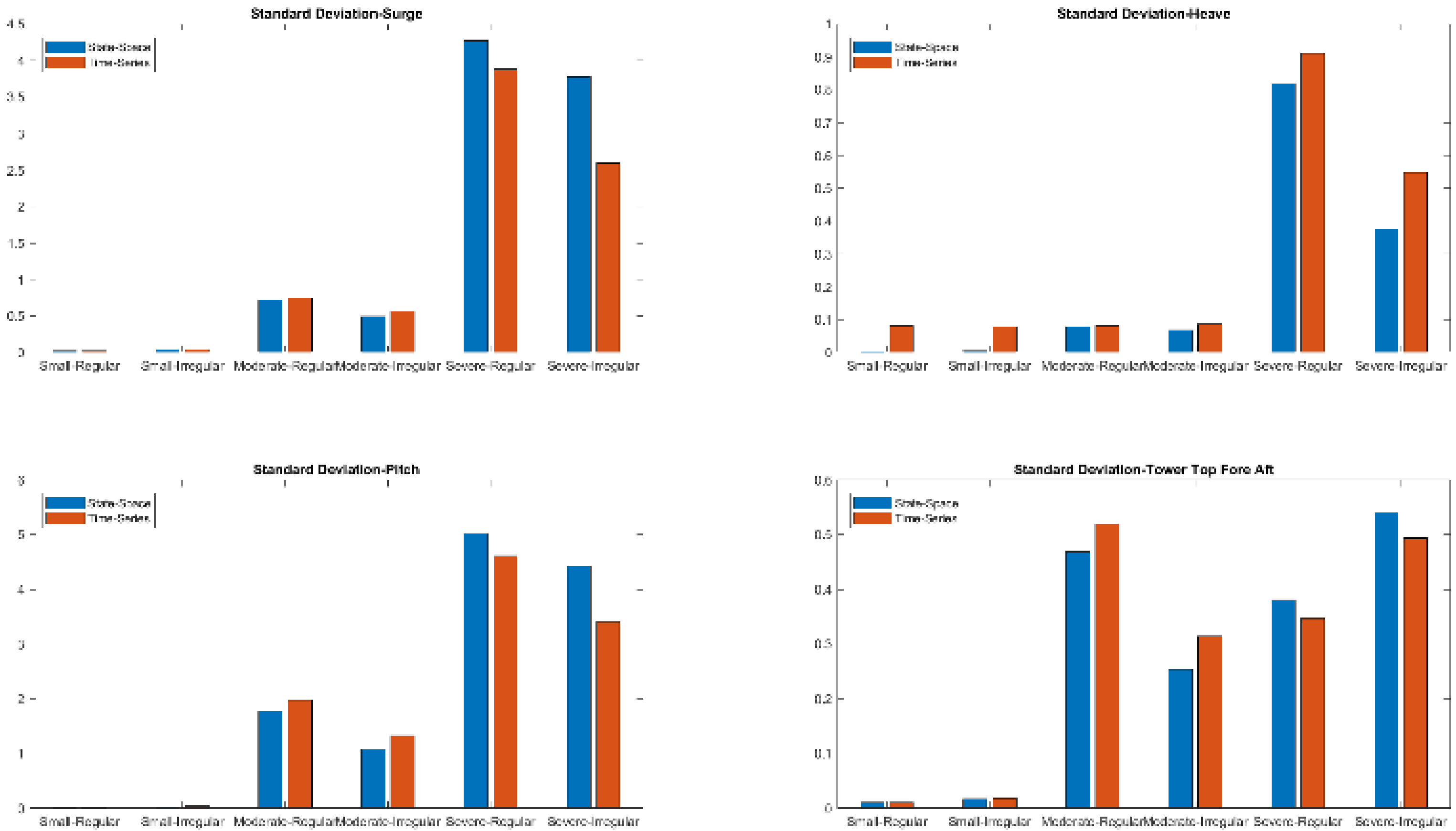
- Modules enabled: **ElastoDyn**, **ServoDyn**, **HydroDyn**, & **MAP++**
- Approach (for each rotor speed): Find periodic steady-state OP → Linearize to find A matrix → MBC → Azimuth-average → Eigenanalysis → Extract freq.s & damping

Results: Campbell Diagram of NREL 5-MW Turbine Atop OC3-Hywind Spar – w/ Aero



- Modules enabled: **ElastoDyn**, **ServoDyn**, **HydroDyn**, **MAP++**, **AeroDyn**, & **InflowWind**
- Approach (for each wind speed): Define torque & blade pitch → Find periodic steady-state OP → Linearize to find A matrix → MBC → Azimuth-average → Eigenanalysis → Extract freq.s & damping

Results: Time Series Comparison of Nonlinear & Linear Models



- Modules enabled: **ElastoDyn**, **ServoDyn**, **HydroDyn**, & **MAP++**
- Nonlinear approach (for each sea state): Time-domain simulation w/ waves
- Linear approach (for each sea state): Find steady-state OP → Linearize to find A, B, C, D matrices → Integrate in time w/ wave-elevation input derived from nonlinear solution

Conclusions & Future Work

- Conclusions:
  - Linearization of underlying nonlinear wind-system equations advantageous to:
    - Understand system response
    - Exploit well-established methods/tools for analyzing linear systems
  - Linearization functionality has been expanded to FOWT w/n **OpenFAST**
  - Verification results:
    - Good agreement in natural frequencies between **OpenFAST** & **FAST** v7
    - Damping differences impacted by trim solution, frozen wake, perturbation size on viscous damping, wave-radiation damping
    - Nonlinear versus linear response shows impact of structural nonlinearites for more severe sea states
- Future work:
  - Improved OP through static-equilibrium, steady-state, or periodic steady-state determination, including trim
  - Eigenmode automation & visualization
  - Linearization functionality for:
    - Other important features (e.g. unsteady aerodynamics of **AeroDyn**)
    - Other offshore functionality (**SubDyn**, etc.)
    - New features as they are developed

Carpe Ventum!

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Approach & Methods: Operating-Point Determination

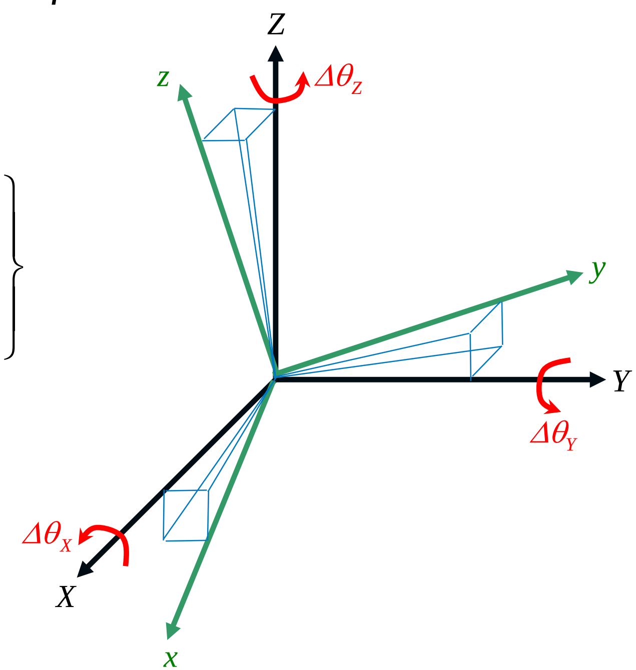
- A linear model of a nonlinear system is only valid in local vicinity of an operating point (OP)
- Current implementation allows OP to be set by given initial conditions (time zero) or a given times in nonlinear time-solution
- Note about rotations in 3D:
  - Rotations don't reside in a linear space
  - **FAST** framework stores module inputs/outputs for 3D rotations using 3×3 DCMs (Λ)
  - Linearized rotational parameters taken to be 3 small-angle rotations about global *X*, *Y*, & *Z* ($\Delta\vec{\theta}$)

$$u = u\big|_{op} + \Delta u \quad \textit{for most variables}$$

$$\Lambda = \Lambda\big|_{op}\,\Delta\Lambda \quad \textit{for rotations}$$

with

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \Lambda \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix}$$



$$\Delta\Lambda = \begin{bmatrix} \frac{\Delta\theta_X^2\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}+\Delta\theta_Y^2+\Delta\theta_Z^2}{\left(\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2\right)\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}} & \frac{\Delta\theta_Z\left(\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2\right)+\Delta\theta_X\Delta\theta_Y\left(\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}-1\right)}{\left(\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2\right)\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}} & \frac{-\Delta\theta_Y\left(\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2\right)+\Delta\theta_X\Delta\theta_Z\left(\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}-1\right)}{\left(\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2\right)\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}} \\ \frac{-\Delta\theta_Z\left(\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2\right)+\Delta\theta_X\Delta\theta_Y\left(\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}-1\right)}{\left(\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2\right)\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}} & \frac{\Delta\theta_X^2+\Delta\theta_Y^2\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}+\Delta\theta_Z^2}{\left(\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2\right)\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}} & \frac{\Delta\theta_X\left(\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2\right)+\Delta\theta_Y\Delta\theta_Z\left(\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}-1\right)}{\left(\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2\right)\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}} \\ \frac{\Delta\theta_Y\left(\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2\right)+\Delta\theta_X\Delta\theta_Z\left(\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}-1\right)}{\left(\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2\right)\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}} & \frac{-\Delta\theta_X\left(\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2\right)+\Delta\theta_Y\Delta\theta_Z\left(\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}-1\right)}{\left(\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2\right)\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}} & \frac{\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}}{\left(\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2\right)\sqrt{1+\Delta\theta_X^2+\Delta\theta_Y^2+\Delta\theta_Z^2}} \end{bmatrix}$$

$$\Delta\Lambda \approx \begin{bmatrix} 1 & \Delta\theta_Z & -\Delta\theta_Y \\ -\Delta\theta_Z & 1 & \Delta\theta_X \\ \Delta\theta_Y & -\Delta\theta_X & 1 \end{bmatrix} \qquad \Delta\vec{\theta} = \begin{Bmatrix} \Delta\theta_X \\ \Delta\theta_Y \\ \Delta\theta_Z \end{Bmatrix}$$

Approach & Methods: Module Linearization

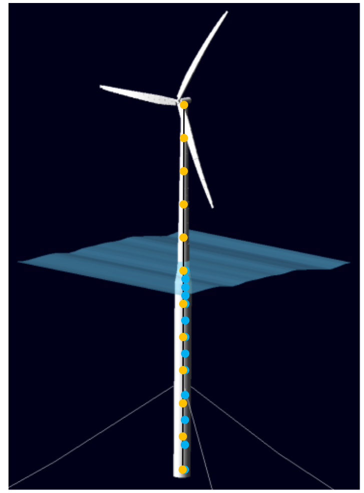
Module	Linear Features	States (*x*, *z*)	Inputs (*u*)	Outputs (*y*)	Jacobian Calc.
ElastoDyn (ED)	• Structural dynamics of: o Blades o Drivetrain o Nacelle o Tower o Platform	• Structural degrees-of-freedom (DOFs) & their 1st time derivatives (continuous states)	• Applied loads along blades & tower • Applied loads on hub, nacelle, & platform • Blade-pitch-angle command • Nacelle-yaw moment • Generator torque	• Motions along blades & tower • Motions of hub, nacelle, & platform • Nacelle-yaw angle & rate • Generator speed • User-selected structural outputs (motions &/or loads)	• Numerical central-difference perturbation technique*
HydroDyn (HD)	• Wave excitation • Wave-radiation added mass • Wave-radiation damping • Hydrostatic restoring • Viscous drag	• State-space-based wave-excitation (continuous states) • State-space-based radiation (continuous states)	• Motions of platform • Wave-elevation disturbance	• Hydrodynamic applied loads along platform • User-selected hydrodynamic outputs	• Analytical for state equations • Numerical central-difference perturbation technique* for output equations
MAP++ (MAP)	• Mooring restoring	• Mooring line tensions (constraint states) • Positions of connect nodes (constraint states)	• Displacements of fairleads	• Tensions at fairleads • User-selected mooring outputs	• Numerical central-difference perturbation technique*

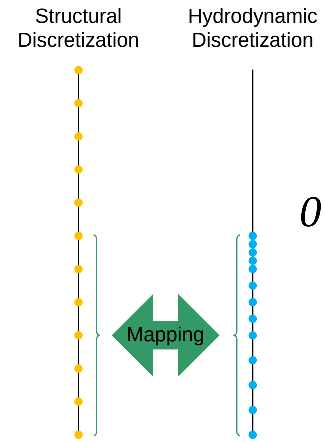
**Numerical central -difference perturbation technique (see paper for treatment of 3D rotations)*

$$\left.\frac{\partial X}{\partial x}\right|_{op} = \frac{X\left(\left.x\right|_{op} + \Delta x, \left.u\right|_{op}, \left.t\right|_{op}\right) - X\left(\left.x\right|_{op} - \Delta x, \left.u\right|_{op}, \left.t\right|_{op}\right)}{2\Delta x} \quad etc.$$

Approach & Methods: Glue-Code Linearization

- Module inputs & outputs residing on spatial boundaries use a mesh, consisting of:
  - Nodes & elements (nodal connectivity)
  - Nodal reference locations (position & orientation)
  - One or more nodal fields, including motion, load, &/or scalar quantities
- Mesh-to-mesh mappings involve:
  - Mapping search – Nearest neighbors are found
  - Mapping transfer – Nodal fields are transferred
- Mapping transfers & other module-to-module input-output coupling relationships have been linearized analytically





$$0 = U\left(u, y\right)$$

$$0 = \left.\frac{\partial U}{\partial \tilde{u}}\right|_{op} \Delta u + \left.\frac{\partial U}{\partial y}\right|_{op} \Delta y$$

with

$$\Delta u = \begin{Bmatrix} \Delta u^{(IfW)} \\ \Delta u^{(SrvD)} \\ \Delta u^{(ED)} \\ \Delta u^{(BD)} \\ \Delta u^{(AD)} \\ \Delta u^{(HD)} \\ \Delta u^{(MAP)} \end{Bmatrix}$$

$$\left.\frac{\partial U}{\partial \tilde{u}}\right|_{op} = \left.\begin{bmatrix} I & 0 & 0 & 0 & \frac{\partial U^{(IfW)}}{\partial \tilde{u}^{(AD)}} & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & \frac{\partial U^{(ED)}}{\partial \tilde{u}^{(BD)}} & \frac{\partial U^{(ED)}}{\partial \tilde{u}^{(AD)}} & \frac{\partial U^{(ED)}}{\partial \tilde{u}^{(HD)}} & \frac{\partial U^{(ED)}}{\partial \tilde{u}^{(MAP)}} \\ 0 & 0 & 0 & \frac{\partial U^{(BD)}}{\partial \tilde{u}^{(BD)}} & \frac{\partial U^{(BD)}}{\partial \tilde{u}^{(AD)}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial U^{(AD)}}{\partial \tilde{u}^{(AD)}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\partial U^{(HD)}}{\partial \tilde{u}^{(HD)}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}\right|_{op}$$

etc.