North Sea met-ocean data analysis using copula for lumping of offshore wind turbine fatigue load cases

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Introduction

This research was done because
- Joint measurements of wind and wave data are not available everywhere at the North Sea
- Cost-efficient design of offshore wind turbines for fatigue damage needs joint met-ocean data
- Planning of marine installation and maintenance-operation needs joint met-ocean data

This research used:
- Copula that isolates the marginal properties from the dependence structure of random variables
- Copula + Marginal = Generating joint distribution
- Lumping to reduce a full-sea-state to some load cases by weighting wind and wave data

This research was done by:
- Collecting long-term joint wind and wave data at four different locations at the North Sea
- Calculating emprical copula and emprical marginal at all location
- Combining copula at one location with wave heigth marginal at another location
- Using the generated joint distribution to lump wind speed
- Comparing the generated lumped wind speed with real data lumped wind speed
- Comparing the fatigue damage caused by lumped wind speed and real lumped wind speed

Applied theory

(H_S, W_S)	Wave height and wind speed	Pairs of two stochastic random variable measured jointly between 16 to 24 years 4 station at North Sea
$U =$	(1) $F_{H_S}(\mathrm{h_S}) = P(H_S \le \mathrm{h_S})$	Empirical Cumulative Distribution function of Hs (marginal)
V =	(2) $F_{W_S}(w_S) = P(W_S < w_S)$	Empirical Cumulative Distribution function of Ws(marginal)
$F_{H,W} =$	(3) $C\left(F_{H_S}(\mathrm{h_S}), F_{W_S}(w_S)\right) = P\left[F_{W_S}(w_S) \le u \cap F_{W_S}(w_S) \le v\right]$	Joint cumulative distribution C is copula which is a function of only marginal
$C_n(u,v)$	(4) $\frac{1}{n}\sum_{i=1}^{n} 1\left(\frac{R_i}{n+1} \le u, \frac{S_i}{n+1} \le v\right)$	Empirical copula ;R is the Rank of Wave height ; S is the rank of Wind speed; n is the number of measurements
Lumping method: Preservation of wave height distribution and lumping wind speed		
$W_{s,i} =$	(5) $\frac{\sum_{j=1}^{m} P_{i,j} W_{s_{i,j}}}{\sum_{j=1}^{m} P_{i,j}}$	Lumped wind speed; P is the probability of occurrence; i, j are scatter diagram cell number
Fatigue damage can be simply estimated using the relation based on quasi static response		
D	(6) $D \propto \Delta\sigma \propto H_s^{\mu}$ (7) $D \propto \Delta\sigma^{\mu} \propto W_s^{\mu}$	D is fatigue damage; $\Delta\sigma$ is the stress range, T_z is the wave period; μ is the S-N curve slope

Data gathering and analysis

Figure1. Data set locations

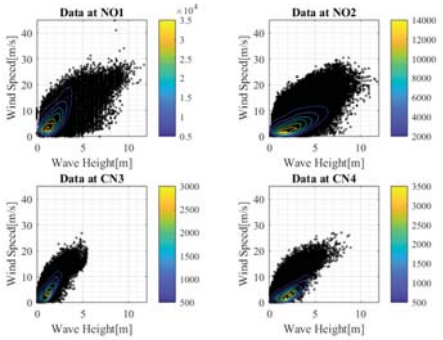

Figure 2. Scatter plot of long-term measurements at different stations

- W_s and Hs are measured every 10 minutes at No1 and No2 and every 3 hours at CN3 and CN4
- Copula is calculated by ranking W_s and Hs and using the formula above
- Small value is added to data to avoid repetitive numbers

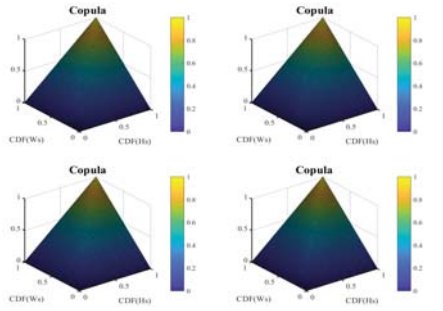

Figure 3. Copula at different stations

Method

- Copula domain, $[0,1]^2$ is a 100x100 mesh grid
- While Copula is calculated at the nodes copula density and wave heigth and wind speed are calculated in the centre of each cell.

$$c_{i,j}(U,V) = \frac{dC}{dUdV} = \frac{C_{ia1,j+1} - C_{i+1,j} - C_{i,j+1} + C_{i,j}}{dU.dV} = f(X,Y)$$

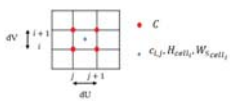

Figure 4. Numerical Stencil of copula mesh grid, dU=dV=0.01

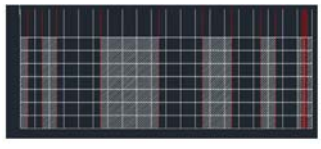

Figure 5. Import marginal to copula domain

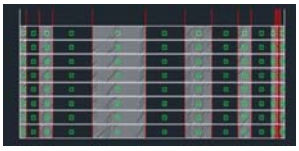

Figure 6.. Average bins based on copula density

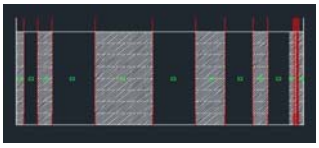

Figure 7. Lumping bin data

- White lines represent copula mesh grid
- Red lines are imported wave heigth bins transformed to [0,1] domain using $CDF(H_S)$.
- Copula density of bin is summation of copula density of cells inside each bin.
- Wind speed is lumped using formula in applied theory where $P_{I,j}$ equals to $c_{I,j}$ in each row of copula mesh grid.

Results

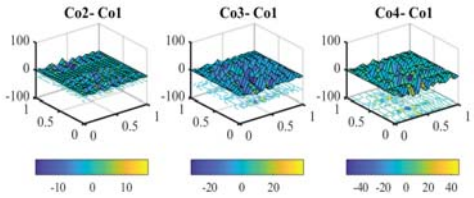

Figure 8. Copula difference

- Copula that is calculated at No1 is subtracted from the copula at other locations.
- the average copula difference is less than15 %.

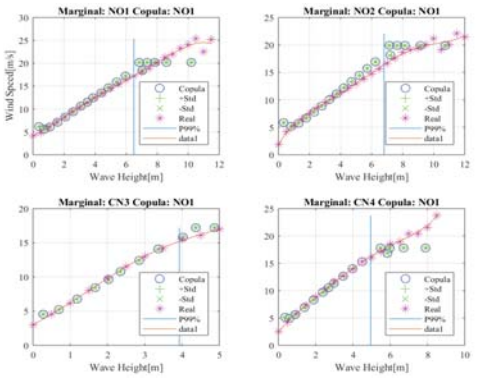

Figure 9. Copula generated lumped data versus real lumped data

- Generating lumped met-ocean data at different sites using copula calculated at NO_1.
- Comparing lumped real data with lumped generated data.
- The difference between stars and circles show how well copula at NO1 can predict the joint behaviour in other locations in the North Sea.
- The blue line represents the upper tale of copula density domain and calculation of extreme values with $P > 99\%$ is not accurate

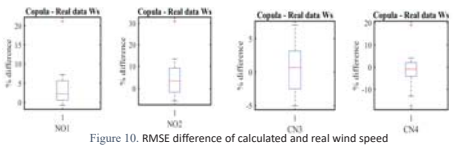

Figure 10. RMSE difference of calculated and real wind speed

- The RMSE calculated and shows the mean difference of lumped data is less than 5%.
- The upper tail is excluded from calculations.

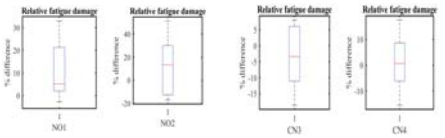

Figure 11. Fatigue damage difference between real and generated lumped data cause by wind speed

- Damage caused by each lumped loads calculated using formula (6) &(7)
- Maximum mean difference of data is less than 12%,

Conclusion and further research

- This research examines effectiveness of combining bivariate Copula of W_s and Hs at one location in the North Sea with wave heigth at other location to generate lumped wind speed
- Copula difference at stations close to each other shows an average difference of less than 10%. An increase in the distance of measurement locations show that the average copula difference is increased up to 15%.
- The average difference of real lumped data from copula generated lumped data is less than 5% which suggests lumped data are predictable using Copula.
- The average difference of fatigue damage by real lumped W_S from copula generated lumped W_S is less than 12%.
- The similarity of copula at different locations around the North Sea suggests that joint behaviour of wind speed and wave height in the North Sea is predictable using a same copula. Therefore, it is recommended to find a family of analytical copula that fits the joint behaviour of wind speed and wave height at the North Sea.

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