An approach to linear analysis of wind power plant dynamics, stability, and control

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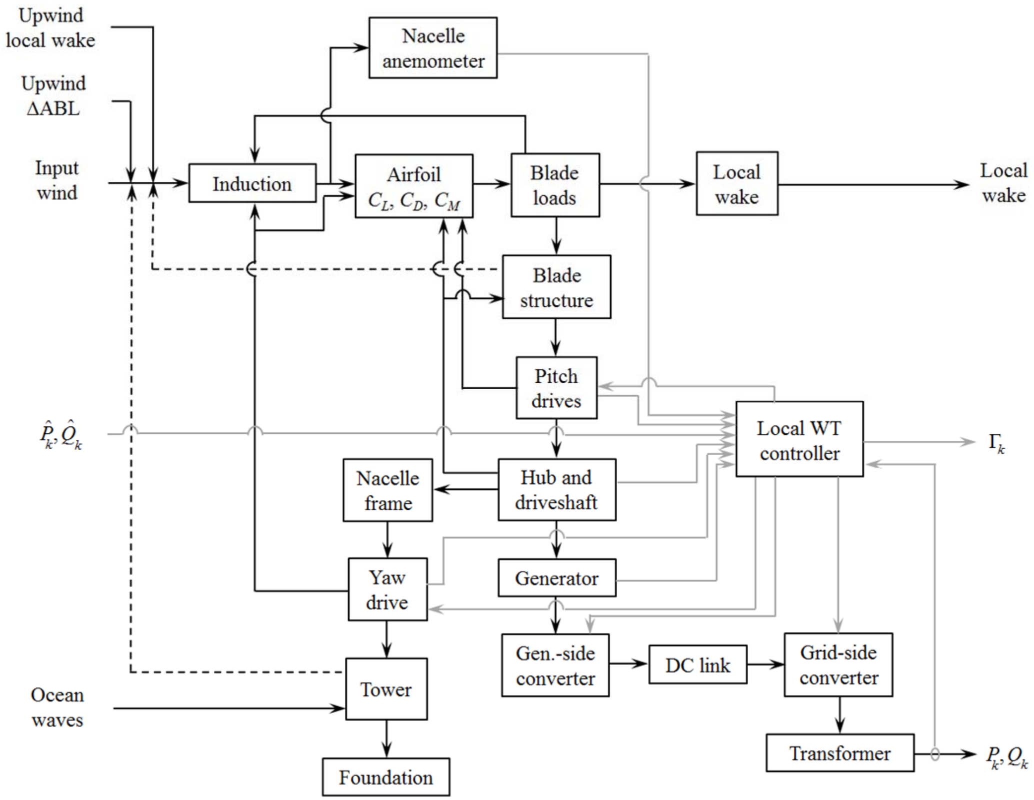
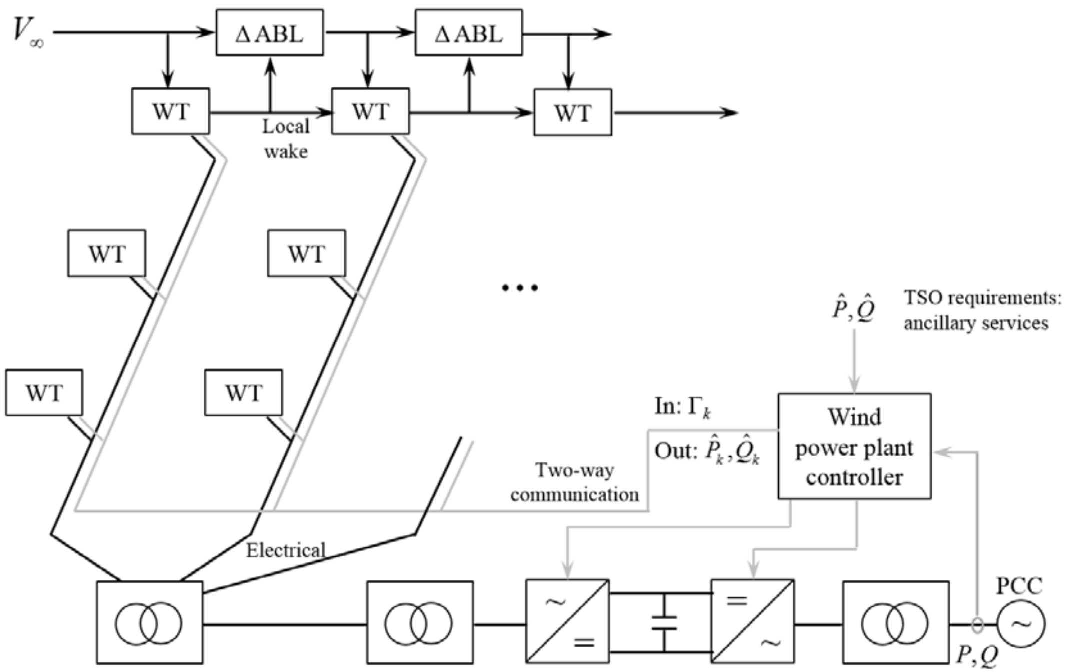
STAS WPP: Unified state-space model of a wind power plant





$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, \mathbf{u}, t)$$

$$\mathbf{y} = \mathbf{g}(\mathbf{x}, \mathbf{u}, t)$$

Nonlinear trajectory

$$\frac{d\mathbf{x}_1}{dt} = \mathbf{f}(\mathbf{x}_1, \mathbf{u}_1, t_1)$$

$$\mathbf{y}_1 = \mathbf{g}(\mathbf{x}_1, \mathbf{u}_1, t_1)$$

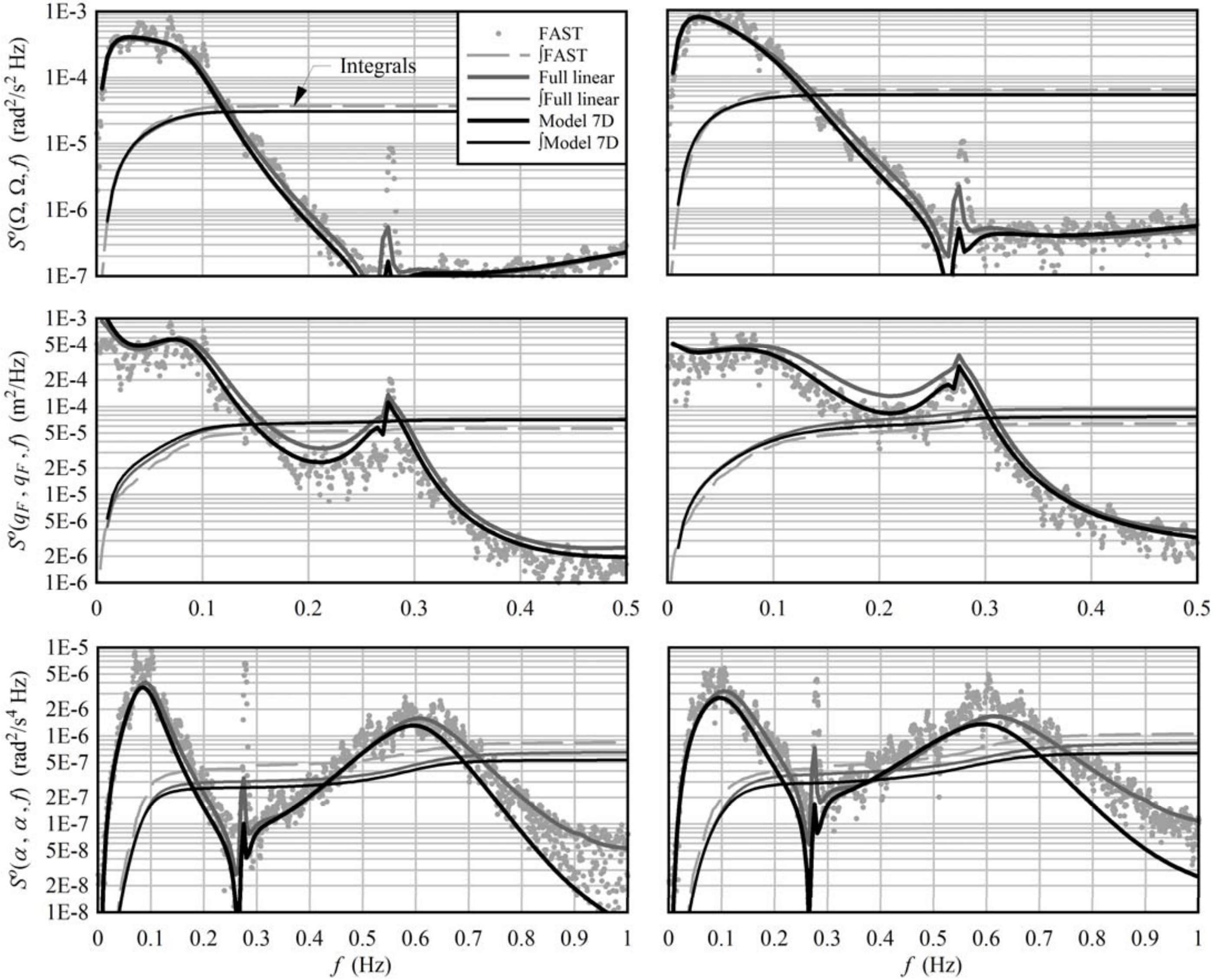
Initial condition

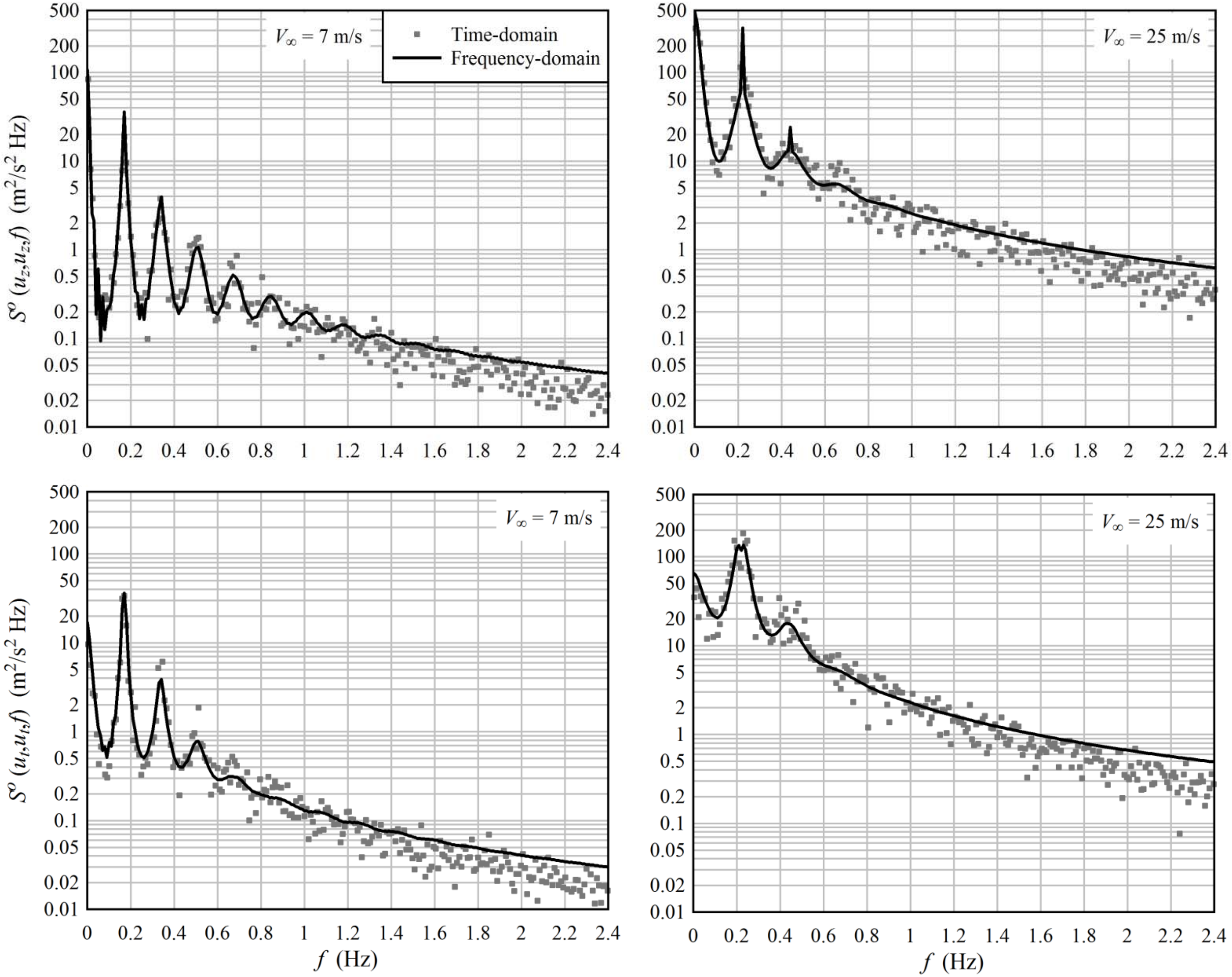
$$\frac{d(\delta\mathbf{x})}{dt} = \left.\frac{\partial\mathbf{f}}{\partial\mathbf{x}}\right|_1 \delta\mathbf{x} + \left.\frac{\partial\mathbf{f}}{\partial\mathbf{u}}\right|_1 \delta\mathbf{u} + \left.\frac{\partial\mathbf{f}}{\partial t}\right|_1 \delta t = \mathbf{A}\,\delta\mathbf{x} + \mathbf{B}\begin{bmatrix}\delta\mathbf{u}\\ \delta t\end{bmatrix}$$

$$\delta\mathbf{y} = \left.\frac{\partial\mathbf{g}}{\partial\mathbf{x}}\right|_1 \delta\mathbf{x} + \left.\frac{\partial\mathbf{g}}{\partial\mathbf{u}}\right|_1 \delta\mathbf{u} + \left.\frac{\partial\mathbf{g}}{\partial t}\right|_1 \delta t = \mathbf{C}\,\delta\mathbf{x} + \mathbf{D}\begin{bmatrix}\delta\mathbf{u}\\ \delta t\end{bmatrix}$$

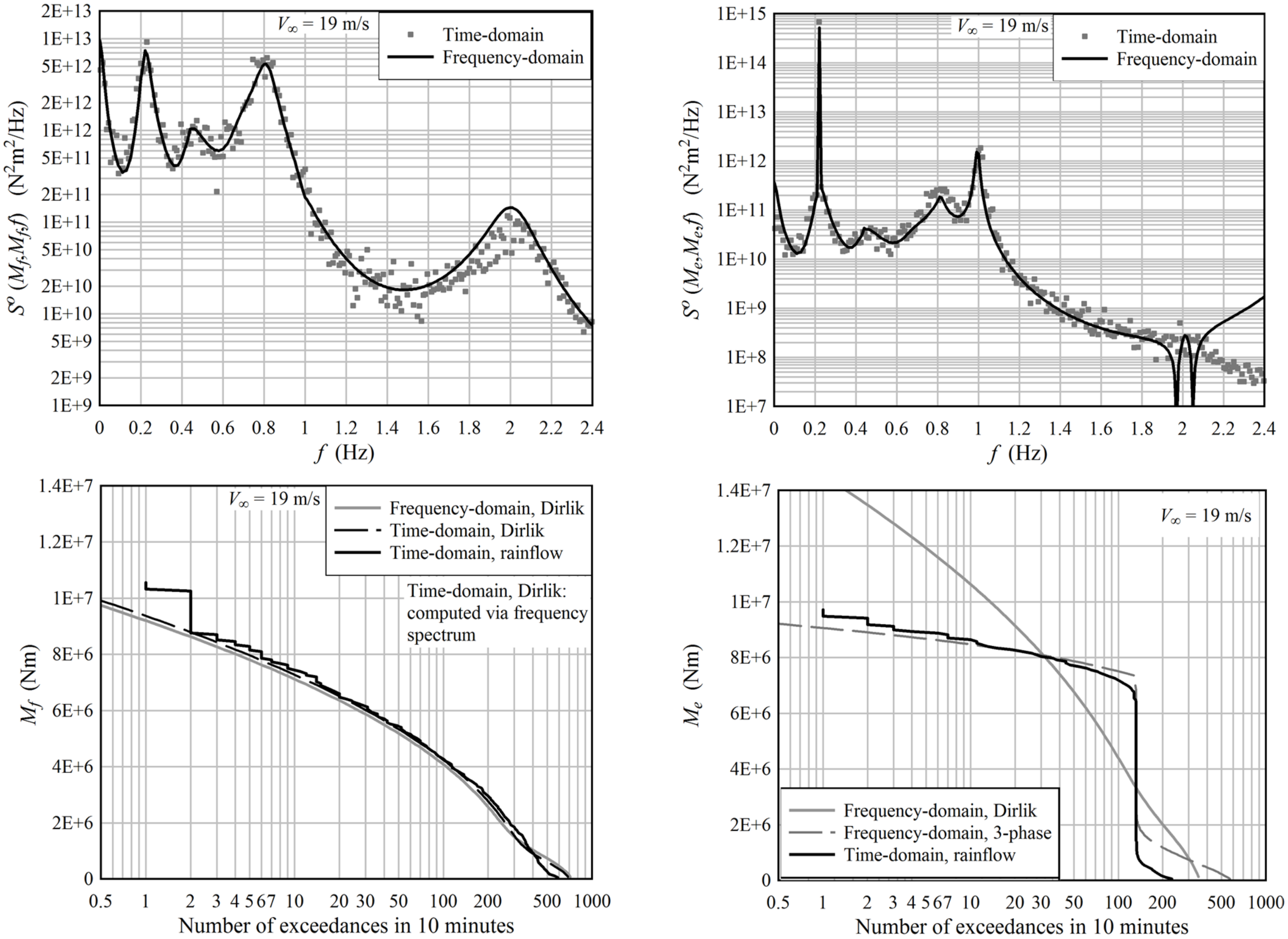
Perturbation

$\mathbf{x}_1$, $\mathbf{u}_1$, t_1 is an initial condition. It does not need to be an equilibrium point.


FAST
∫FAST
Full linear
∫Full linear
Model 7D
∫Model 7D
Integrals
$S^o(\Omega, \Omega, f)$ (rad^2/s^2 Hz)
$S^o(q_F, q_F, f)$ (m^2/Hz)
$S^o(\alpha, \alpha, f)$ (rad^2/s^4 Hz)
f (Hz)
f (Hz)

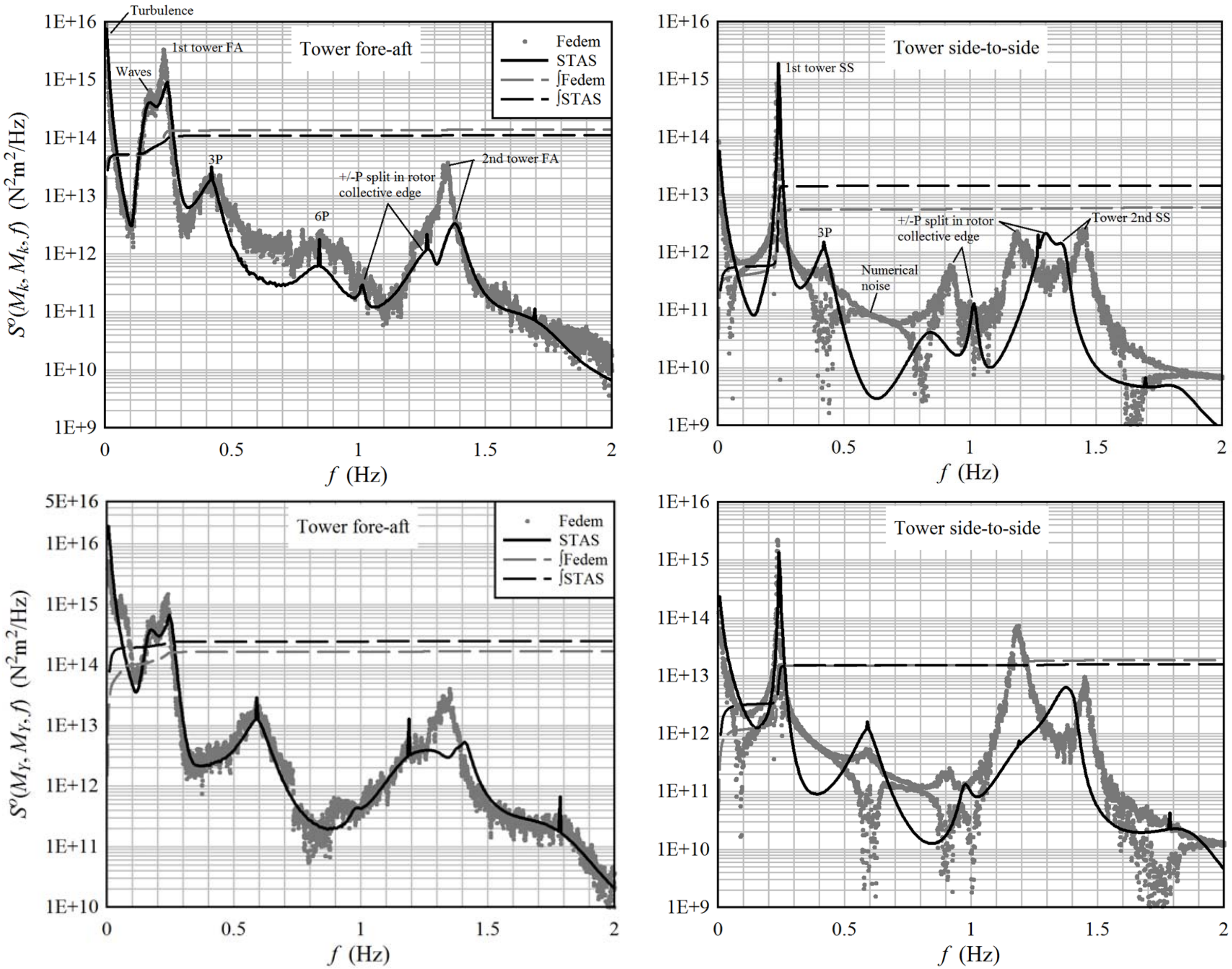


Time-domain
Frequency-domain
$V_\infty = 7$ m/s
$V_\infty = 25$ m/s
$V_\infty = 7$ m/s
$V_\infty = 25$ m/s
$S^o(u_z, u_z, f)$ (m^2/s^2 Hz)
$S^o(u_t, u_t, f)$ (m^2/s^2 Hz)
f (Hz)
f (Hz)


$V_\infty = 19$ m/s
Time-domain
Frequency-domain
$S^o\ (M_f, M_f, f)$ (N^{2}m^2/Hz)
f (Hz)
$S^o\ (M_e, M_e, f)$ (N^{2}m^2/Hz)
Frequency-domain, Dirlik
Time-domain, Dirlik
Time-domain, rainflow
Time-domain, Dirlik: computed via frequency spectrum
M_f (Nm)
Number of exceedances in 10 minutes
Frequency-domain, 3-phase
M_e (Nm)

Two different codes, models built by two different analysts.

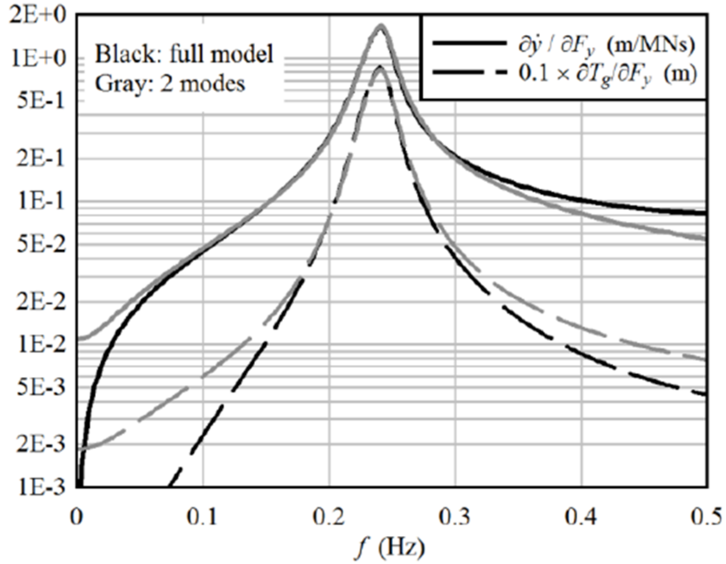
What is nonlinearity and what is modelling error?



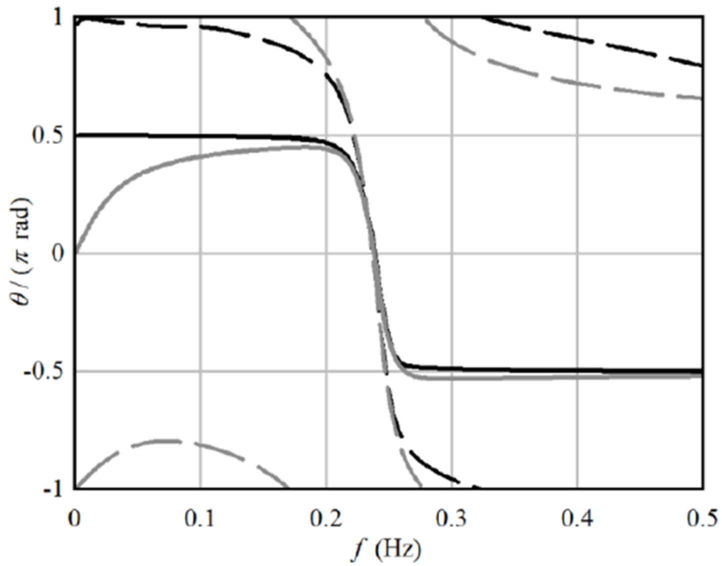
The new version of STAS provides nonlinear and linear equation sets that agree, at the point of linearization, to machine precision. Perturbed solutions will show explicitly the influence of nonlinearity.

	Tower side-to-side mode $\lambda = -0.049 \pm i\,1.508$		(Tower fore-aft mode) $\lambda = -0.095 \pm i\,1.557$		SS damping filter mode $\lambda = -0.090 \pm i\,1.449$	
States	$\lvert a\rvert$	θ/π	$\lvert a\rvert$	θ/π	$\lvert a\rvert$	θ/π
$\dot{q}_S$	1.000	0.000	0.244	-0.966	1.000	0.000
$\dot{q}_F$	1.230	0.428	1.000	0.000	1.487	-0.931
Ω	0.039	0.410	0.003	0.707	0.032	0.670
$\overline{\Omega}$	0.035	0.239	0.003	0.524	0.028	0.465
β	0.016	-0.344	0.005	-0.414	0.025	0.030
$\bar{v}_S$	1.432	-0.065	0.396	0.759	2.178	0.189
$\bar{v}_F$	1.695	0.579	2.565	0.020	1.780	-0.562
Outputs	$\lvert a\rvert$	θ/π	$\lvert a\rvert$	θ/π	$\lvert a\rvert$	θ/π
$\hat{P}_e$	8.048	-0.025	2.233	0.800	12.174	0.229
T_g	8.889	-0.092	2.405	0.753	13.182	0.175
P_e	8.034	-0.120	2.217	0.705	12.004	0.140
T_a	2.982	-0.657	2.565	0.826	3.809	0.016
F_T	0.134	-0.899	0.181	0.790	0.097	0.130

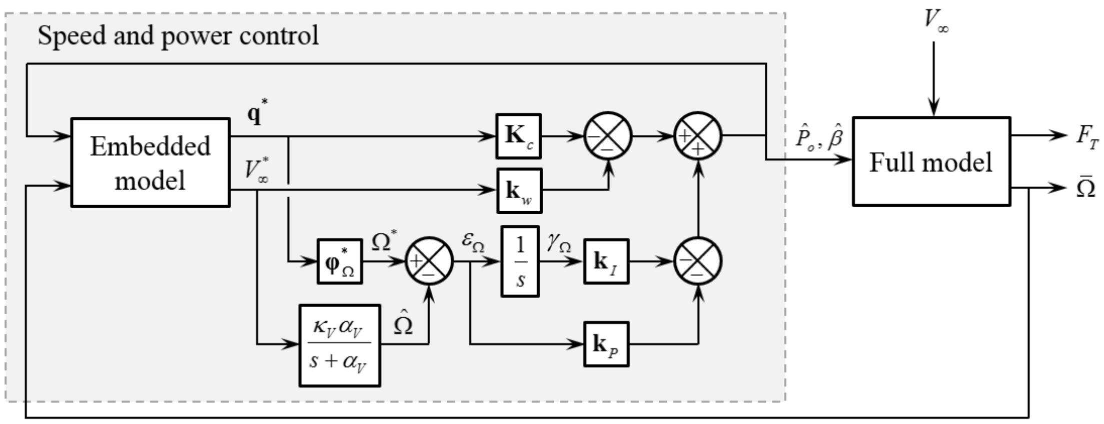
$\dot{q}_S$: Side-to-side velocity of the nacelle (m/s). $\dot{q}_F$: Fore-aft velocity of the nacelle (m/s). Ω: Rotor speed, measured at the generator (rad/s). $\bar{v}_S$: Band-pass filtered side-to-side nacelle velocity (m/s). $\hat{P}_e$: Commanded electrical power (MW). T_g: Generator air gap torque (MN). P_e: Electrical power at the network-side terminals of the wind turbine's transformer (MW).

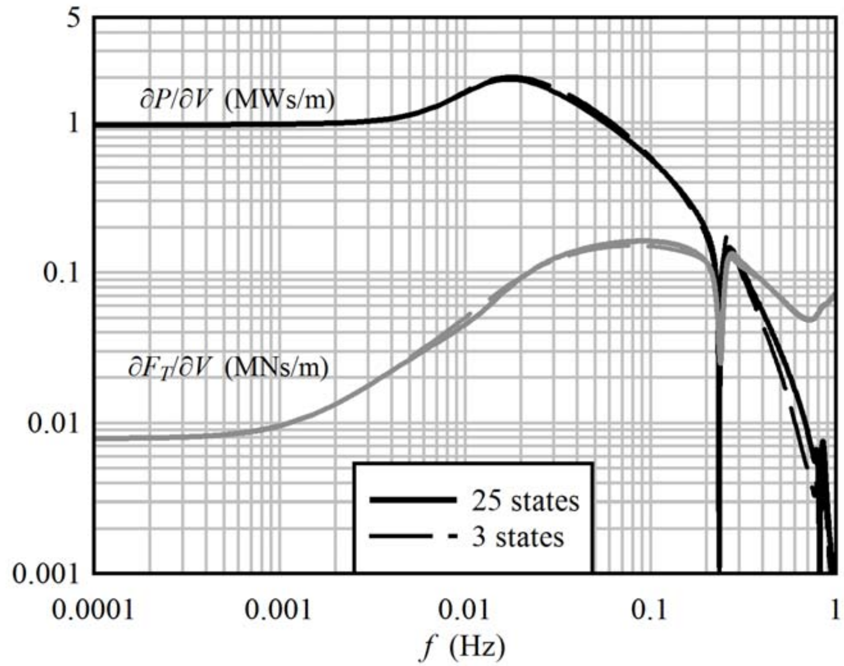


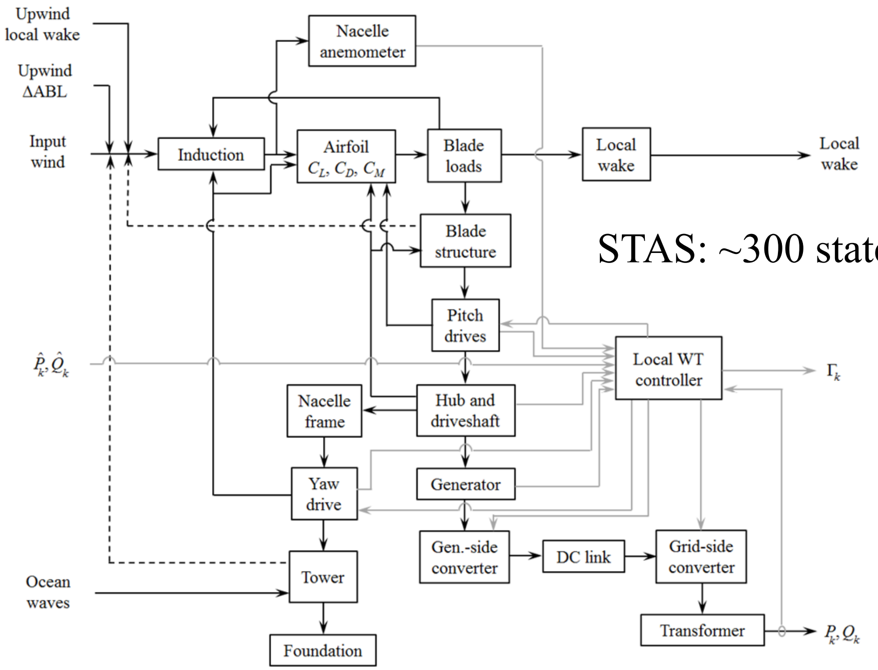
	Tower side-to-side mode		Tower fore-aft mode		Damping filter mode	
K_s (MN)	f_n (Hz)	ζ	f_n (Hz)	ζ	f_n (Hz)	ζ
0	0.236	0.007	0.245	0.052	0.236	0.100
1	0.237	0.011	0.245	0.054	0.236	0.094
2	0.237	0.016	0.245	0.055	0.235	0.087
5	0.240	0.028	0.247	0.059	0.231	0.069
10	0.244	0.030	0.251	0.056	0.224	0.065
40	0.247	0.036	0.263	0.033	0.207	0.068
5 + lead	0.240	0.032	0.248	0.061	0.231	0.062



Formal model reduction



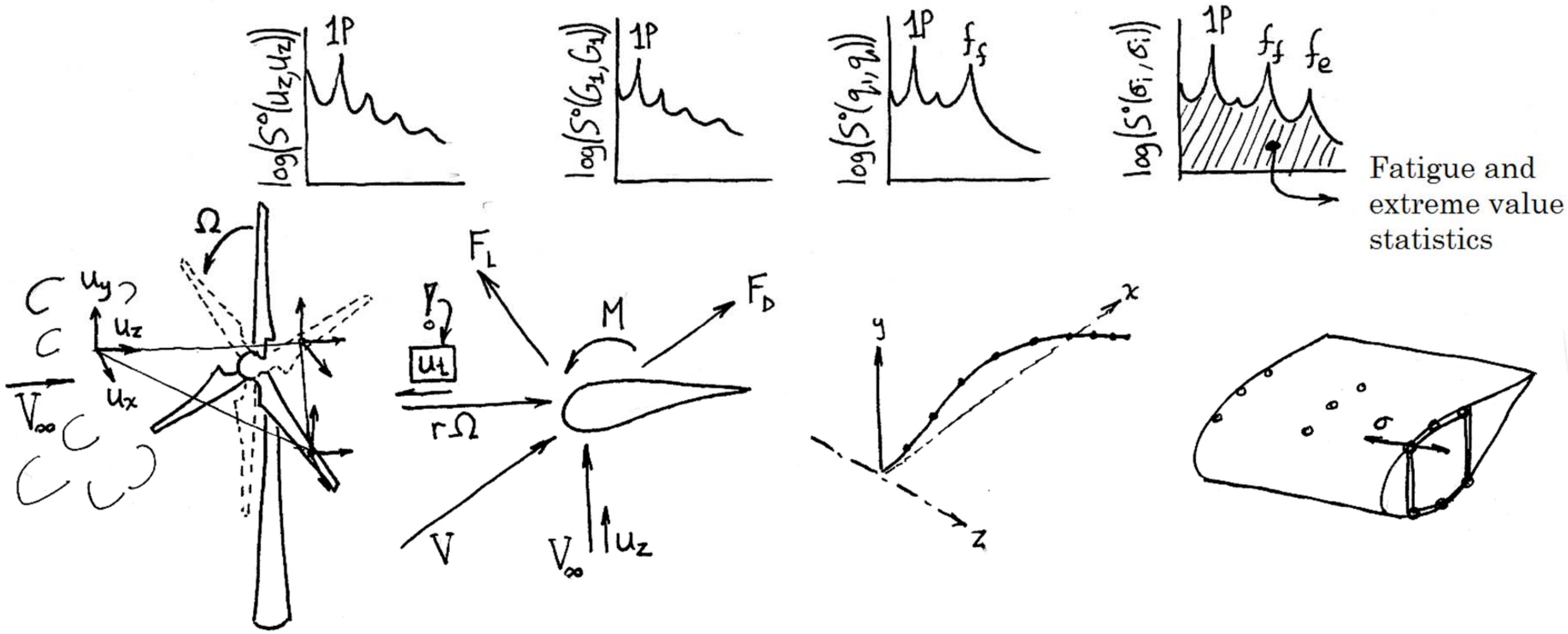




STAS: ~300 states

→ Modal reduction: 22 states → LQG optimal controller: 25 states → **Reduced-order controller: 3 states**

Starting high-resolution and reducing, we can check that we don't miss any important dynamics.



In the frequency domain, stochastic loads and fatigue cycle counts are numerically smooth and deterministic: no random numbers!

Tangent dynamics: applications in the optimization of wind power plants

$$\Pi = \int_0^T P(\mathbf{x}, \mathbf{u})\, dt \qquad \frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, \dots, t)$$

$$\Pi = \int_0^T P + \boldsymbol{\lambda}^T \left(\frac{d\mathbf{x}}{dt} - \mathbf{f} \right) dt$$

$$\delta_u \Pi = \int_0^T \left(\frac{\partial P}{\partial \mathbf{u}} - \boldsymbol{\lambda}^T \frac{\partial \mathbf{f}}{\partial \mathbf{u}} \right) \delta \mathbf{u} - \underbrace{\left(\frac{d\boldsymbol{\lambda}^{\mathbf{T}}}{dt} + \boldsymbol{\lambda}^T \frac{\partial \mathbf{f}}{\partial \mathbf{x}} - \frac{\partial P}{\partial \mathbf{x}} \right)}_{\text{Define } \boldsymbol{\lambda}} \delta \mathbf{x}\, dt + \boldsymbol{\lambda}^T \delta \mathbf{x} \Big|_0^T$$

$$\frac{d\boldsymbol{\lambda}}{dt} = -\left(\frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right)^T \boldsymbol{\lambda} + \left(\frac{\partial P}{\partial \mathbf{x}} \right)^T \qquad \frac{\partial P}{\partial \mathbf{u}} = \boldsymbol{\lambda}^T \frac{\partial \mathbf{f}}{\partial \mathbf{u}}$$

Evolution equation for the gradient of the cost with respect to some parameters **u**.