Reduced Order Modeling of lift characteristics of NACA0015 using van der Pol equation

M. Salman Siddiqui[1], Adil Rasheed[2], Trond Kvamsdal[1,2]

[1]NTNU, Dept. of Mathematical Sciences, NO 7491 Trondheim. [2]SINTEF ICT, Dept. of Applied Mathematics, NO 7465 Trondheim.

INTRODUCTION

The ability to accurately predict vortex shedding around wind turbine blades is paramount, particularly at high Reynolds number. We employed RANS approach with the use of three turbulence models (Spalart-Allmaras, k-ϵ and k-ω Shear Stress Transport model) to investigate the vortex shedding pattern on a NACA0015 airfoil. Spectral analysis is performed over the time history of aerodynamic coefficients to identify the dominant frequencies along with their even and odd harmonics. A reduced-order model based on van der Pol equation is proposed for the aerodynamic lift calculation. The model is also tested in a predictive setting, and the results are compared against the full order model solution.

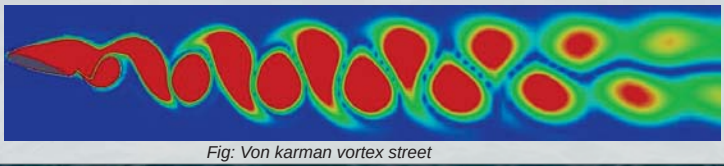


Fig: Von karman vortex street

METHODOLOGY

A multiblock approach has been adapted to allow more control over the generation of computational mesh. Quality orthogonal cells are clustered due to the presence of sharp gradients arising from the rapid changes in the flow physics on the surface and the wake region of the airfoil.

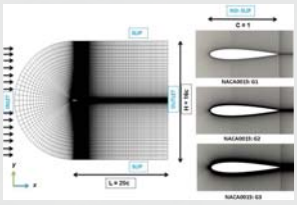


Fig: Mesh domain

No transverse flow distribution is observed, which is considered a prime reason for similar flow pattern in the third spatial dimension. Over the entire span of angle of attack, three-dimensional results consistently matched well with the two-dimensional predictions.

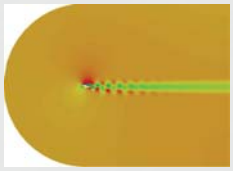


Fig: 2D simulation

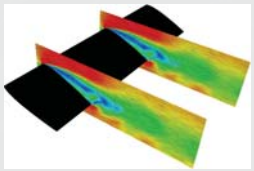
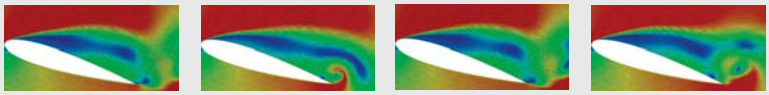
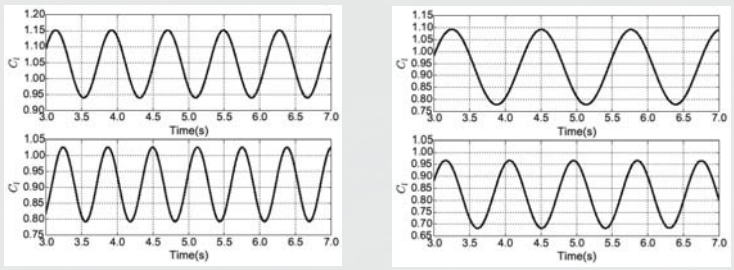


Fig: 3D simulation

RESULTS AND DISCUSSION



(a) (b) (c) (d)



k-ω SST turbulence model k-ϵ turbulence model

Fig: Lift coefficient oscillations

Van der Pol ROM model

Based on the high fidelity solution and spectral decomposition of the time history of coefficients a ROM is developed to model lift.

$$\ddot{C}_l + \varpi^2 C_l = \upsilon \dot{C}_l - \Gamma C_l \dot{C}_l - \varrho C_l^2 \dot{C}_l$$

Spectral analysis

Spectral analysis is performed on the time series of the aerodynamic list coefficient to extract the dominant frequencies. A strong quadratic and cubic couplings is observed in the frequency harmonics. The magnitude of the fundamental frequency at aoa 17 is 0.9 and 1.5 for k-ϵ and k-ω SST models respectively. The second harmonic is exhibited at the quadratic frequency of 1.8 and 3.0 ($f_s + f_s = 2\ f_s$), whereas cubic coupling of the frequency is seen at 3 f_s . Both models have shown distinct magnitudes and peaks for the fundamental frequency and its quadratic and cubic couplings.

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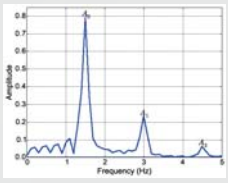


Fig: Frequency spectrum k-ω SST turbulence model

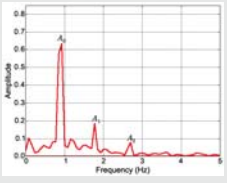
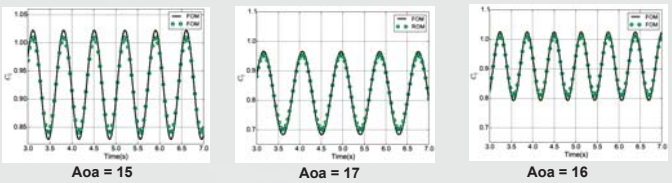


Fig: Frequency spectrum k-ϵ turbulence model

Fig : Frequency plots

Rom in predictive settings

The obtained result from ROM is compared with FOM. The proposed ROM model is further analyzed in a predictive setting to access its validity. Lift is computed at aoa = 16, using both high-fidelity simulation models and ROM approach.



Aoa = 15 Aoa = 17 Aoa = 16

Fig : ROM vs FOM

CONCLUSION

- Flow separation and vortex shedding pattern of NACA0015 is investigated at high Reynolds number over different angles of attack.
- Spalart-Allmaras, k- ϵ, k-ω Shear Stress Transport model turbulence models are investigated in two and three-dimensional spatial setting.
- Spectral analysis results show the even and odd frequencies harmonics in the temporal coefficients.
- A reduced-order model (ROM) of lift based on van der Pol equation is proposed.
- ROM model is tested in a predictive setting, and the results are compared against the full order model solution.

The authors acknowledge the financial support from the Norwegian Research Council and the industrial partners of the FSI-WT-project (216465/E20) and NOWITECH-project (Grant No.:193823/S60)

Contact: muhammad. siddiqui@math.ntnu.no