Fast divergence-conforming reduced order models for flow

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Problem: Repetitive solutions of parametrized flow problems (see left) can be quite demanding, each solution involving up to $10^5$ degrees of freedom and hours or days of computational time.

Answer: Reduced Order Modelling (ROM) offers solutions with lower accuracy but dramatic speedups. When tied to a divergence-conforming high-fidelity method, the gains can be even greater.

Problem specifics
High fidelity simulations of stationary Navier-Stokes were performed of flow around a NACA0015 airfoil with chord length of 1 m. The inflow velocity $u_\infty$ varied from 1 to 20 m/s, and the angle of attack $\varphi$ varied from $-35$ to $35^\circ$. The viscosity was fixed at $1\times10^{-5}$. Snapshots were evaluated at the $15\times15$ Gauss points on the parameter domain, and reduced models created with $N = 10, 20, \ldots, 50$ degrees of freedom.

The system matrix (size $2N$) will usually have a rank-deficient velocity-pressure block (VP, indicated with dashed lines). Enriching the velocity space with so-called supremizers ensures a full-rank system matrix with size $3N$. A divergence-conforming method will produce a fully divergence-free basis, so the VP-block vanishes, giving a block-triangular system, solvable as two size-$N$ systems instead of one size-$3N$ system.

**Discussion**

- ROMs are able to deliver results within two to three orders of magnitude at dramatic speedups.
- Divergence-conforming ROMs can deliver higher speeds, up to one order of magnitude faster in the present examples, by exploiting specific properties of the velocity bases.