Stochastic Particle Trajectories in the Wake of Large Wind Farm

M Bakhoday Paskyabi, A. Valinejad
Geophysical Institute, University of Bergen, Bergen, Norway

Introduction

The main goal of this study is to investigate pollutant diffusions with carrier flow and their temporal-spatial evolution in the wake regions of a large wind farm. The important feature of current study can be explained by its ability to focus on non-linear interactions between farm, passive tracers, and surface gravity waves by the means of the stochastic diffusion. Here, we specify a wind farm with a characteristic length, L, and assuming an analytical 2D U-shaped wake profile based on educated knowledge of wind deficit behind farm. For the numerical simulation, we modify 2D shallow water wave equations by including wave breaking and wave-current interaction effects. With progressive wave energy evolution and stochastic wave orbital motions, we solve Lagrangian equations of motions for pollutants. Then, we compare the particle trajectories in the wind-generated symmetrical range-dependent dipoles to highlight the temporal-spatial tendency of passive tracers with and without wave forcing with those calculated by vanishing farm contribution. Results also confirm the role of stochastic modeling of pollutants to capture more realistically the underlying physics by reducing the related uncertainties, especially during the strong oceanic upwelling and downwelling that influence marine life strongly, by bringing colder, nutrient rich water to the surface zone that there is enough light to provide appropriate conditions for growing and reproduction of phytoplankton.

Large Wind turbine and Wind Stress

- **By vertical integrating momentum and continuity equations in the presence of wave effect, the following differential equations are obtained**

$$\frac{\partial(uh)}{\partial t} - f(v+vs) + \mathbf{F}_{ds}^{x} = -\frac{\partial(u^2h+0.5gh^2)}{\partial x} - \frac{\partial(uvh)}{\partial y} + \frac{1}{\rho_w}\left(\tau_x - \tau_x^w - \tau_B^x\right)$$

$$\frac{\partial(vh)}{\partial t} + f(u+us) + \mathbf{F}_{ds}^{y} = -\frac{\partial(uvh)}{\partial x} - \frac{\partial(v^2h+0.5gh^2)}{\partial y} + \frac{1}{\rho_w}\left(\tau_y - \tau_y^w - \tau_B^y\right)$$

$$\frac{\partial h}{\partial t} + \left(\frac{\partial(uh)}{\partial x} + \frac{\partial(vh)}{\partial y}\right) = 0$$

- **where u and v are the mass transports in the x and y directions, By assuming a thin layer of fluid with density ρ_0 and thickness *h* overlying a deep, motionless abyssal layer, assuming constant wind and wave characteristics, and by ignoring bottom friction and wave-induced momentum redistribution term $\mathbf{F}_{ds}$, we can obtain another simplified expression.**

Finite Volume Technique

The conservation form of Eq. (1) can be written as

$$\frac{\partial\theta}{\partial t} + \frac{\partial F(\theta)}{\partial x} + \frac{\partial G(\theta)}{\partial y} = S(t)$$

where source term is given as

$$\mathbf{S}(t) = \frac{1}{\rho_w}\begin{bmatrix} 0 \\ \tau_x - \tau_{in}^x - \tau_B^x \\ \tau_y - \tau_{in}^y - \tau_B^y \end{bmatrix} + \begin{bmatrix} 0 \\ f_{cor}(v+v_s) - \mathbf{F}_{ds}^x \\ -f_{cor}(u+u_s) - \mathbf{F}_{ds}^y \end{bmatrix}$$

We use Lax-Friedrichs technique as a member of finite volume (FV) to discretize homogenous version of Eq. (3) as

$$\boldsymbol{\theta}_{i,j}^{n+1} = \boldsymbol{\theta}_{i,j}^{n} - \frac{\Delta t}{\Delta x}\left(\mathbf{F}_{i+\frac{1}{2},j}^{n+\frac{1}{2}} - \mathbf{F}_{i-\frac{1}{2},j}^{n+\frac{1}{2}}\right) - \frac{\Delta t}{\Delta y}\left(\mathbf{G}_{i,j+\frac{1}{2}}^{n+\frac{1}{2}} - \mathbf{G}_{i,j-\frac{1}{2}}^{n+\frac{1}{2}}\right)$$

in which

$$\mathbf{F}_{i+\frac{1}{2},j}^{n++\frac{1}{2}} = \frac{\mathbf{F}(\boldsymbol{\theta}_{i,j}^{n}) + \mathbf{F}(\boldsymbol{\theta}_{i+1,j}^{n})}{2} - \frac{1}{2}\left|\lambda\left(\frac{\boldsymbol{\theta}_{i,j}^{n} + \boldsymbol{\theta}_{i+1,j}^{n}}{2}\right)\right|\left(\boldsymbol{\theta}_{i+1,j}^{n} - \boldsymbol{\theta}_{i,j}^{n}\right)$$

λ **is non-linear advection speed. The external force is imposed to technique by following ordinary differential equation**

$$\frac{\partial\boldsymbol{\theta}}{\partial t} = \mathbf{S}(t)$$

Stochastic Lagrangian Particle Trajectories

The particle trajectory can be estimated using stochastic differential equation as

$$dX_t = f(X_t, t)\,dt + dW_t = \varepsilon f(X_t, t)\,dt + dW_t$$

where f is the orbital , W_t is the Brownian motion with $dW_t \sim N(0, \sigma^2 dt)$
$\varepsilon = kA/\omega$, and $dW_t \sim N(0, \sigma^2 dt)$

If $(x(t), y(t))$ is the particle trajectory

$$\frac{dx}{dt} = u, \qquad \frac{dy}{dt} = v,$$

a perturbation series of X_t for the small parameter ε:

$$X_t = X_t^{(0)} + \varepsilon X_t^{(1)} + \varepsilon^2 X_t^{(2)} + O(\varepsilon^3)\,.$$

Numerical Results

- **In this study for constant wind and wave the following analytical expression is proposed [1,3]:**

$$\Lambda = \Lambda_{init} - \Delta\Lambda_{*}\,\mathbf{P}(X, Y)$$

in which *X* and *Y* show the horizontal axes, Λ is wind-wave forcing vector, $\Delta\Lambda_*$ is wind-wave forcing fluctuation, and $\mathbf{P}$ gives the distribution of forcing behind wind farm. Wind and wave forcing are determined based on introduced shape function (Fig. 1) [3].

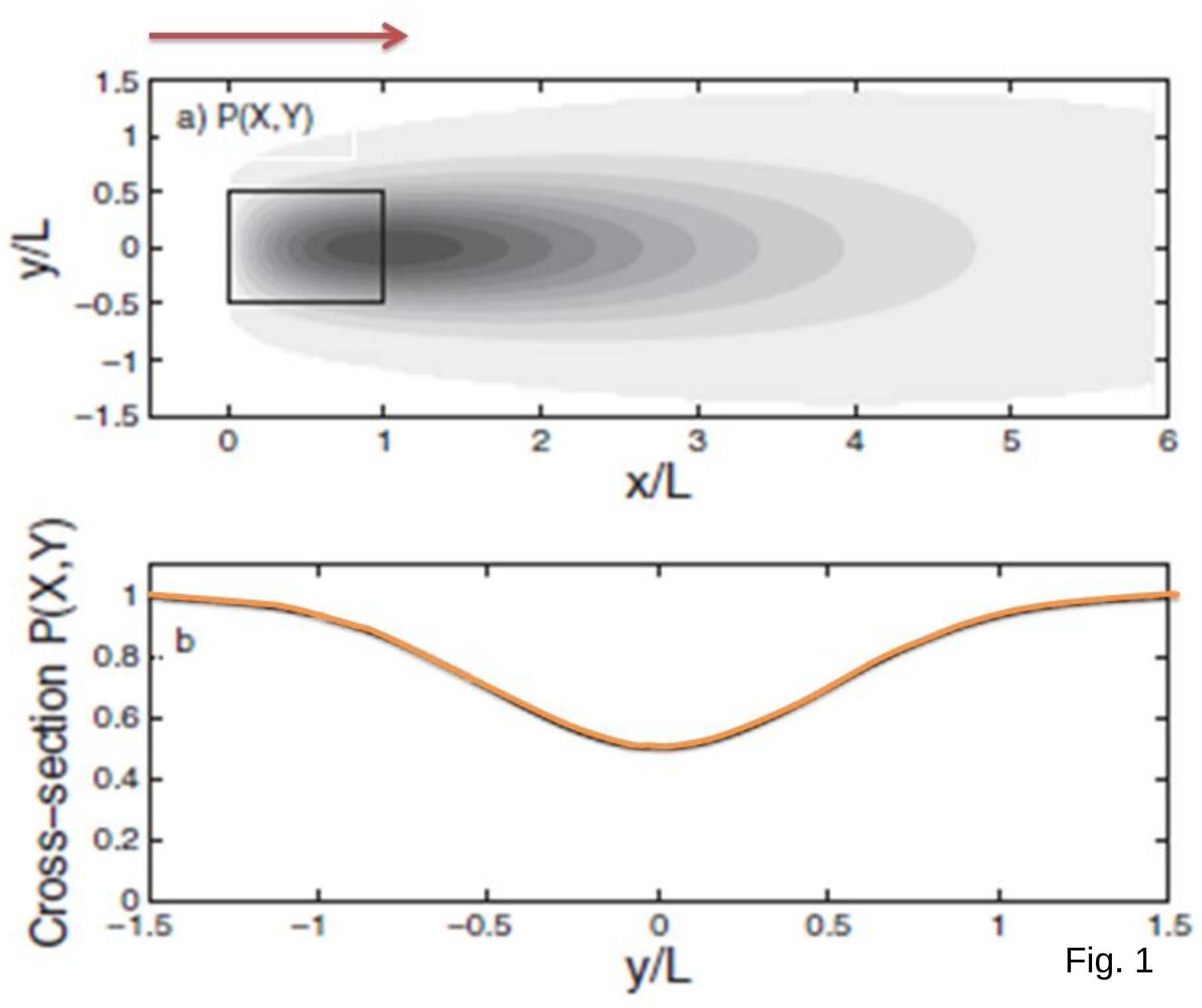

Fig. 1

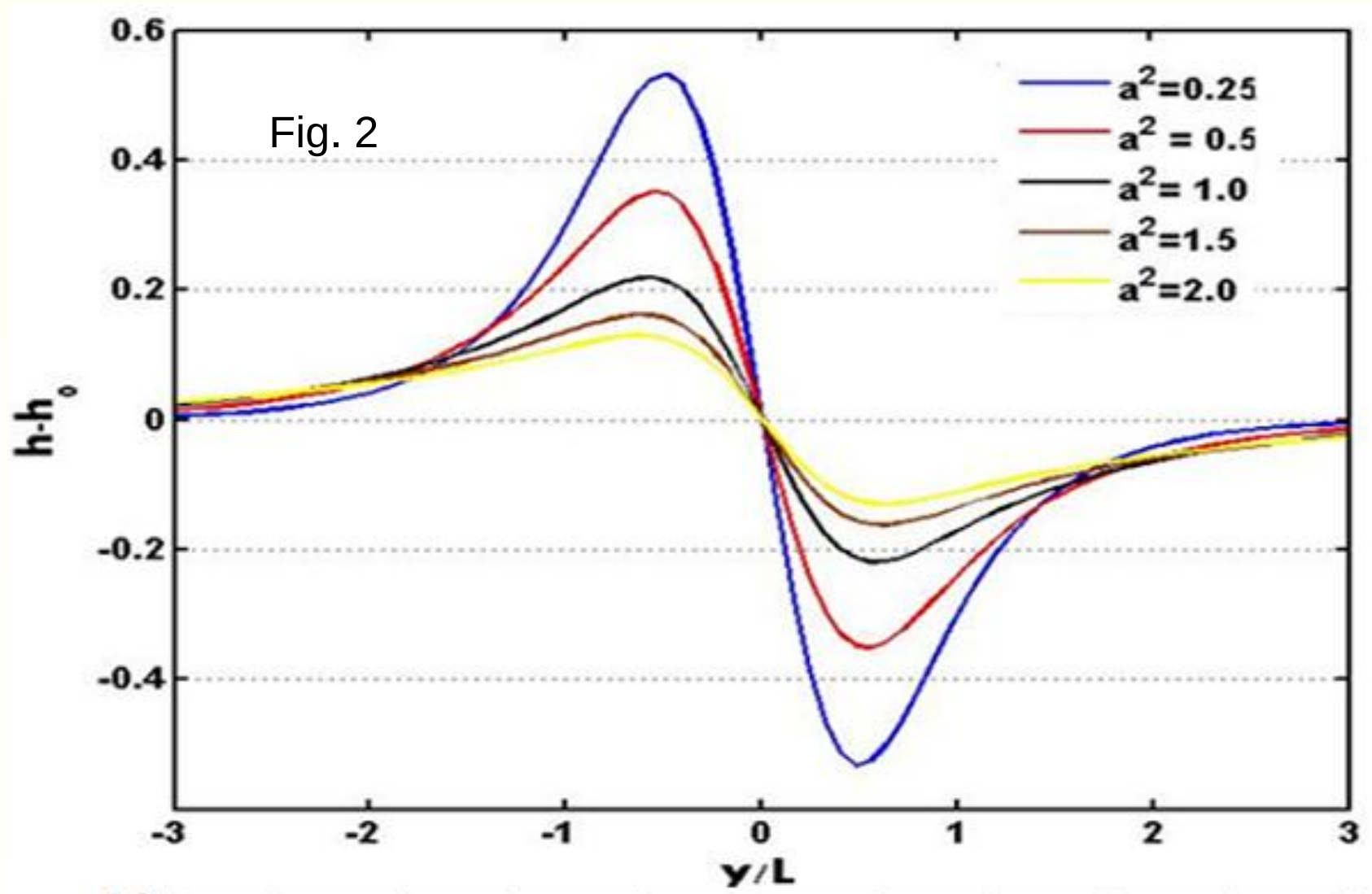

Fig. 2

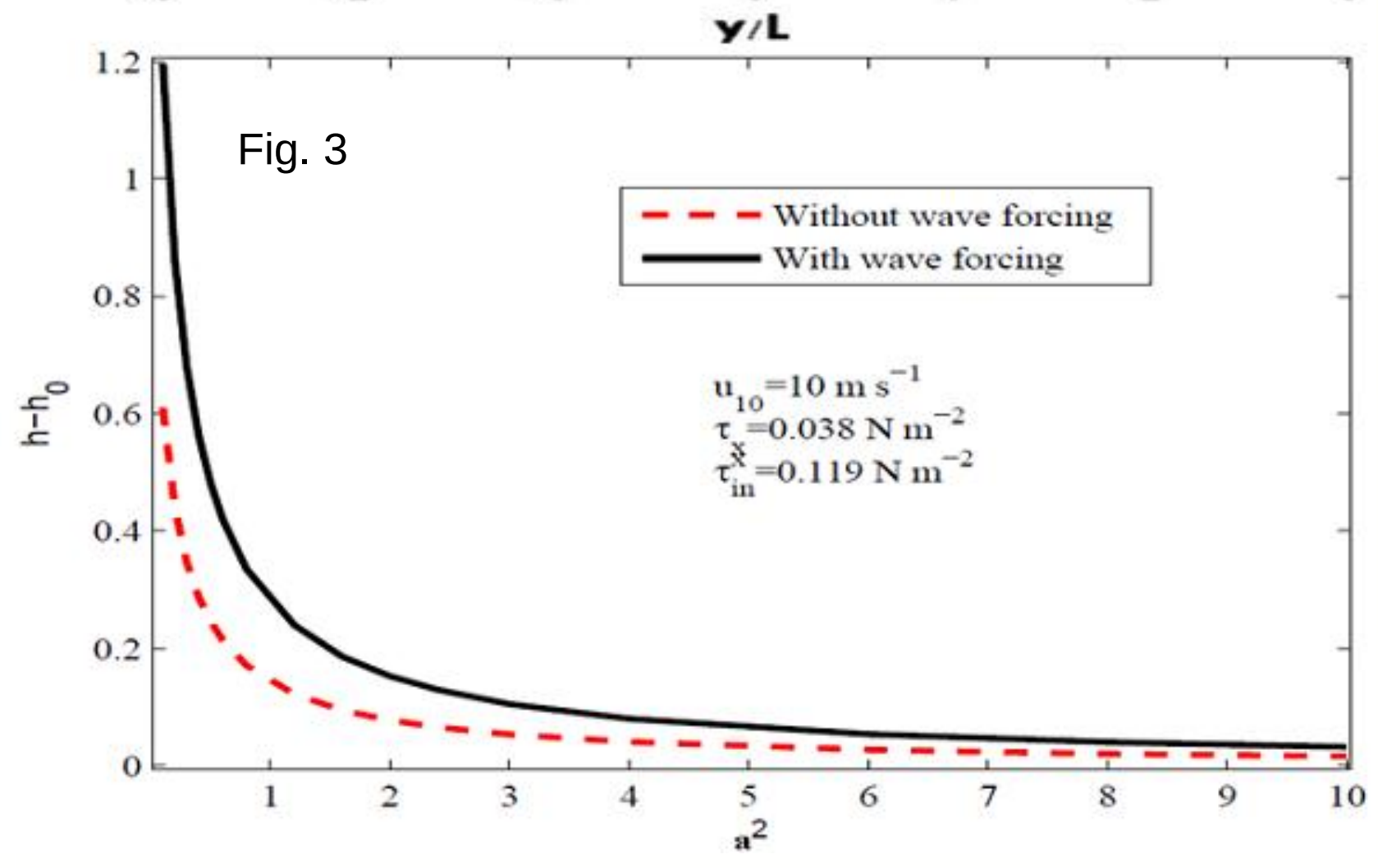

Fig. 3

- **In fact, this parameter states how large the internal deformation radius is compared to the size of wind turbine farm (Fig. 2). The maximum value of pycnocline and the strength of upwelling as a function of *a* is shown in figure 3. It can be seen that the amplitude of response decreases rapidly with *a* that highlights the role of physical size of wind wake in upper ocean response [3].**

- **Figure 4 shows the rising of pycnocline in the southern side of wind farm and corresponding falling due to geostrophic adjustment on the northern side. Further, including wave effect modifies ocean response by larger amplitude of pycnocline height.**

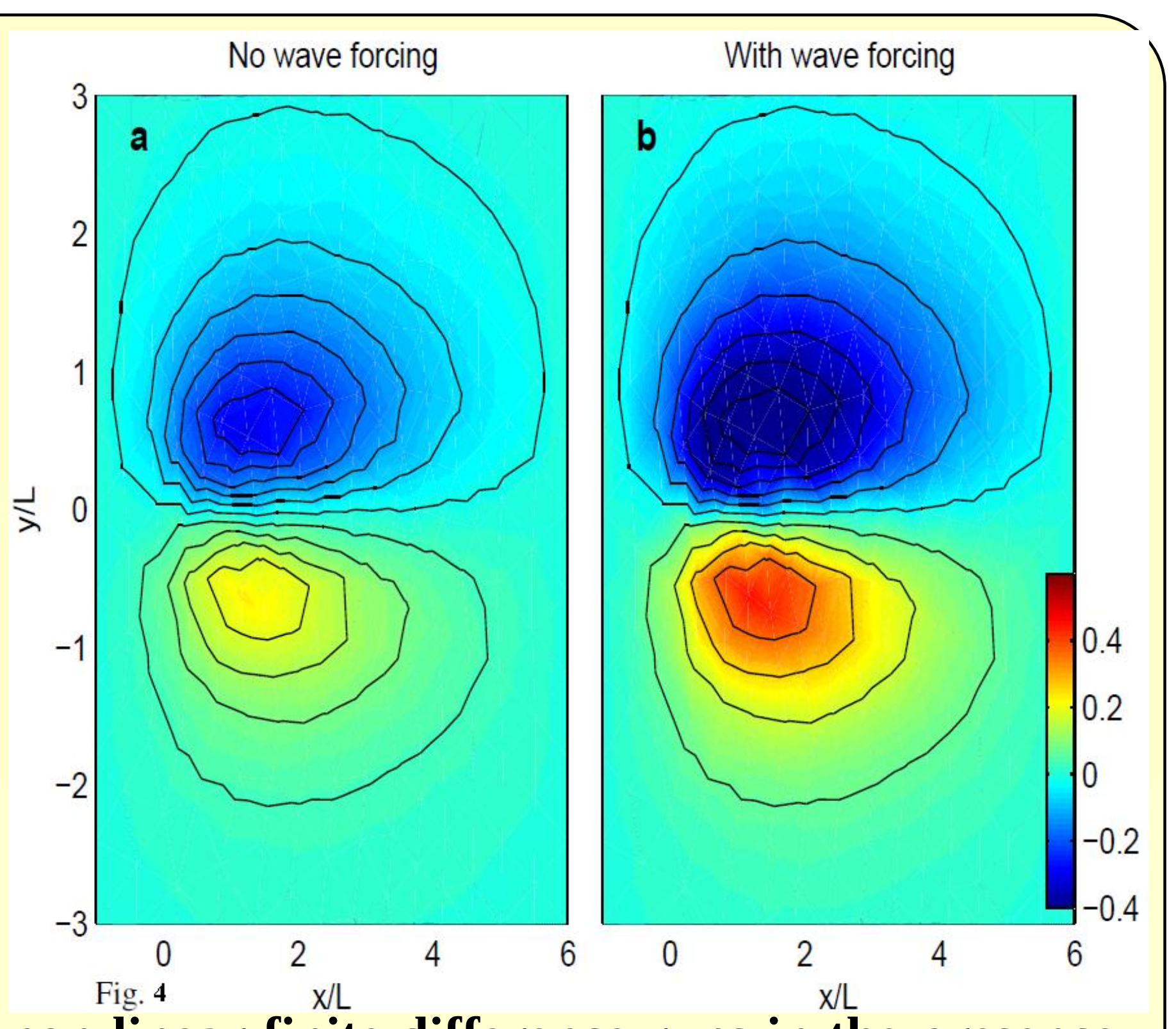

Fig. 4

- **Here, we consider 5000 stochastic Particles.**

Figures 5 and 6 shows the linear FV runs, non-linear finite difference runs in the presence of bottom friction and advection term, and ROMS model results [3].

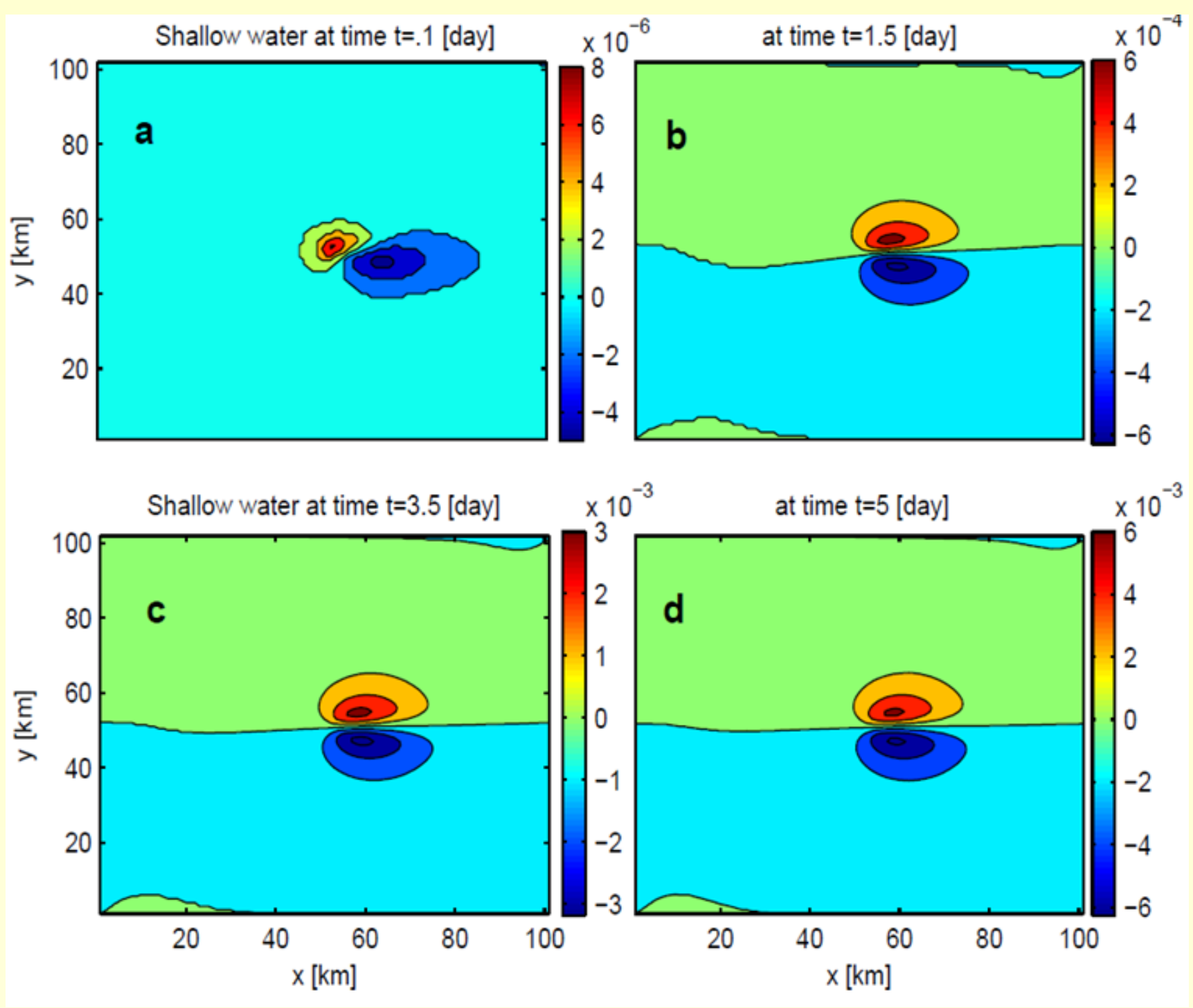

Fig. 5

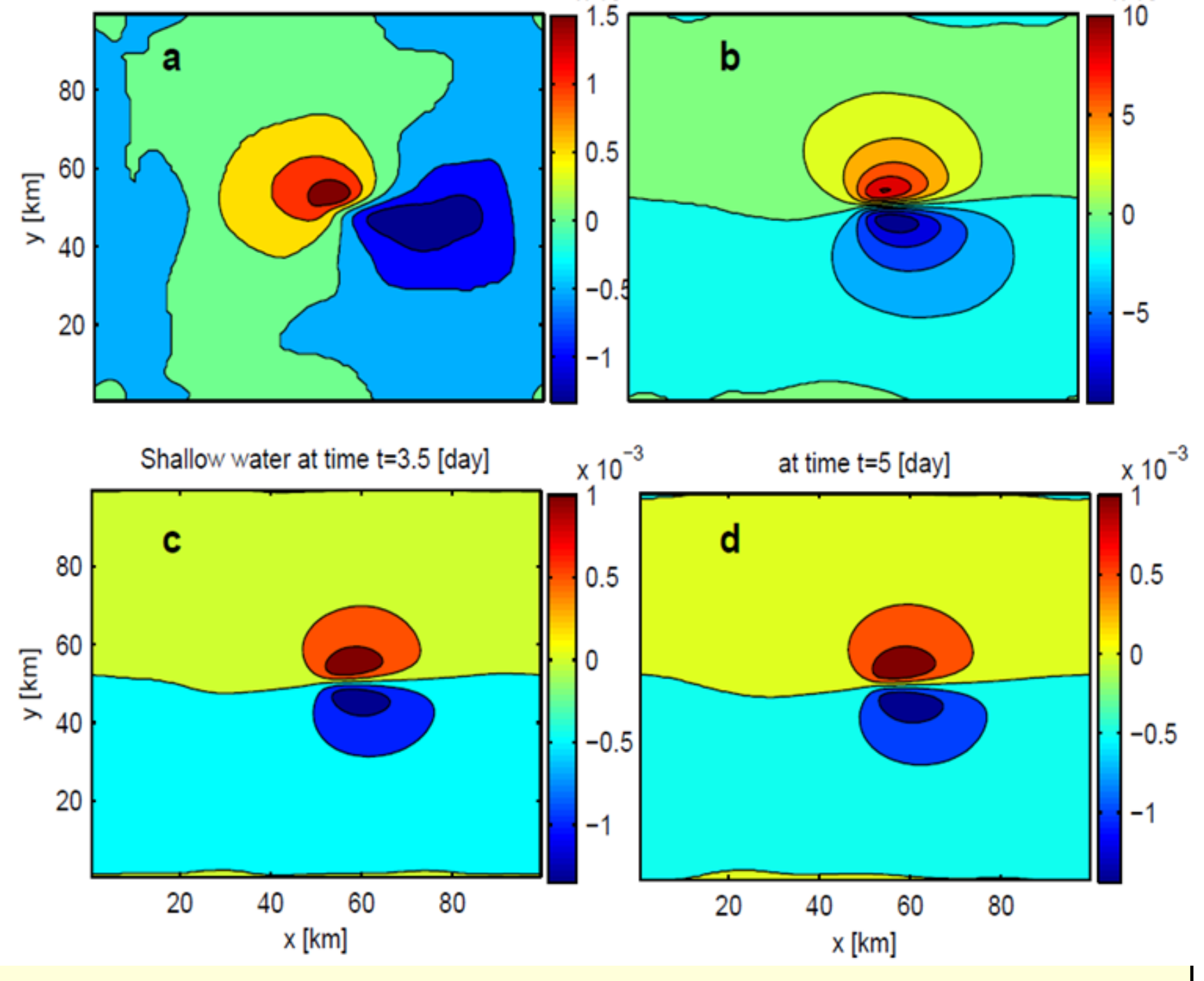

Fig. 6

- **Temporal and spatial evolutions of Particles are shown in Fig. 7 for the scenario presented in Figs 8 and 9. To highlight the particles' trajectories in more details, we marked a moving particle for t=0, 1.5, 3.5, and 5 days, Respectively.**

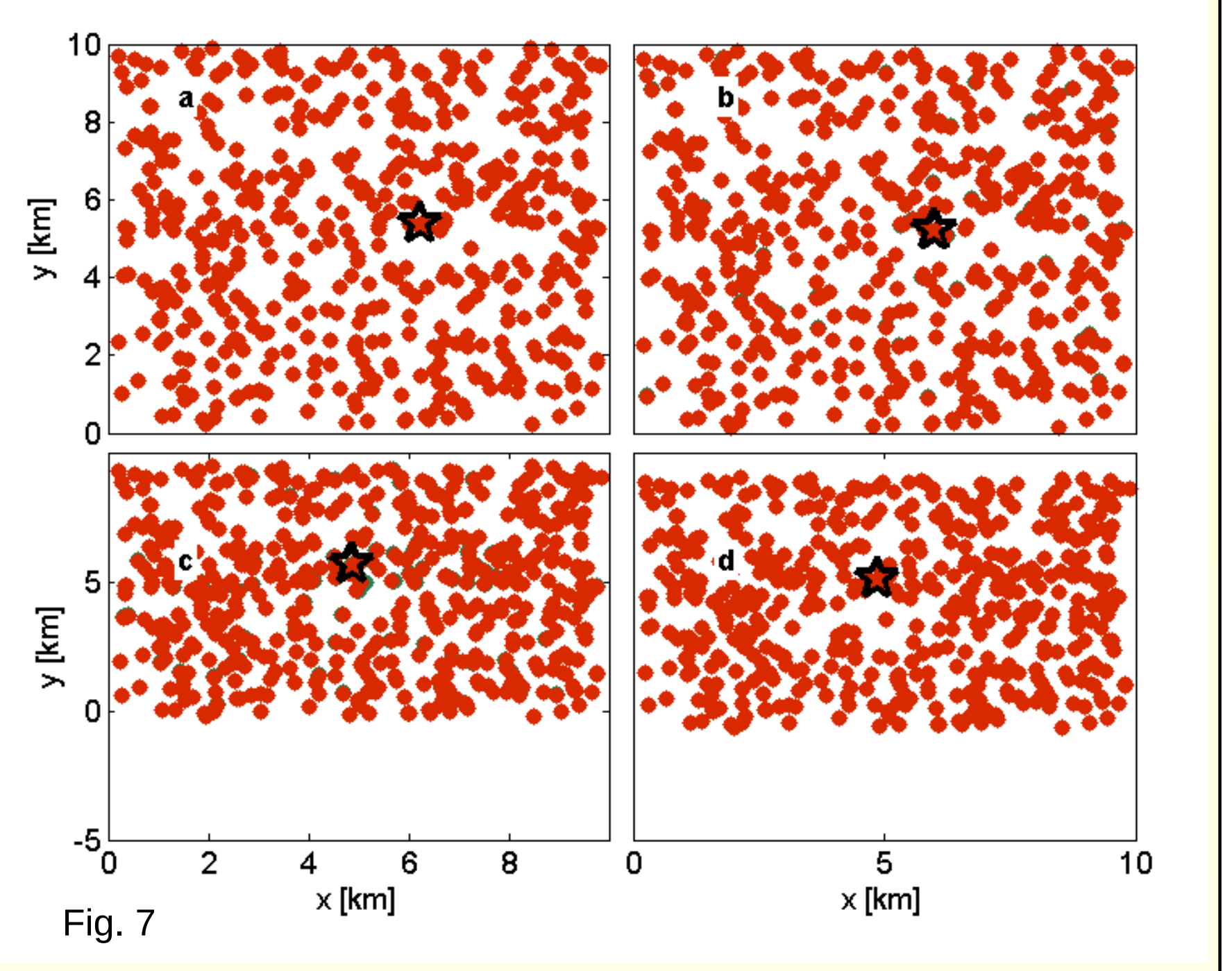

Fig. 7

- **In Figs. 8 and 9, we show the trajectorie 4 stochastic particles in the presence of**

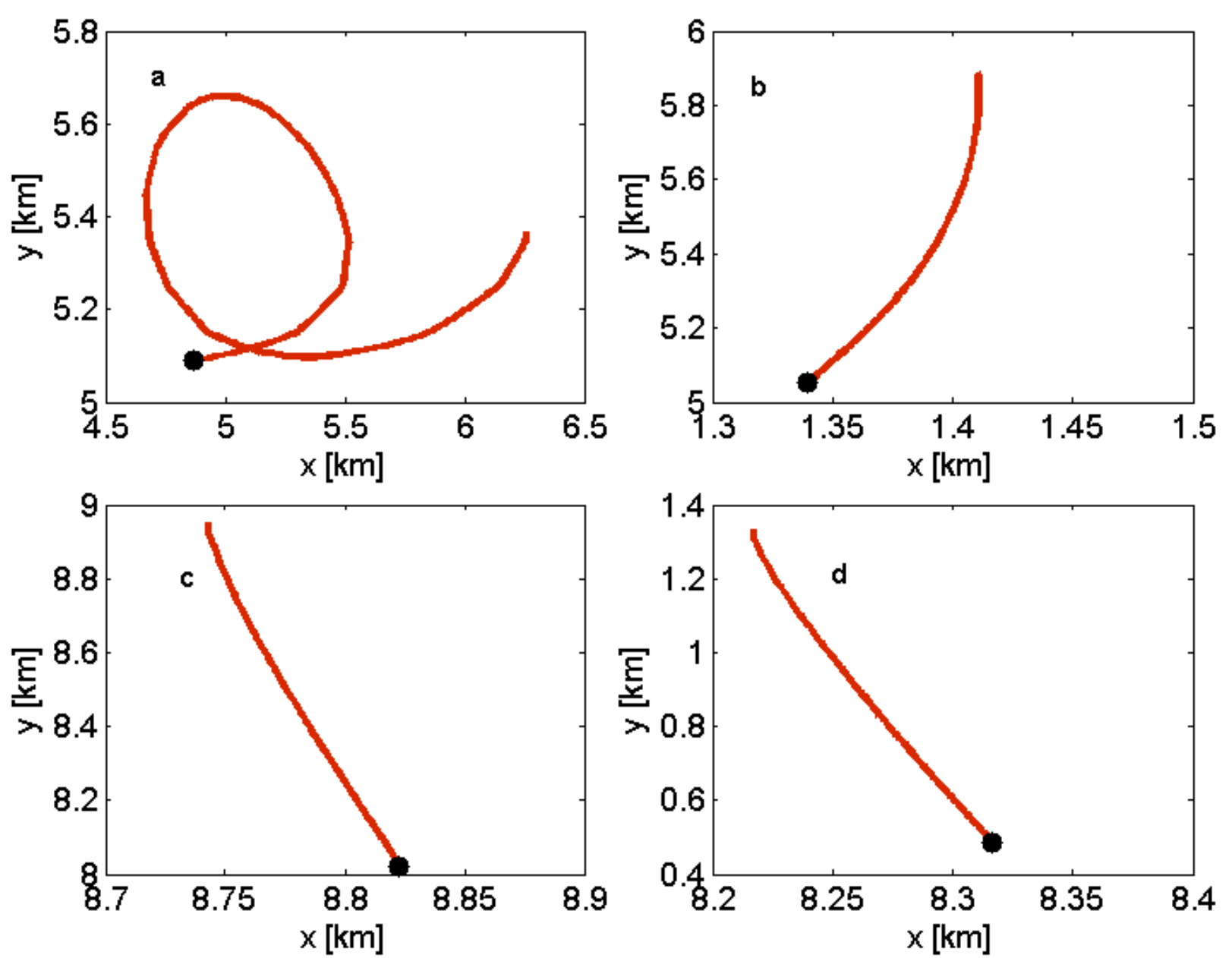

Fig. 8

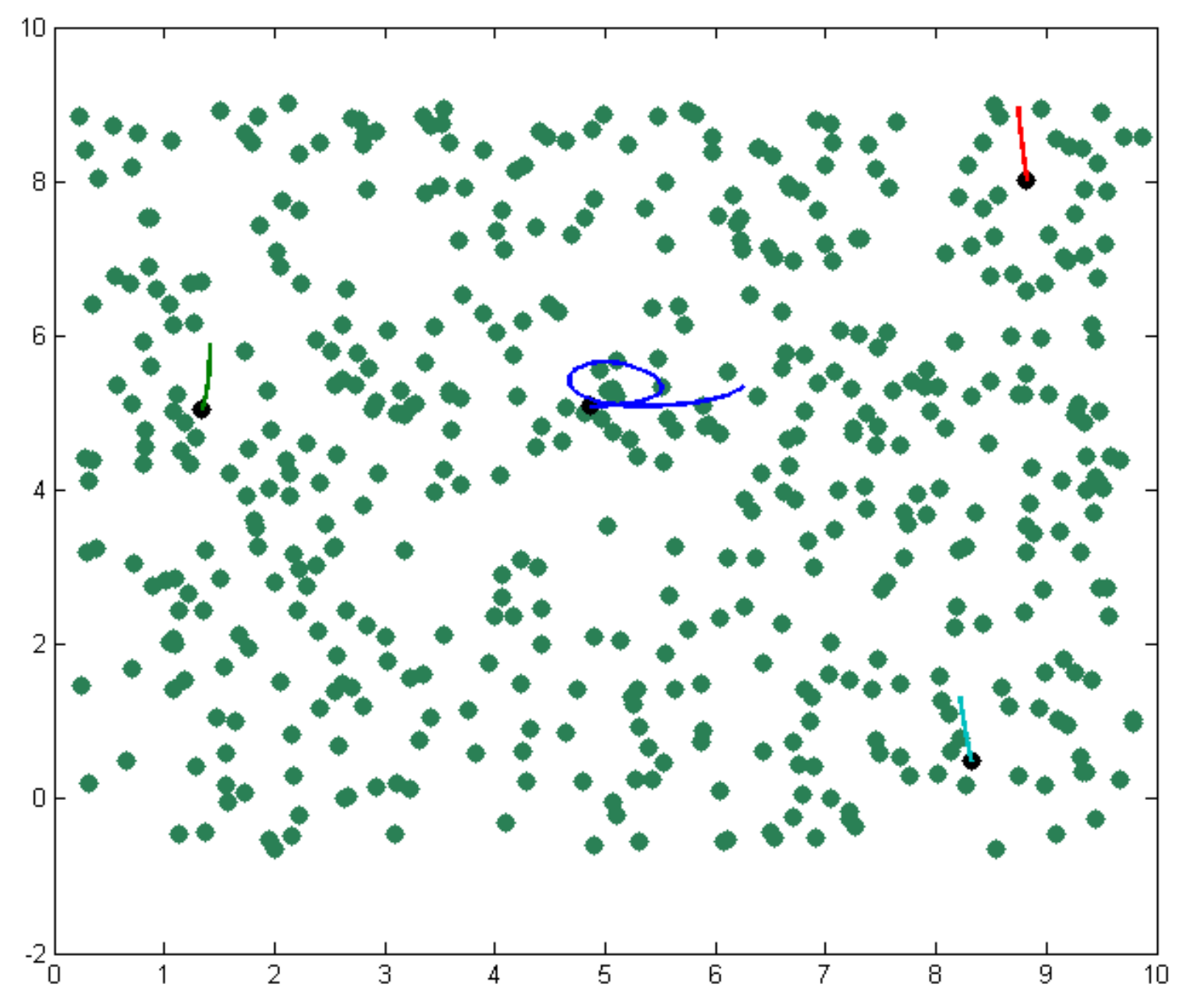
Fig. 9

trajectories have been shown in Fig. 8. These Figures show that wind farm modify the tracers trajectories, especially at the center of diploes.

Acknowledgement

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Summary

Growing the offshore wind industry necessitates investigations of different aspects of interaction between large wind farms and atmosphere, as well as ocean. Regarding to the later, upper ocean reveals direct but slow response to the wake strength and vertical extent of wind profile behind farm. All kind of variations in the atmospheric forcing conditions influence the wake pattern and structure downstream of wind farm and increase complexity of studying interaction between upper ocean and large farm. Among different issues of interest about this interaction, environmental effects of wind farm getting more important as a result of continual technological advances in design, installation, maintenance, and transport of energy from wind farm to power markets.

We showed that the max amplitude of pycnocline height with the wave effect is greater than that in no-wave case and this height approach to zero when *a* goes to infinity in both cases. Including non-linear term, horizontal diffusion, and the bottom friction led to decreasing of the strength of eddies. But, the amplitude of disturbances in the lee regions of the farm becomes weaker after almost three days. Furthermore, the wind turbine effects on the passive tracers have been studied in terms of stochastic lagrangian technique. We showed introductory results of these interactions suggesting small contribution of wind farm in distribution of surface particles. The results are preliminary and we plan to further study this interaction.

References

[1] Brostrom G. (2008): On the influence of large wind farms on the upper ocean circulation, *Journal of Marine Systems*, 74, 585-591.
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