Collaborative Research Center SFB559
Modeling of Large Logistic Networks
Project M8 - Optimization

Rich Vehicle Routing Problems – Challenges and Prospects in Exploring the Power of Parallelism

Andreas Reinholz

1st COLLAB Workshop

Andreas Reinholz
Optimization Consulting

Structure

- Motivation and context
- Metaheuristics as Iterative Variation Selection Procedures
- Elementary and composed Neighborhood Generating Operators
- Modeling concepts and constraint handling
- Neighborhood Generating Operators for Vehicle Routing Problems
- Acceleration techniques and efficient data structures
- Decomposition methods
- Closer to the real world: Modeling uncertainty, flexibility and risk
- Conclusions and outlook

Features and Challenges of Logistics Optimization Tasks

Mixed – Integer Optimization Problems with

- Various **constraints**
- **Multiple objectives**
- Range from **Strategic Planning** to **Online-Optimization**
- **Open** or **Disturbed Systems**, imprecise or incomplete data, noise
- **Dynamic Optimization** tasks with moving optima
- **Hierarchies** of complex optimization problems
- Integration in “**Interactive Decision Support Systems**”
- Evaluation model could be a **Simulation Model** or a “**Black Box**”

Andreas Reinholz Optimization Consulting

Metaheuristics

- Neighborhood Search (NS)
- Variable Neighborhood Search (VNS)
- Iterative Local Search (ILS)
- (Recursive) Iterative Local Search (R-ILS)
- Tabu Search (TS)
- Greedy Randomized Adaptive Search Procedure (GRASP)
- Evolutionary Algorithms (EA)
- Ant-Systems, Particle Swarm, …
- Scatter Search
- Adaptive Memory Programming
- Estimation of Distribution Algorithms (EDA)
- Multiple Agent Systems
- Stochastic Local Search (SLS)
- …

The 10 commandments for powerful Hybrid Metaheuristics

1. …
2. …
3. …
4. …
5. …
6. …
7. …
8. …
9. …
10. You shall covet your best competitors procedures, methods and strategies, break them into parts and use them as Local Search.

Andreas Reinholz Optimization Consulting

Scheme of an Iterative Variation Selection Procedure (IVS)

Initialization

…

REPEAT

    Select Candidate Solutions for Modification

    …

    Modify Candidate Solutions

    …

    Select Candidate Solutions for further Iterations

    …

UNTIL Stopping Criteria(GNr, LastImprovingGNr, Threshold, …);

IVS: Horizontal and Recursive Composition

```
IVS( RecLevel, NS_Set, IVS_ParaSet )
Initialization
…
REPEAT
        FOR (HLevel = 0) TO GetMaxHLevel(…) DO
                Select Candidates for Modification
                Modify Candidates
                IVS( RecLevel-1, NS_Set, IVS_ParaSet )
                …
                Select Candidates for further Iterations
UNTIL Stopping Criteria( GNr, LastImprovingGNr, Threshold, Level… );
```

Variation Operators

Systematic modification of decision variables

- **Deterministic** principals
- **Stochastic** principals
- **Local** view (i.e. modify only few variables at each step)
- **Global** view (i.e. Tree Search)
- **Construction**, **destruction** and **modification** schemes
- **Decomposition** strategies (hierarchical, geographical, functional)
- **Combined** or **composed** variation operators (i.e. VNS, Mutation)

Neighborhood Generating Operators

- Elementary Neighborhood Generating Operator =
  Systematic parameterized modification of decision variables
  - One Step Neighborhood
  - Neighborhood Transition Graph (NH-Transition Graph)
  - (Asymmetric) Distance measure, metric
- Neighborhood Search templates
  - Steepest ascent
  - Next ascent
- K-Step Neighborhood
  - Local optima of quality K (iterative or recursive scheme)
  - Discrepancy Search, Local Branching
  - Rapid-Tree Search, Rapid-B&B
  - Probabilistic K-Step Neighborhood (i.e. Mutation-Operator)

Multiple Solution Variation Operators (Recombination)

- Recombination Operator =
  Parents define a subspace or a subset of the search space

  - Standard Crossover = **Randomly selected point** in this subset
  - **Series of points** in this subset using a NH-Transition Graph
    - **Deterministic** principles
      - **Connecting path** between parents (with discrepancies)
      - **Enumerate** the complete subset
      - **Deterministic Sub-Problem Solver**
    - **Probabilistic** principles
      - **Re-Sampling** or **Random Walk**
      - **Connecting random path** (with discrepancies)
      - **Probabilistic Sub-Problem Solver**

Combining IVS Procedures

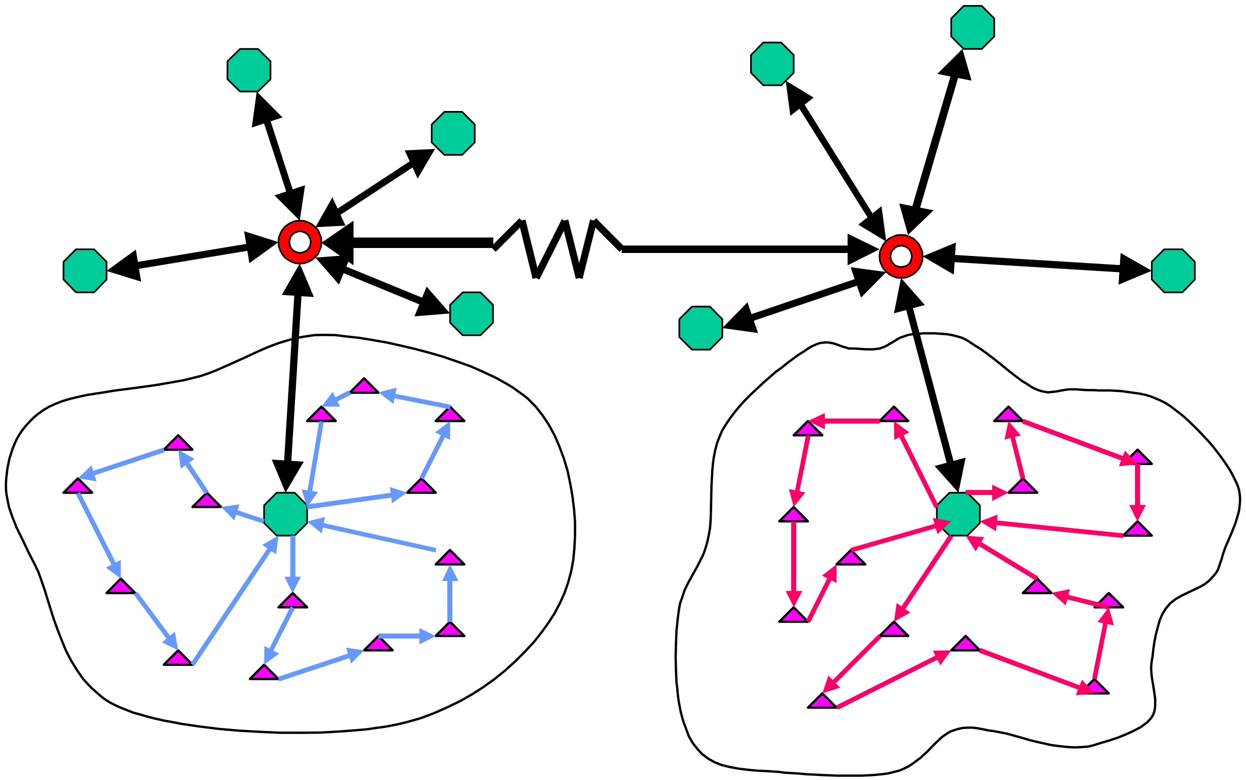
- Variable Neighborhood Search
  - Fixed sequence
  - Probabilistic sequence
  - Adaptive or self-adaptive sequence
- Evolutionary Algorithms
  - Mutation Operator (Probabilistic K-Step Neighborhood)
  - Crossover Operator (Dynamic Sub Problem Search)
- Hybrid Evolutionary Algorithms
  - i.e. Hybrid (1+1) EA = Iterative Local Search
- Multi - Start Metaheuristics
  - Number of runs vs. number of iterations (Multi Start Factor)

ARopt
Andreas Reinholz
Optimization Consulting

Aspects of Iterative Variation Selection Procedures

- Problem specific representation
- Problem specific variation operators
- (Variable) Neighborhood Search techniques
- Accelerated Delta Evaluation of the objective function
- Efficient data structures
- Dynamic Adaptive Decomposition strategies (DADs)
- Biased disruption strategies
- Adaptive or self-adaptive search control
- Population Management

Transportation Logistics: Sub-Problems

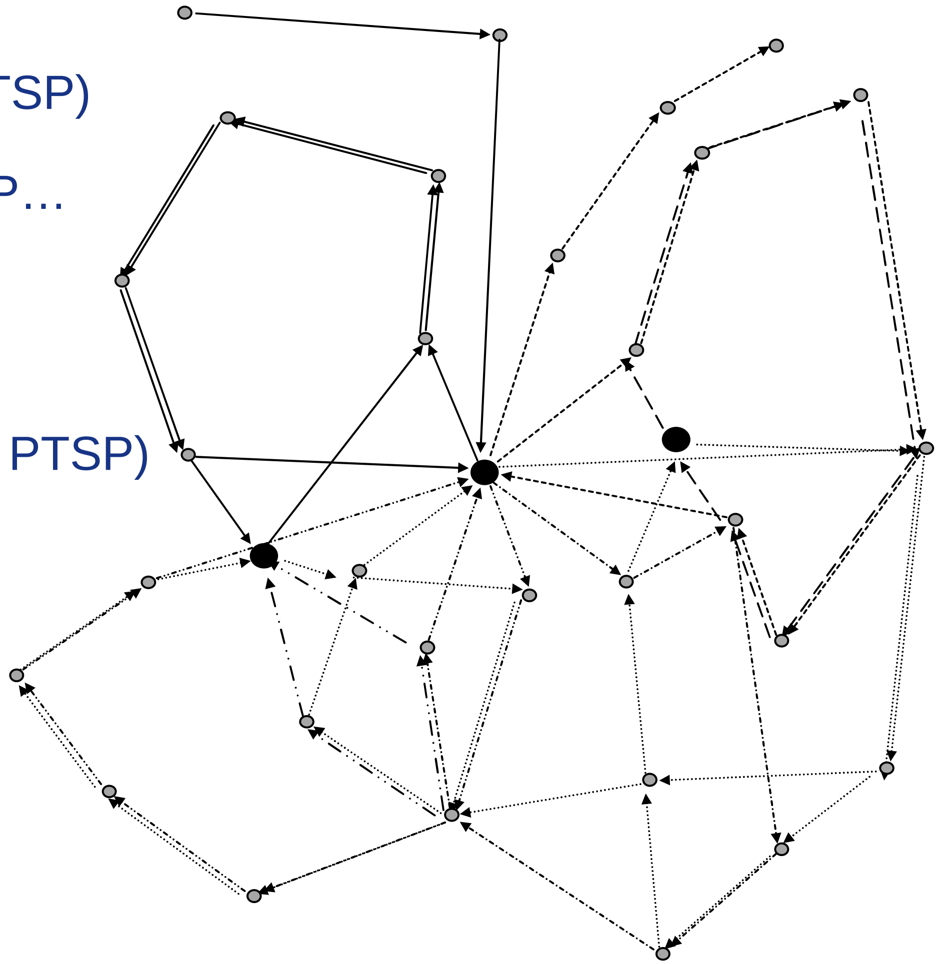


Vehicle Routing Problems (VRP)

- "Traveling Salesman Problem" (TSP)
- CVRP, VRPTW, VRPBH, PDVRP…
- "Open VRP" (OVRP)

- "Periodic TSP and VRP" (PVRP, PTSP)
- "Multiple Depot VRP" (MDVRP)
- "Periodic MDVRP" (PMDVRP)

- Time Windows (TW)
- Split Demands (SD)

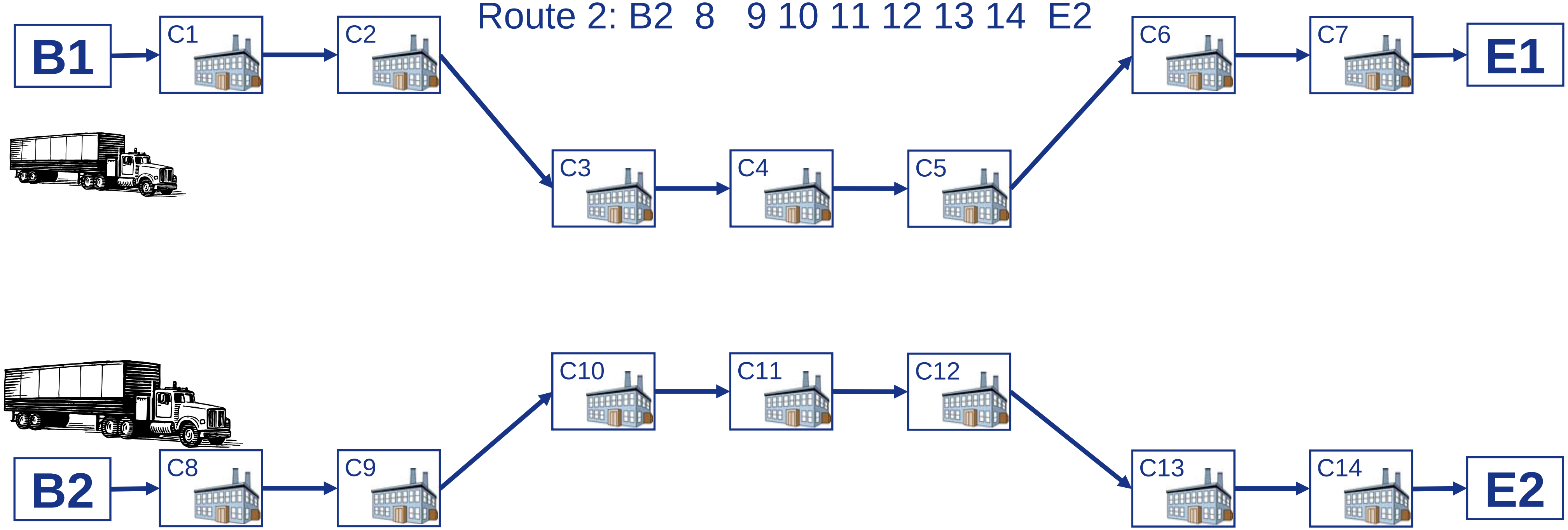


Modeling Concepts

- Process and Vehicle Oriented View
- Resource Consumption Concept, Set of Resources, Status Variables
- (Composed) Transformation Functions, Finite State Machines

Route 1: B1 1 2 3 4 5 6 7 E1

Route 2: B2 8 9 10 11 12 13 14 E2



Modeling of the OVRP: Structural Constraints

Structural constraints that are kept valid by Neighborhood operators:

- All demands are satisfied
- Each customer is visited exactly once
- All tours start at the depot
- Open tours
  - Open tour ends are modeled with an auxiliary ending node (virtual ending depot)
  - No resource consumption for virtual ending depots and their incoming edges (Transformation function is the Identity Function)
  - Standard CVRP-Operators can be used

Modeling of the OVRP: Resource Constraints

Capacity

- Resource initialization at the (starting) depot

  FreeSpace := Capacity

- Transformation

  At node C(i) :

  FreeSpace := *FreeSpace – Demand(C(i))*

Tour length restriction

- Resource initialization at the (starting) depot

  RemainingTourLength := MaxTourLength

- Transformation

  At edges E(i,j):

  RemainingTourLength := RemainingTourLength – DrivingTime(E(i,j))

  At nodes C(i):

  RemainingTourLength := RemainingTourLength – HandlingTime(C(i))

Examples: Constraints and Modeling Aspects

- Capacity limit
- Tour length limit
- Time Windows
- Split Demand, Single Unit VRP
- Pickup and Delivery
- Backhauls
- Heterogeneous fleet
- Multiple compartments, dynamic compartment sizes, load restrictions
- Fixed costs
- Customer dependent costs
- Asymmetric distance and driving costs
- Customer specific service times, back on route times
- Traffic flow factor
- Flexible starting times

Operators / Neighborhoods / Neighborhood Size

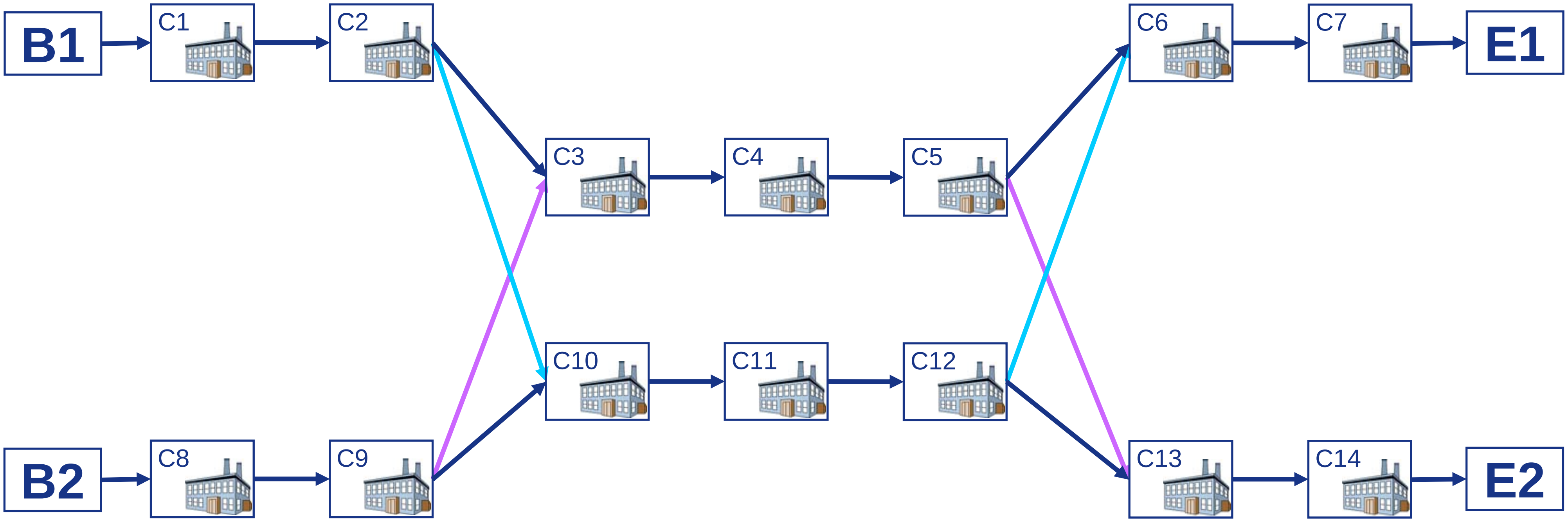
customers = n, # routes = m

- Single Route Operators
  - InsertCustomer, RemoveCustomer: O(1)
  - CheapestInsertCustomer: O(n)
  - 2 – OPT: $O(n^2)$
- **Multiple Route Operators**
  - Single Customer Operators
    - Move: $O(n^2)$
    - Exchange: $O(n^2)$
    - Combined Move/Exchange: $O(n^2)$
  - **Path Operators (Multiple adjacent customers, solution parts)**
    - **Concatenate Tour Pair: $O(m^2)$**
    - **Split Tour: O(n)**
    - **Path Exchange: $O(n^4)$**
    - **Restricted Path Exchange (one end fixed to be a depot): $O(n^2)$**

Operator: Exchange Path

Route 1: B1 1 2 3 4 5 6 7 E1

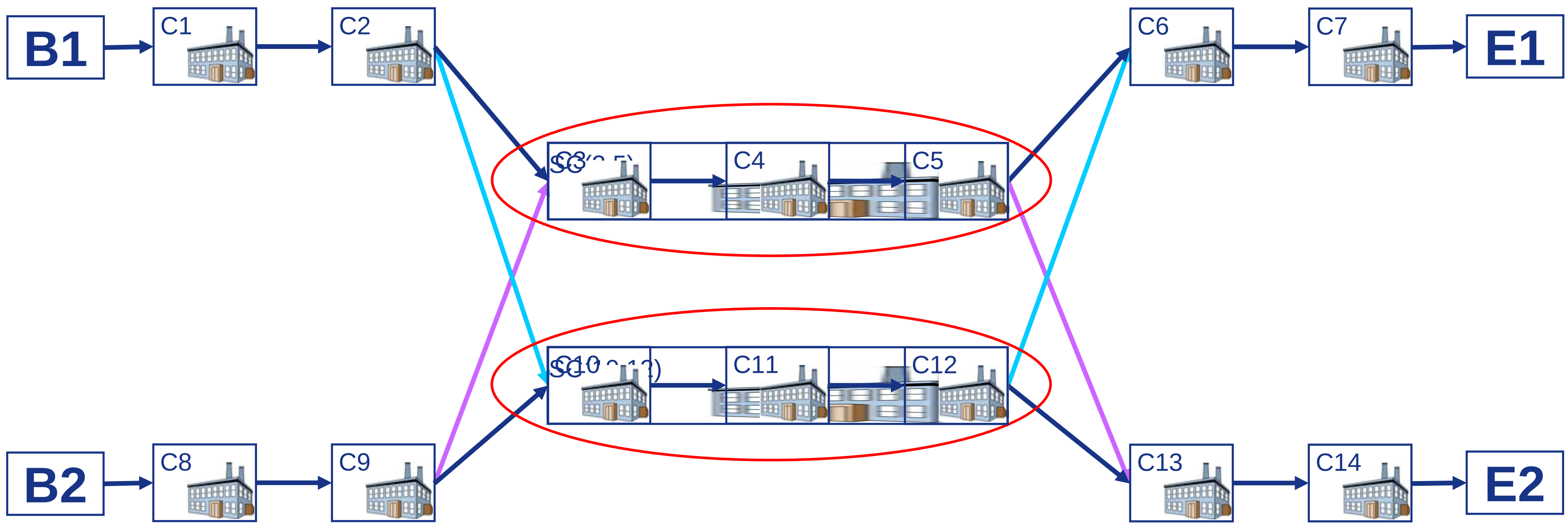
Route 2: B2 8 9 10 11 12 13 14 E2



Operator: Exchange Path = Exchange 2 SuperCustomers

Route 1: B1 1 2 3 4 5 6 7 E1

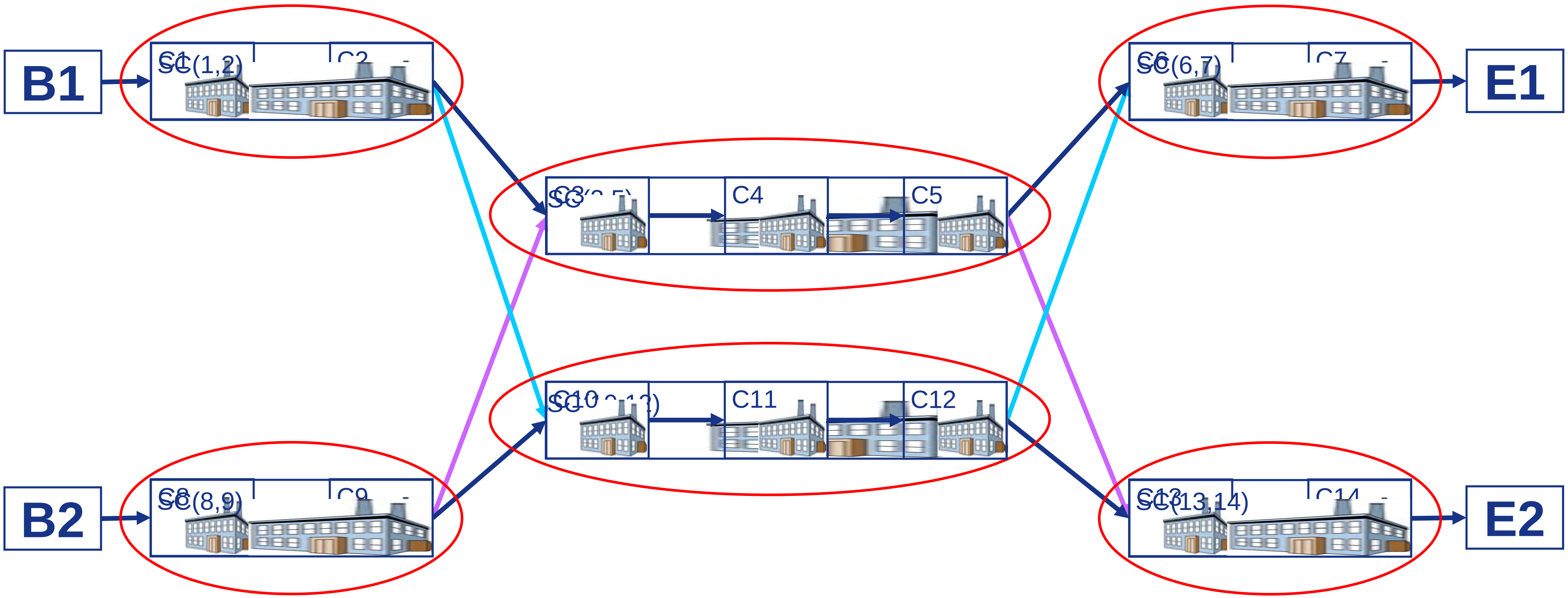
Route 2: B2 8 9 10 11 12 13 14 E2



Exchange Path = Concatenate 2 x 3 SuperCustemers

Route 1: B1 1 2 3 4 5 6 7 E1

Route 2: B2 8 9 10 11 12 13 14 E2



Constraint Handling and Acceleration Techniques

- Super-Customer Concept for Accelerated Delta Function Evaluations of Path Based Neighborhood Generating Operators
- Super-Customer Matrix, Fast Super-Customer Lookup Object or Hash Tables
- Reusing information of already visited Neighborhoods and Sub-Neighborhoods
- Priority Lists
- Static or Dynamic Neighborhood Reduction, Candidate or Tabu Lists
- Efficient Data Structures

Partial Fixing of Decision Variables

- “Neighborhood Specific Local Optima Flags” for parts of the solution:
  - Customers (or subsets of customers)
  - Routes (or subsets of routes)
  - Routes assigned to a depot (or a subset of depots)
  - Routes assigned to a day (or a sub period)
  - Partial solutions according to a decomposition scheme

Decomposition in Sub Problems and Large Neighborhoods

- Hierarchical decomposition
  - PMDVRP, PVRP, PTSP
  - MDVRP, MDTSP
  - CVRP
  - TSP

- Select a series of different subsets of Routes
  - Geographical decomposition
  - Disjoint (Parallelization)
  - Overlapping
  - VNS-Scheme: Increasing number of routes

Scheme of a Hybrid (1+1)-Evolutionary Strategy

```
GNr := 0;
LastImprovingGeneration:= 0;
Initialization( Parent );
VariableNeighborhoodSearch( Parent );
REPEAT
        GNr := GNr+1;
        Child := Mutation( Parent );
        VariableNeighborhoodSearch( Child );
        if ( Fitness( Child ) => Fitness( Parent ) )
                Parent := Child;
                LastImprovingGeneration:= GNr;
UNTIL StoppingCriteria( GNr, LastImprovingGeneration, FitnessThreshhold );
```

Monday

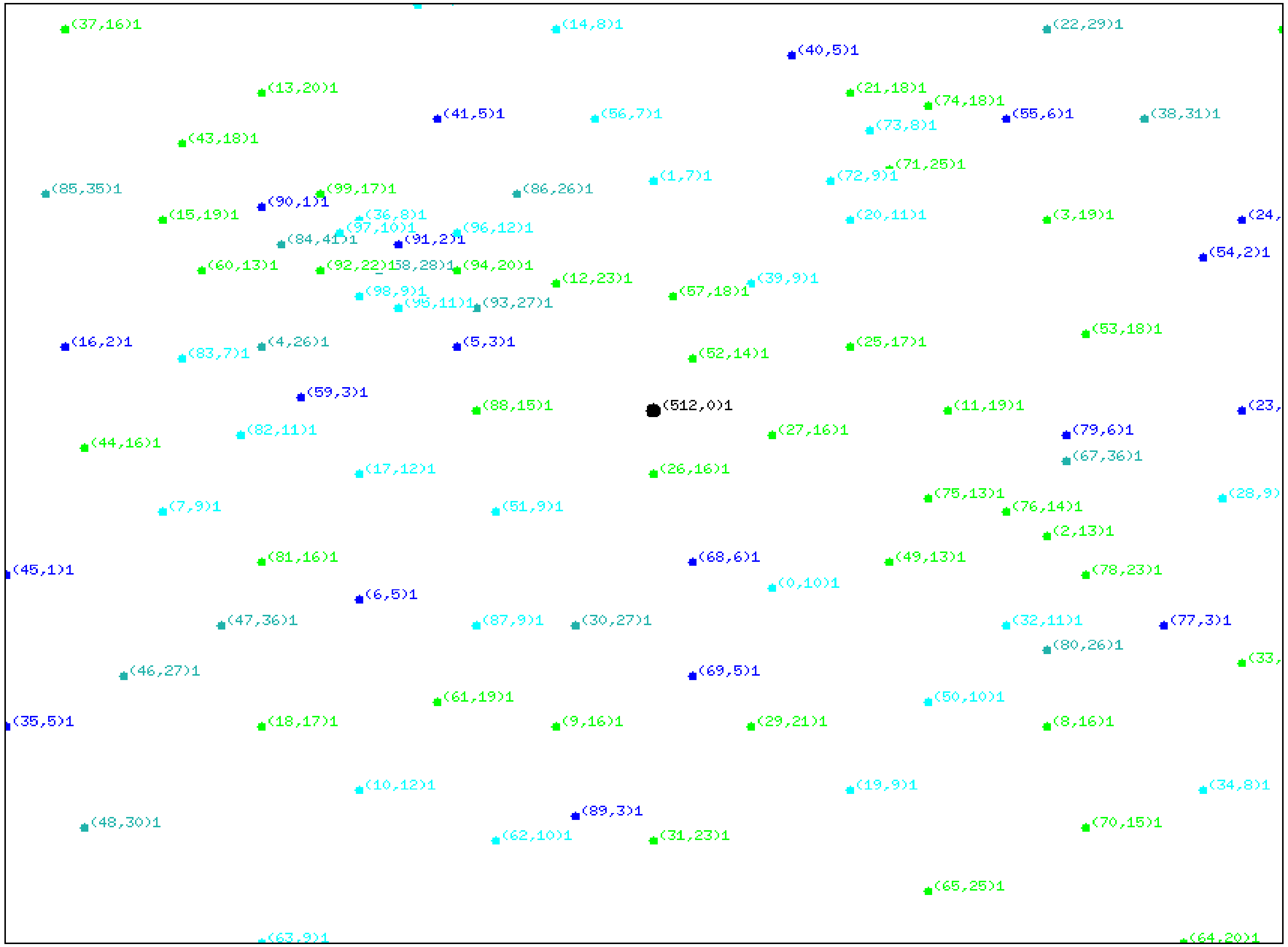
CostN 1492.36 TLEu 1492.36 TLEuF 1492.36 TLEuInt 1483 Cost 1492356 Length: 1492356 D2Best: -80.0

Nr	0	1	2	3	4	5	6	7	8
TourL	213596	220291	210937	162860	143941	184797	179335	115073	61526
TourLC	-213596	-220291	-210937	-162860	-143941	-184797	-179335	-115073	-61526
Costs	213596	220291	210937	162860	143941	184797	179335	115073	61526
VU	194	185	171	172	169	182	179	153	53
TourLEu	213.60	220.29	210.94	162.86	143.94	184.80	179.34	115.07	61.53
OpenTime	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

-87.17% | OpCountMsg

OpMsg | LoopMsg

Mouse | SAVE Tour | RESTORE Tour | Close! | Delay | DrawVU | DrawAngle | DrawTour | LineWidth



Monday

CostN 1592.86 TLEu 1592.86 TLEuF 1592.86 TLEuInt 1585 Cost 1592862 Length: 1592862 D2Best: -92.8

Nr	0	1	2	3	4	5	6	7	8
TourL	189616	149807	213642	178654	240707	210937	164975	130778	113746
TourLC	189616	149807	213642	178654	240707	210937	164975	130778	113746
Costs	189616	149807	213642	178654	240707	210937	164975	130778	113746
VU	193	184	181	166	164	171	172	169	58
TourLEu	189.62	149.81	213.64	178.65	240.71	210.94	164.98	130.78	113.75

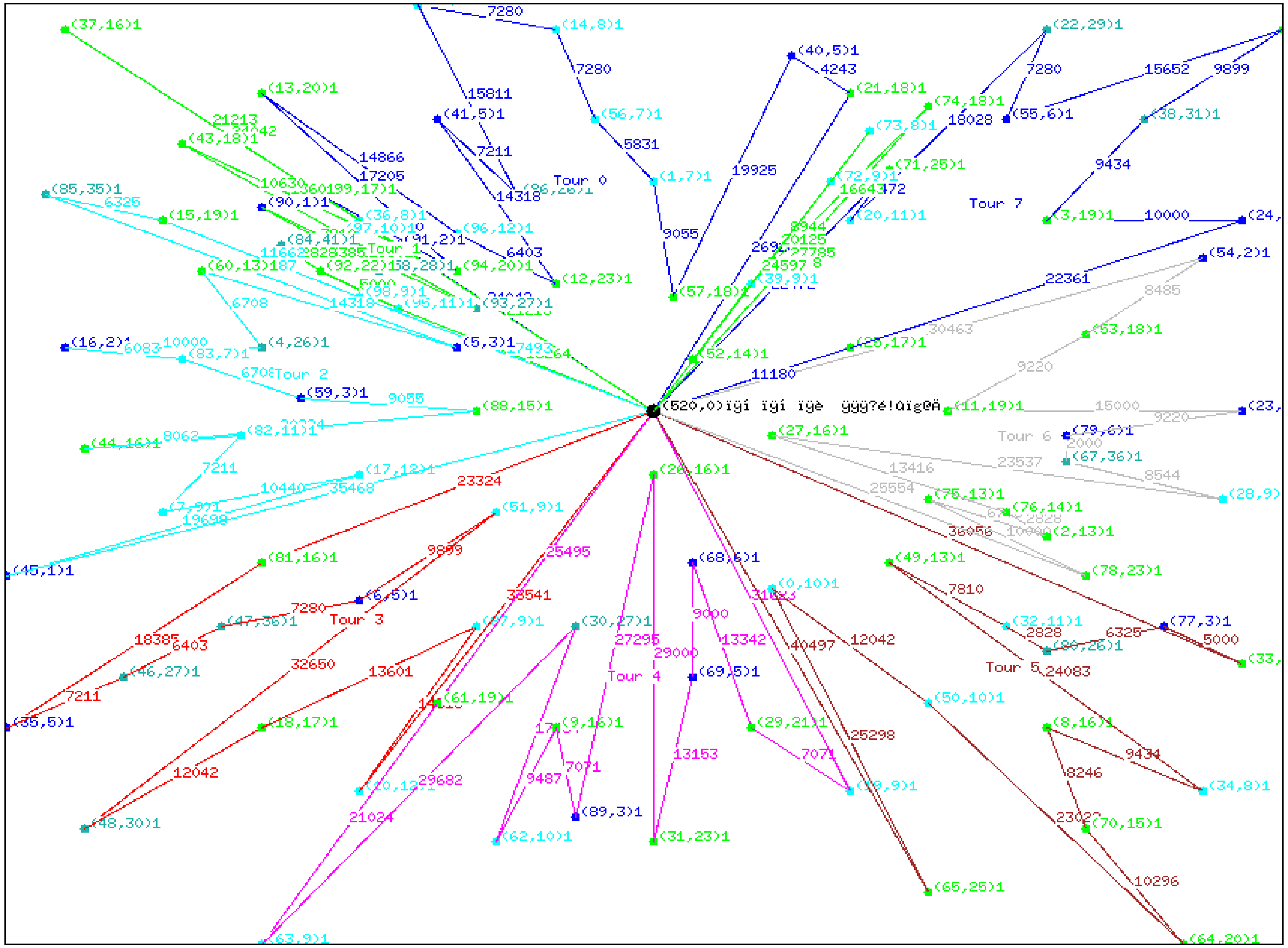
-87.47%

OpCountMsg

OpMsg

LoopMsg

Mouse | SAVE Tour | RESTORE Tour | Close! | Delay | DrawVU | DrawAngle | DrawTour | LineWidth



Andreas Reinholz Optimization Consulting

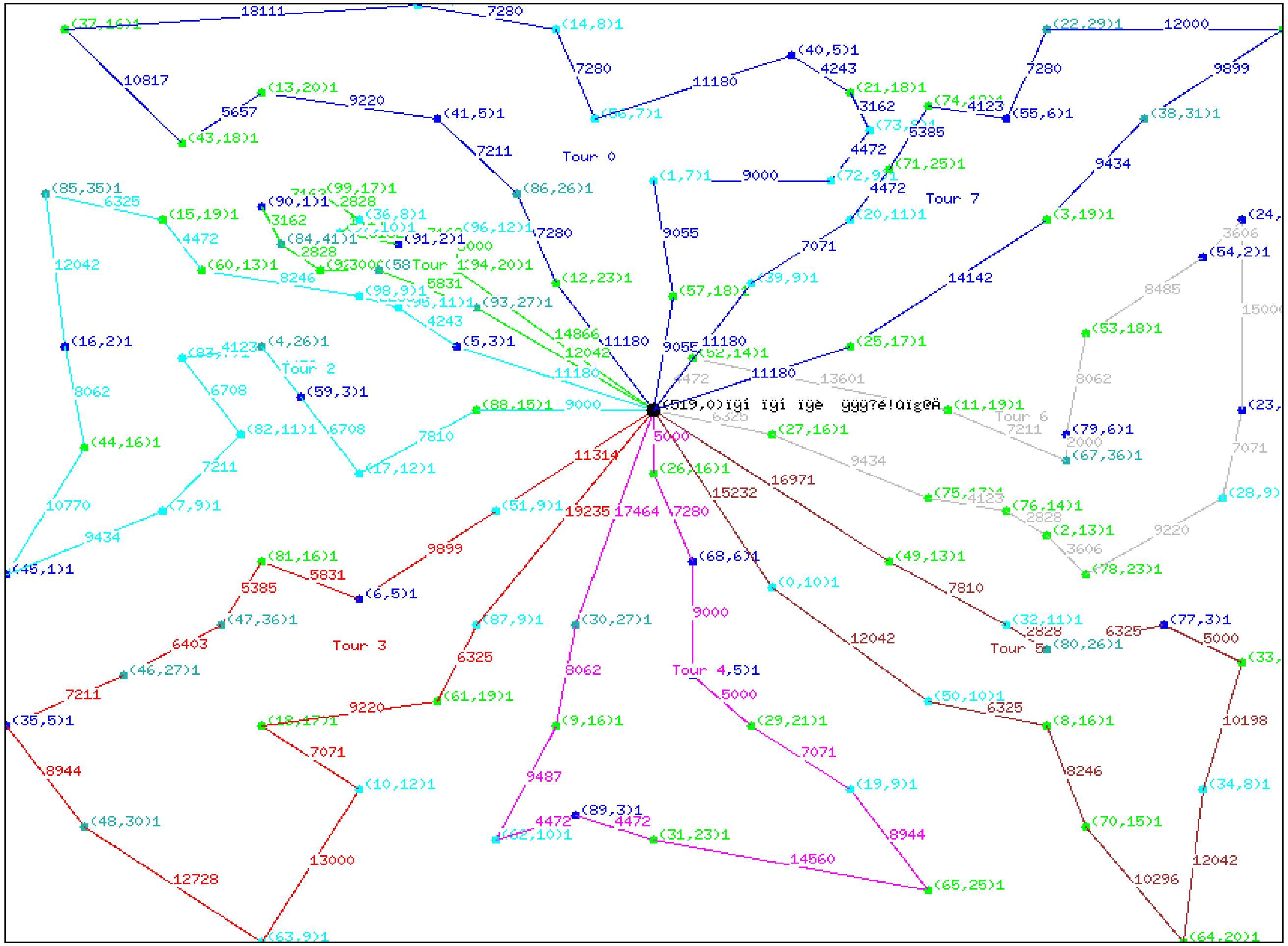
Monday

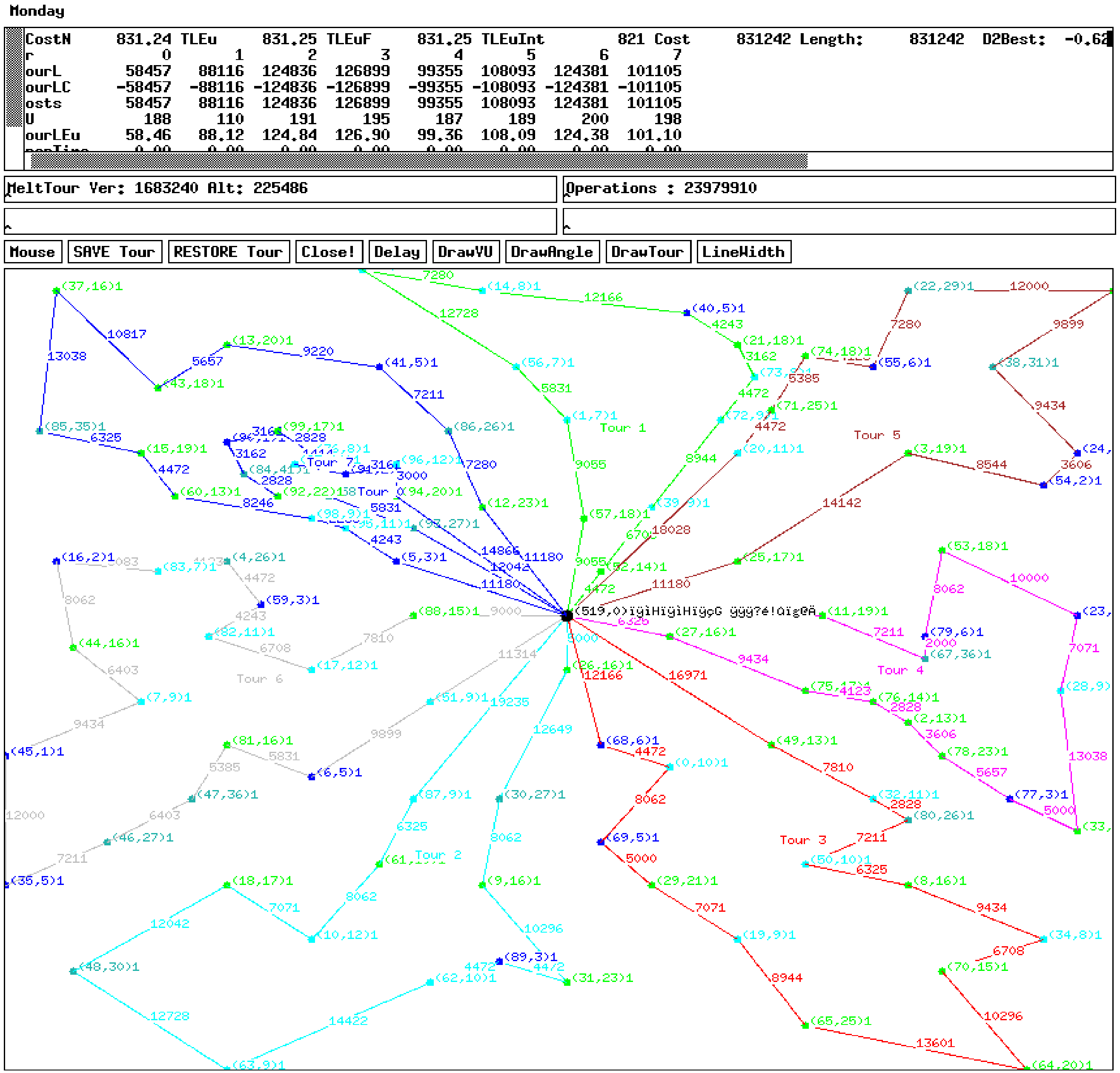
CostN 853,61 TLEu 853,61 TLEuF 853,61 TLEuInt 841 Cost 853605 Length: 853605 D2Best: -3,3

Nr	0	1	2	3	4	5	6	7
TourL	134203	58457	123042	122566	100812	113315	105044	96166
TourLC	134203	58457	123042	122566	100812	113315	105044	96166
Costs	134203	58457	123042	122566	100812	113315	105044	96166
VU	195	188	192	194	161	146	192	190
TourLEu	134,20	58,46	123,04	122,57	100,81	113,31	105,04	96,17
DropTime	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00

Operations : 1312778

Mouse | SAVE Tour | RESTORE Tour | Close! | Delay | DrawVU | DrawAngle | DrawTour | LineWidth



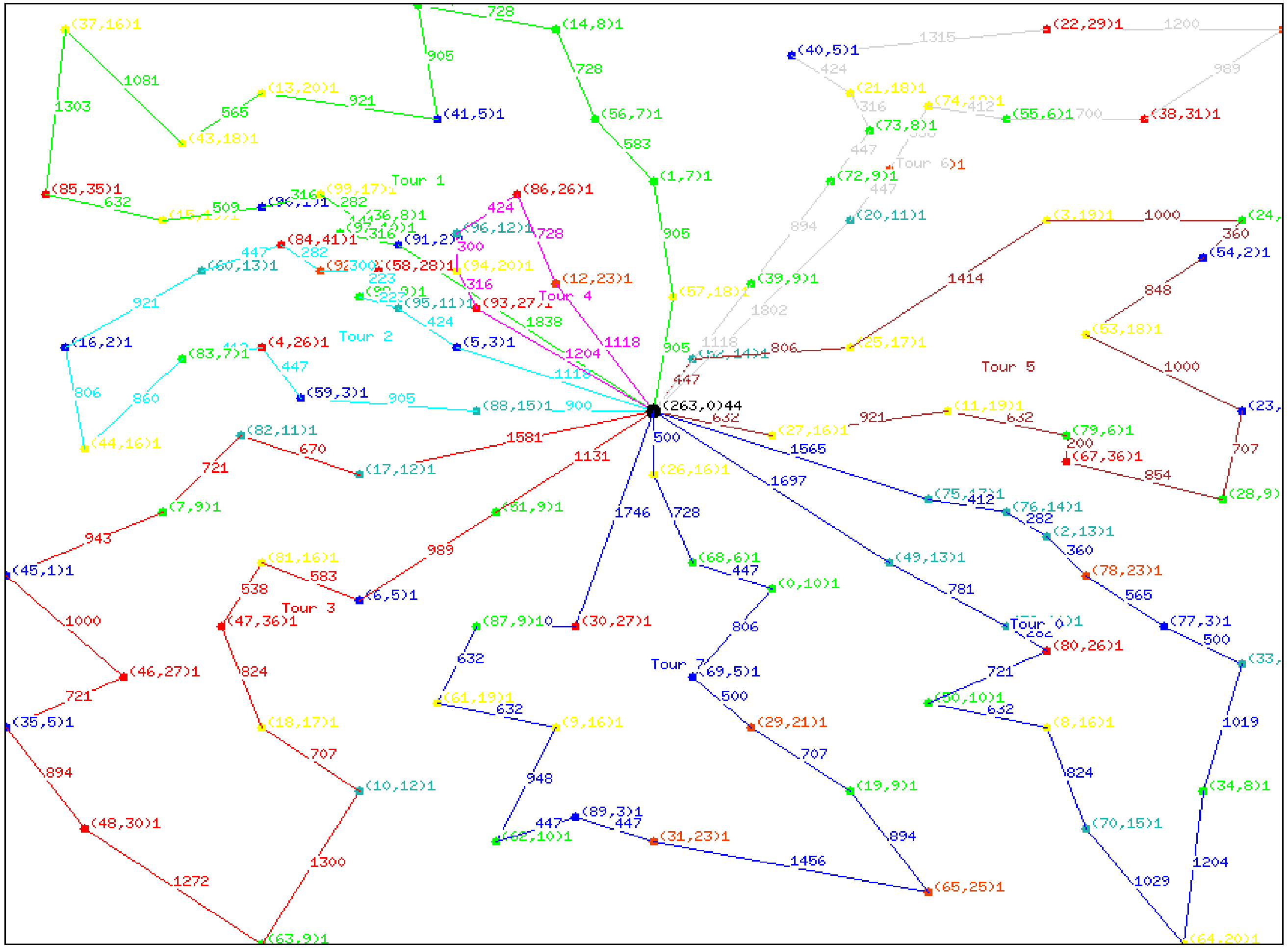

Monday
CostN 831.24 TLEu 831.25 TLEuF 831.25 TLEuInt 821 Cost 831242 Length: 831242 D2Best: -0.62
r 0 1 2 3 4 5 6 7
ourL 58457 88116 124836 126899 99355 108093 124381 101105
ourLC -58457 -88116 -124836 -126899 -99355 -108093 -124381 -101105
osts 58457 88116 124836 126899 99355 108093 124381 101105
U 188 110 191 195 187 189 200 198
ourLEu 58.46 88.12 124.84 126.90 99.36 108.09 124.38 101.10
MeltTour Ver: 1683240 Alt: 225486
Operations : 23979910
Mouse
SAVE Tour
RESTORE Tour
Close!
Delay
DrawVU
DrawAngle
DrawTour
LineWidth
Tour 1
Tour 2
Tour 3
Tour 4
Tour 5
Tour 6

Monday

CostNorm 825.76 TourLEu 826.14 Cost 82576 Length: 82576 VU: 1458

Nr	0	1	2	3	4	5	6	7
TourL	11873	12658	8268	13874	4090	9821	10602	11390
TourLC	11873	12658	8268	13874	4090	9821	10602	11390
Costs	11873	12658	8268	13874	4090	9821	10602	11390
VU	199	198	196	199	108	165	194	199
TourLEu	118.79	126.66	82.73	138.79	40.91	98.25	106.06	113.93

SAVE Tour | RESTORE Tour | Close! | Delay | DrawVU | DrawAngle | DrawTour



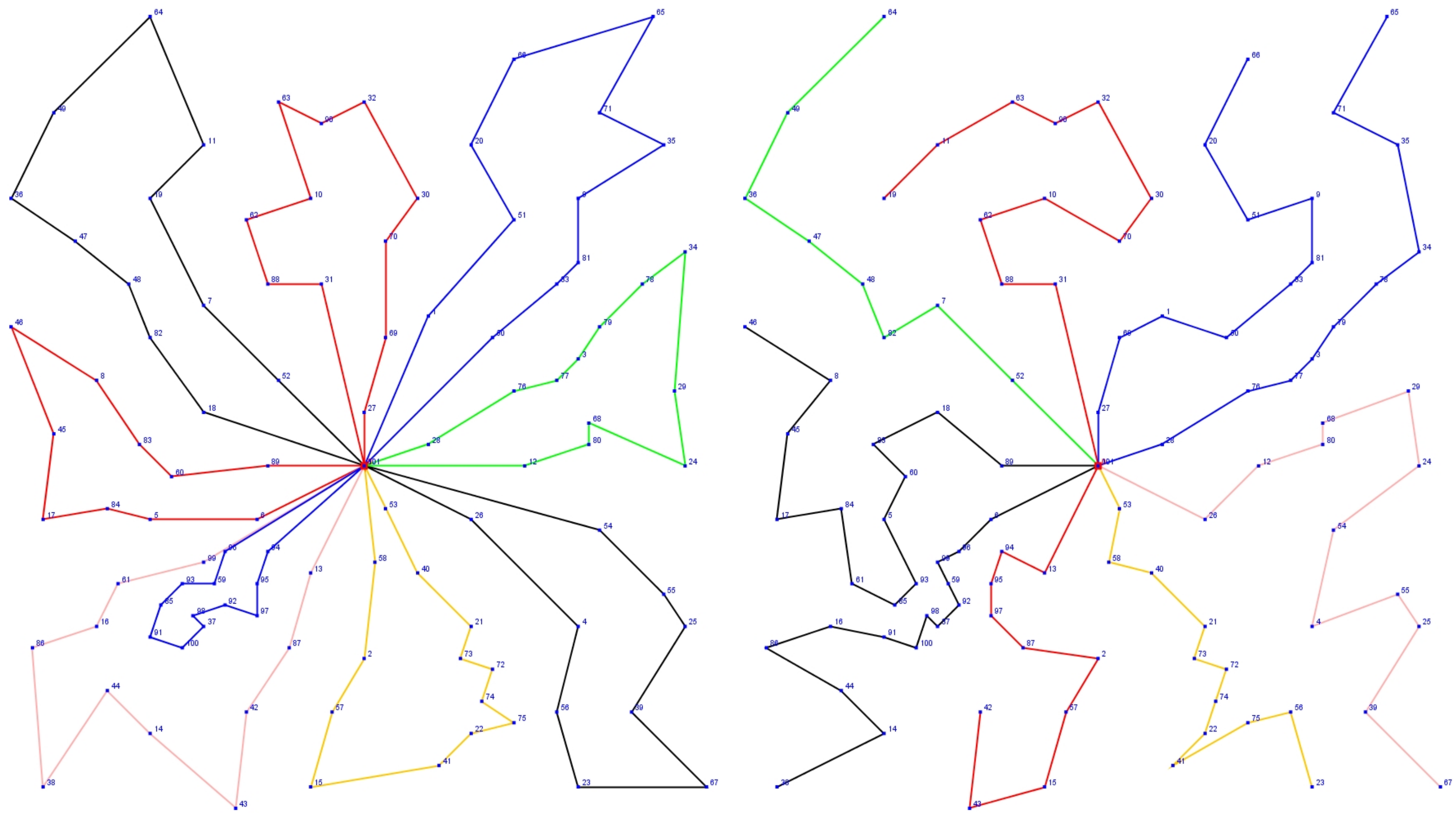
OVRP Benchmarks

14 CVRP instances of Christofides, Mingozzi and Toth

Best known results from Rokpe et al. (2007), Fleszar et al. (2008), Li et al. (2007) and Derigs et al. (2008)

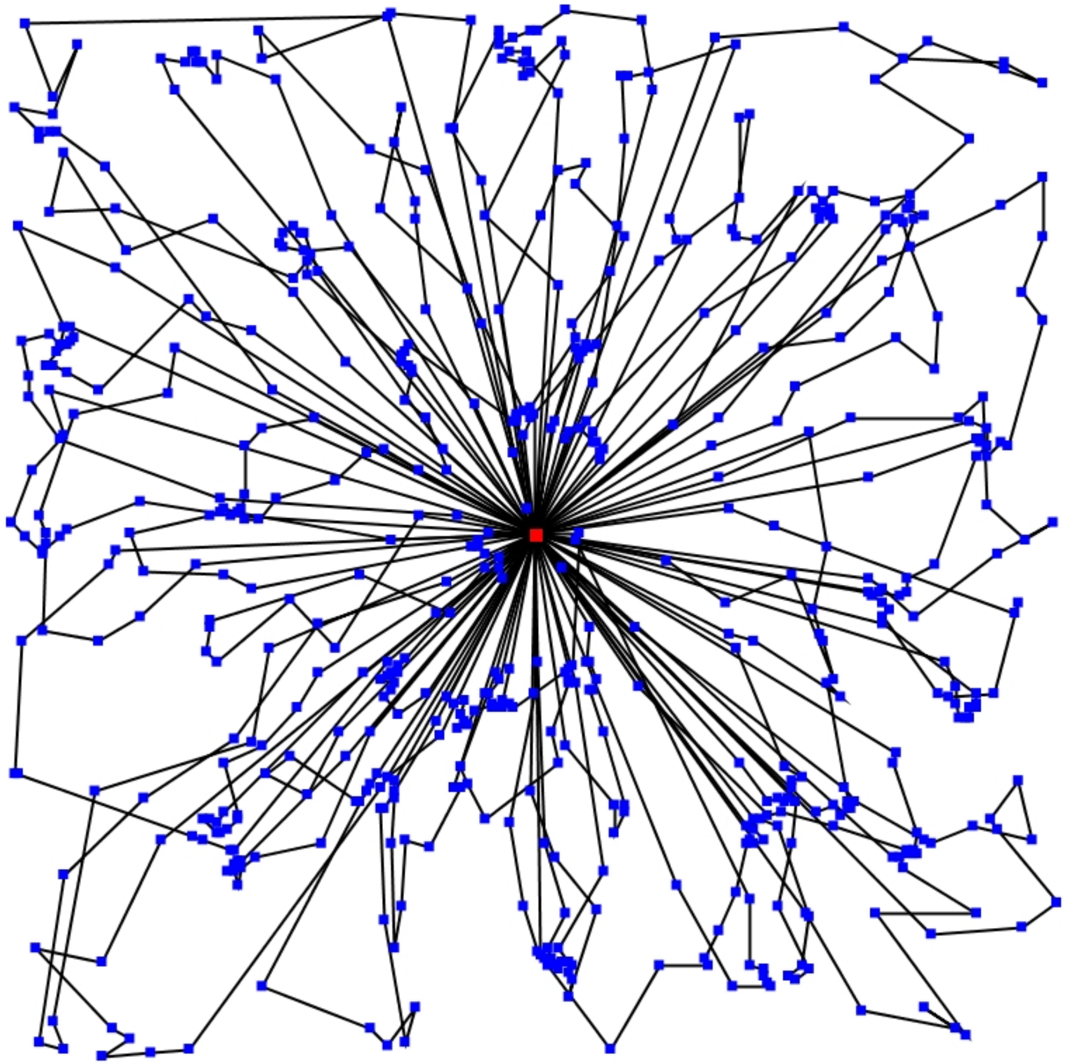
instance name	# customer	max. capacity	max. tour length	handling time	best known results tour count	best known results tour length	Our results tour count	Our results tour length	diff
c1	50	160	-	0	**6**	**412,95**	**6**	**412,95**	0,00%
c2	75	140	-	0	**11**	**564,06**	**11**	**564,06**	0,00%
c3	100	200	-	0	**9**	**639,25**	**9**	**639,25**	0,00%
c4	150	200	-	0	**12**	**733,13**	**12**	**733,13**	0,00%
c5	200	200	-	0	17	869,24	**17**	**868,44**	-0,10%
c6	50	160	200	10	**6**	**412,95**	**6**	**412,95**	0,00%
c7	75	140	160	10	11	568,49	**11**	**566,93**	-0,28%
c8	100	200	230	10	9	644,63	**15**	**640,89**	-0,59%
c9	150	200	200	10	13	757,84	**13**	**741,44**	-2,17%
c10	200	200	200	10	17	875,67	**17**	**871,58**	-0,47%
c11	120	200	-	0	**10**	**678,54**	**10**	**678,54**	0,00%
c12	100	200	-	0	**10**	**534,24**	**10**	**534,24**	0,00%
c13	120	200	720	50	11	869,50	**11**	**836,55**	-3,79%
c14	100	200	1040	90	11	591,87	**11**	**552,64**	-6,63%

CVRP vs. OVRP: C8 with tour length limit (100 customers)

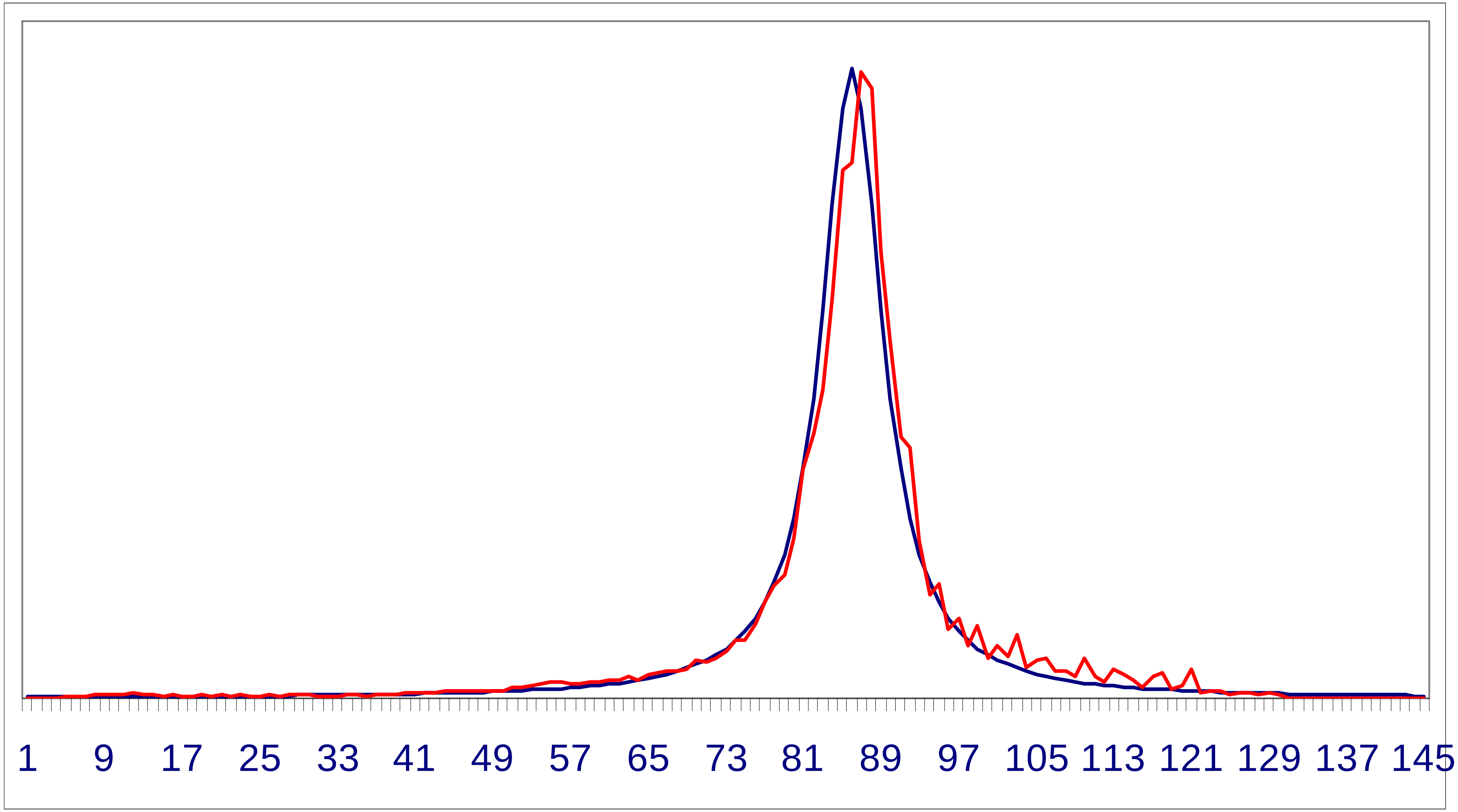


Andreas Reinholz Optimization Consulting

Are highly optimized VRPTW solutions robust enough?

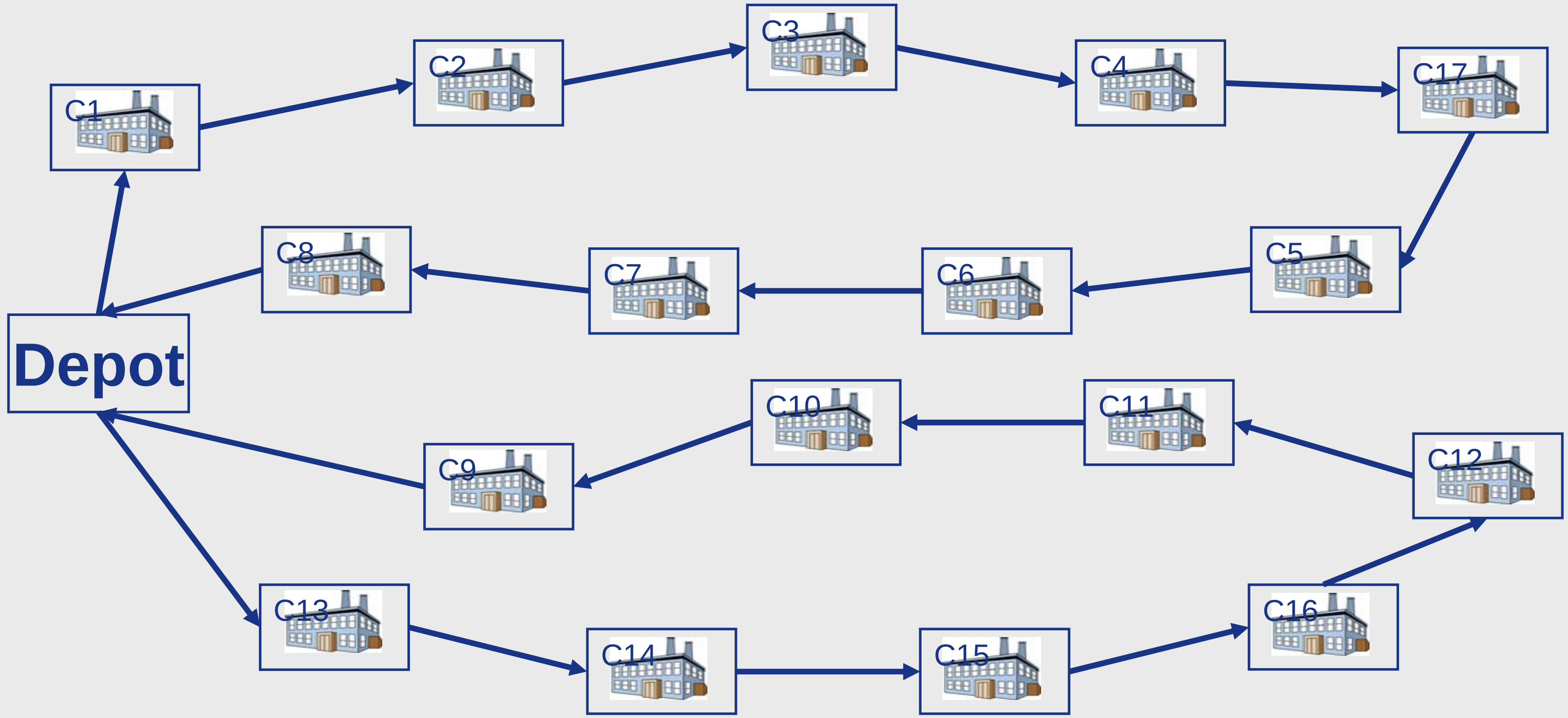


Uncertainty: Fitting Speed with a Cauchy Distribution



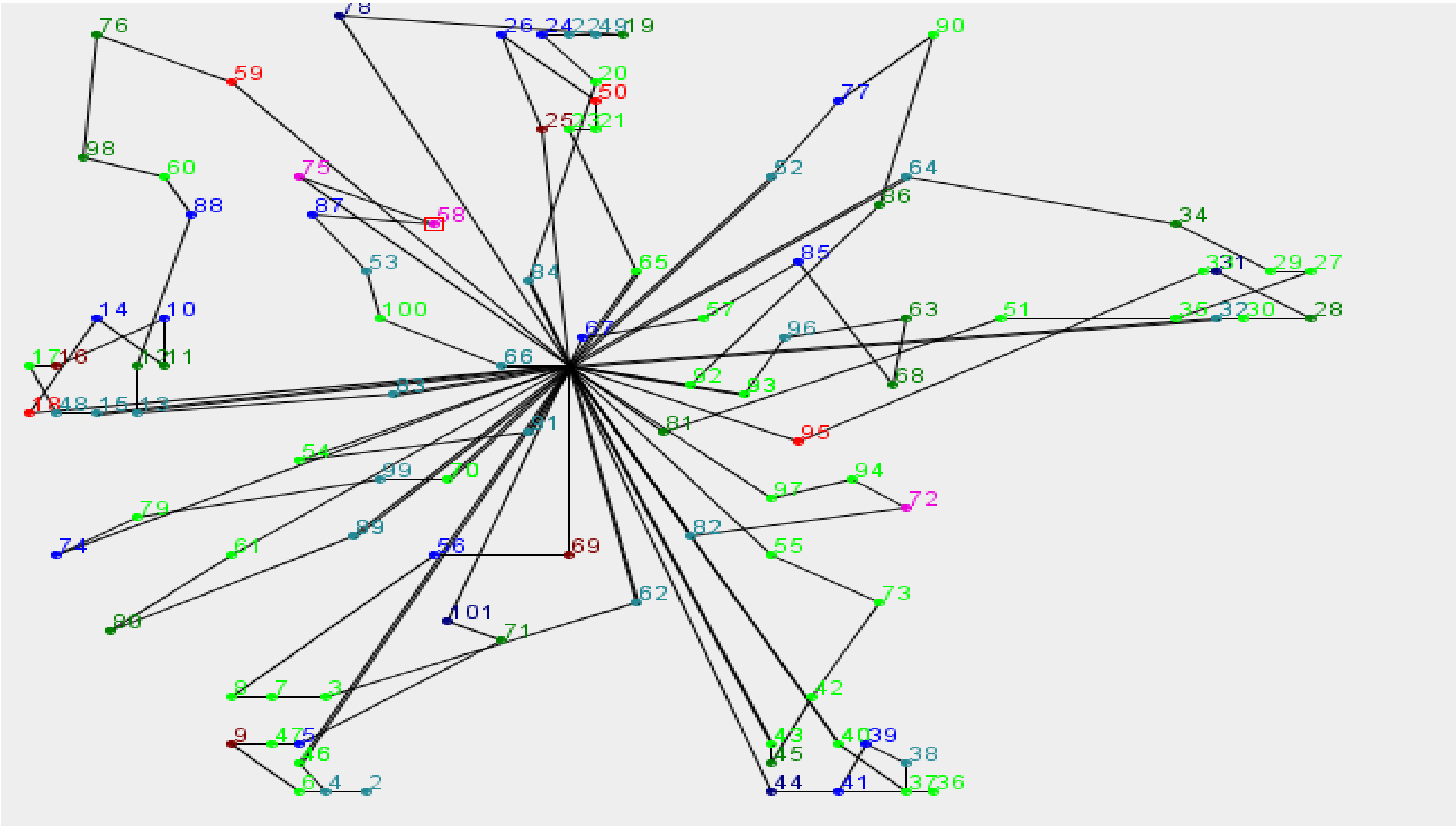
Modeling Uncertainty and Risk with Resources

- Probabilistic Resource Consumption (i.e. speed, driving time, demands, …)
- Series of Conditional Probability Distributions

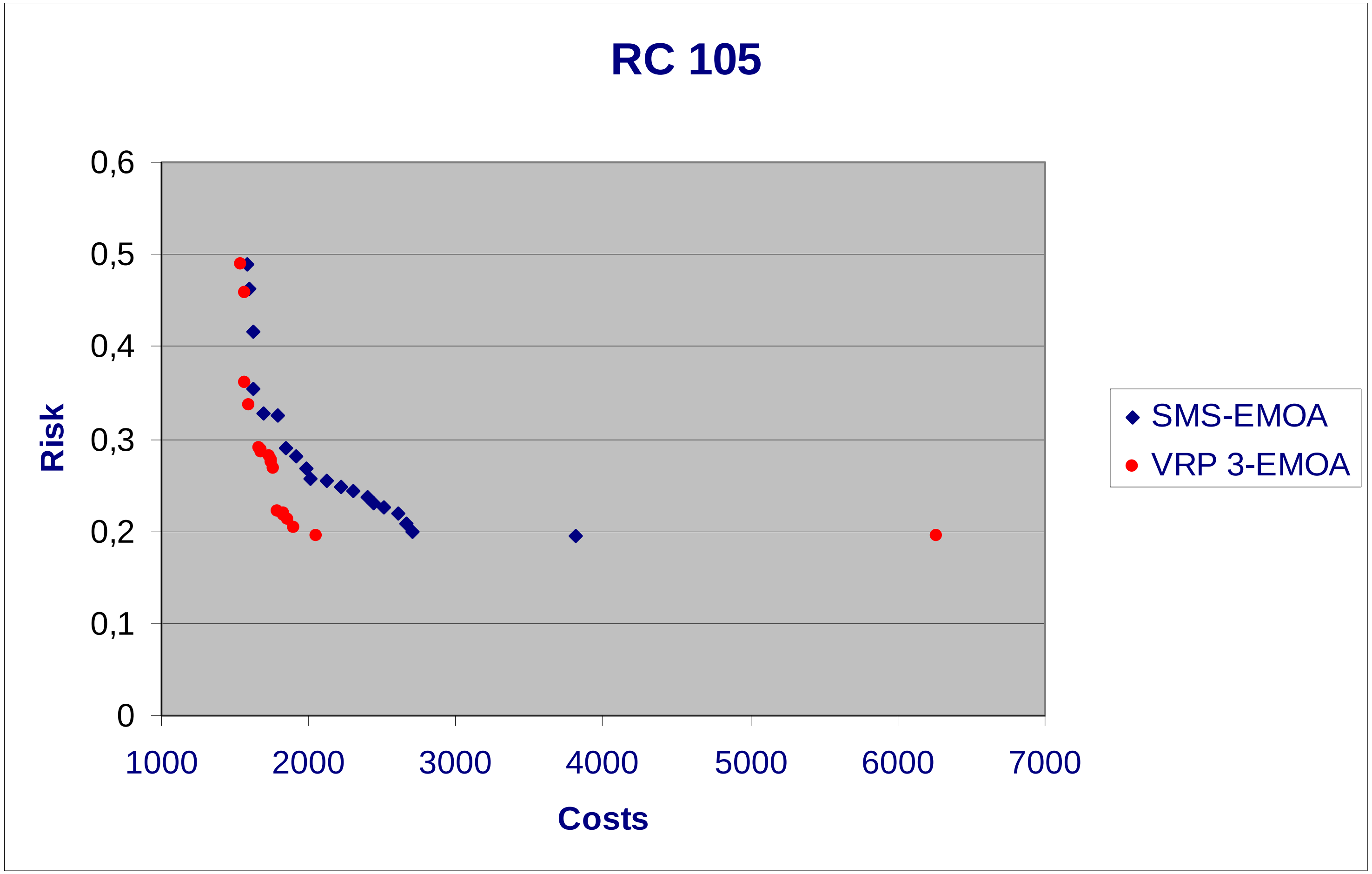


Andreas Reinholz
Optimization Consulting

Risk Assignment and Visualization



Tradeoff between Costs and Risk



Complexity

Size of the Search Space: Exponential

Algorithmic Complexity: NP-Hard

Complexity Management and Handling:

- Local Modification, Neighborhood Generating Operators, Local Search
- Decomposition of
  - Objective Functions in Sub-Functions
  - Neighborhoods into Sub-Neighborhoods
  - Problems into Sub-Problems
- Re-use of already done partial work
  - (Common) Sub-Functions
  - Fast Delta Function Evaluation (according to a Neighborhood)
  - Overlapping Neighborhoods
  - (Common) Sub-Neighborhoods
  - (Common) Sub-Problems

Parallelization Levels

- Decomposition Level
  - Sub-Functions
  - Sub-Neighborhoods
  - Sub-Problems
  - Sub-Networks

- Multiple Strategy Level
  - Multiple Parameter Sets
  - Multiple Neighborhoods
  - Multiple Metaheuristics
  - Multi Agent Systems (MAS)

Summary and Challenges

- Abstract and generalized view on Metaheuristics
- Flexible modeling and optimization concept for Rich VRP
- Acceleration techniques for path based Neighborhood Search methods
- A simple way how to quantify risk and to use it in a bi-criteria search method

Thank you for your attention!

- Noise, incomplete and uncertain data, robustness, risk management
- Dynamic and Online Optimization, Multi-criteria Optimization
- Enhanced variation mechanisms, adaptive disruption strategies
- Parallel methods, adaptive strategy control mechanisms (Species, Agents)
- More complex and hierarchical nested optimization problems
- …