## Parallel Solution Methods for the VRP

- Recent survey by Theo Crainic, chapter in "The Vehicle Routing Problem – Latest Advantages and New Challenges" by Golden, Raghavan, Wasil (2008)
- 80 references
- Main strategies, exact and approximative methods
- Promising directions



## Some of Theo's observations/conclusions Motivation

#### VRP research

- Iarger instances
- more difficult, richer formulations
- more focus on real-time solving
- need for "robustness"
- -> motivates utilization of parallel computing
- Shift in architecture of commodity computers



# Some of Theo's observations/conclusions Research

Relatively little parallel optimization work targeted at the VRP
 Most work done after year 2000



## Some of Theo's observations/conclusions Types of parallel computing

- Functional (task, control) parallelism
- Data parallelism
- Fine-grained
- Coarse-grained
- Multi-search
- Shared memory vs. separate memory
- Synchronous vs. asynchronous communication



## Some of Theo's observations/conclusions Performance measures

- Speedup (wall-clock time)
  - exact methods
  - approximative methods
- Robustness

Heterogeneous computing: utilization of special processor



## Some of Theo's observations/conclusions Parallel metaheuristics - categorization

#### Search Control Cardinality

- 1-control (1C), e.g. master-slave
- p-control (pC), e.g. multi-search
- Search Control and Communication
  - Rigid Synchronization (RS), e.g. independent multi-search
  - Knowledge Synchronization (KS), synchronized exchange of information
  - Collegial (C), e.g. asynchronous cooperative multi-thread search
  - Knowledge Collegial (KC), Collegial + procedures for information extraction and synthetization
- Search Differentiation
  - Same initial Point/population, Same search Strategy (SPSS)
  - Same initial Point/population, Different search Strategies (SPDS)
  - Multiple initial Point/population, Same search Strategy (MPSS)
  - Multiple initial Point/population, Different search Strategies (MPDS)



### **Parallel metaheuristics: Hierarchical levels** (Talbi: Metaheuristics – from design to implementation, Chapter 6)

### Algorithmic level

- independent or cooperating self-contained metaheuristics
- problem independent
- Iteration level
  - each iteration of a metaheuristic is parallelized
  - speed-up by reducing search time
  - problem independent
- Solution level
  - parallel handling of a single solution
  - speed-up by reducing search time
  - problem dependent
  - flexible evaluation



## Some of Theo's observations/conclusions - Parallel metaheuristics (1)

- fine-grained parallelization of neighborhoods (1C/RS/SPSS) has limited value
- may be interesting as low-level part of hierarchical parallel methods
- parallelization of classical multi-start methods may work well
- cooperative strategies seem to offer better performance than independent search
- asynchronous methods seem to be superior to synchronous methods
- information communication and exchange mechanisms are critical elements



# Some of Theo's observations/conclusions - Parallel metaheuristics (2)

- Controlled, parsimonius, timely exchange of information is the clue
  - Direct (e.g. island model of Genetic Algorithms) vs.
  - Indirect, through memory (solution pool, warehouse, blackboard, ..)

- Partial solutions (adaptive memory, vocabularies) vs.
- Complete solutions (elite, diverse)
- Multi-level cooperative search is very interesting
  - works well on graph partitioning and network design etc.
  - VRP:



## A survey of papers in parallel VRP a.r.t. - (meta)heuristics

Olli Bräysy, Wout Dullaert, Pasi Porkka, Geir Hasle

#### Parallel algorithms

- 44 technical papers (1989-2007)
- 3 surveys (2003, 2007, 2008)
- Cooperative (collaborative) search and multi-level search
  - 20 papers (1998-2007)
- Search reduction
  - 14 technical papers (2002-2008)
- Decomposition
  - 19 technical papers (1993-2008)



## **Ideas from the literature**

#### Rochat and Taillard (JoH 1, 1995)

- Probabilistic Diversification and Intensification in Local Search for Vehicle Routing
- Tabu Search threads
- Adaptive Memory: tours of good solutions
- TS threads pick tours randomly from AM
- later developments: Set covering thread
- Gehring and Homberger (JoH 8, 2002)
  - Parallelization of a Two-Phase Metaheuristic for Routing Problems with Time Windows
  - concurrent searches
  - differently configured
  - two phase: (min # tours, min total distance)
  - exchange of solutions through blackboard
- Berger and Berkaoui (C&OR 31, 2004)
  - A parallel hybrid genetic algorithm for the VRPTW
  - GA with two populations
    - minimize total distance
    - minimize violation of TW-constraints



## Ideas for our rich VRP solver - General / Media product distribution

- Heterogeneous computing
- Better iteration level parallelization



## **General Ideas**

Higher level parallelization with heterogeneous computing

Cooperation exact methods and approximative methods

Parallelization of LNS

Multi-level search

