An Integrative Cooperative Search approach for rich vehicle routing problems

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Plan

❖ Goals, objectives, inspiration / fundamental ideas
❖ The Integrative Cooperative Search approach
❖ Rich VRP
❖ Illustration: ICS for MDPVRP
❖ Perspectives
Rich Problem Setting

ág Large number of interacting attributes (characteristics)
á Larger than in “classical” (“academic”) settings
á Problem characterization
á Objectives
á Uncertainty
á That one desires to (must ☻) address simultaneously
Design of Wireless Networks

Simultaneously determine the
- Appropriate number of base stations
- Location
- Height & power
- Number of antennas/station
- Tilt & orientation of each antenna
- Optimize cost, coverage & exposure to electrosmog
Rich VRP

_vehicle routing in practice_

 Attributes of several generic problems

- Time windows
- Vehicle capacities
- Route duration & length
- Topology of routes
- Multi-compartment
- Multi-depot
- Multiple periods

- Pick up at (small but numerous) suppliers
- Deliveries at (few) plants (very different in capacity)
- Mixing pickups and deliveries (sometimes)
- Multiple tours
- Complex “cost” functions

Plan now by firm, execute repetitively later by carriers
What Methodology for Rich Problems?

- Simplify (!!!)
- Series of simpler problems
- Simultaneous handling of multiple attributes
- A more complex problem to address!
- Propose a “new” approach
Sources of Inspiration

 Decomposition
   - Major methodological tool in optimization
   - Domain decomposition in parallel computation
   - Solution reconstruction? “Partition” modification?

 Simplification
   - Fixing part of a rich problem may yield an easier case to address

 Cooperative (parallel) multi-search meta-heuristics successful in harnessing the power of different algorithms (guidance mechanisms)
Indirect, Memory-based Cooperation

Memory, Pool
Reference Set
Elite Set
Data Warehouse

Search \(i\) → \(k\) → Search \(l\)

Search \(j\) → \(k\) → Search \(l\)
ICS Fundamental Ideas & Concepts

- Decomposition by attribute
- Concurrent population evolution
- Solver specialization
- Cooperation with self-adjusting and guidance features
Shared-Memory ICS

- Integrator
- Partial Solutions (set A)
- Partial Solutions (set B)
- Complete Solutions
- Partial Solutions (set B)
- Partial Solver (set A)
- Partial Solver (set B)
- Global Search Coordinator
Decomposition by Attribute

- Simpler settings by **fixing** (ignoring) variables or constraints
  - “Eliminating” variables or constraints might yield the same sub-problems but impair the reconstruction of solutions

- Yields
  - Well-addressed, “classical” variants with state-of-the-art algorithms
  - Formulations amenable to efficient algorithmic developments
  - Our idea: be opportunistic!
Decomposition by Attribute (2)

- Each subproblem = Particular fixed attribute set
  - Addressed using effective specialized methods
    → Partial Solvers
- Partial Solvers focus on the unfixed attributes
  → Partial Solutions
- Multiple search threads
  - One or several methods for each subproblem
  - Meta-heuristic or exact
- Central-memory cooperation
Decomposition by Attribute (3)

 반드시 and challenges

 - Homogeneous vs. heterogeneous population
 - Purposeful evolution of partial solutions
 - Reconstruction & improvement of complete solutions
Reconstructing Complete Solutions

- Recombining Partial Solutions to yield complete ones
- **Integrator** search threads/operators: *Partials* → *Complete*
  - Select and forward
  - Population-based methods: GA & Path Relinking
  - Mathematical programming-based models
- Issues and challenges: **Selection** of
  - Partial solutions for combination
  - Complete solutions for the central-memory population
    - Which subproblems?
    - What quality? What diversity?
Improving Complete Solutions

- Search threads – **Solvers** *(to come)*
- Evolve & improve complete solution **population**
- These are difficult problems!
  - If they were “easy”, we would not need ICS!
- Should modify solutions differently from partial solvers
  - Post-optimization
  - Large neighbourhoods for “few” iterations
  - Select few solutions and solve an exact problem
  - Local branching on variables from different attribute sets
Global Search Coordinator

- A “richer” role: purposeful evolution
  - Reconstruct / approximate global status of the search
  - Avoid “heavy-handed” process control: cooperation
  - Information exchange mechanisms
  - Guidance for Partial Solvers & global search

- Monitor
  - Pools / populations
    - Complete solutions (direct)
    - Partial solutions (& integrators) (direct or indirect)
  - Exchanges / communications (possibly)
Global Search Coordinator – Monitoring

- Classical (central) memory statistics
- A “richer” memory through analysis of solutions & communications
  - Quality of solution & evolution of population
  - Impact on population (quality & diversity)
  - Presence of solutions or solution elements in various types/classes of solutions
    - Arcs, paths, …
    - Good, average, poor solutions
    - Solutions with particular attribute sets, …
Global Search Coordinator – Monitoring (2)

- The process – Partial Solver, Integrator, Solver – that produced the solution
- Search space covering
  - Attribute values (combinations of) corresponding to visited search space regions (how often, quality measures)
- …
Global Search Coordinator – Guidance

❖ Partial Solvers cooperate and communicate according to their own internal logic
❖ Based on monitoring results, “instructions” (solutions, often) are sent GSC → Partial Solver (pool) or Integrator
   ❖ Enrich quality or diversity
   ❖ Modify the values of the fixed attributes ⇔ Moving the search to a different region
   ❖ Modify the sets of attributes defining the partition
   ❖ Modify parameter values or method or replace method
ICS Concurrent Evolution

- Selective migration
- Guiding migration
- Evolution
- Integrative evolution

- Set $S_1$ of partial solvers
- Population $P_1$ of partial solutions
- Population $P'_1$ of elite partial solutions
- Set $I$ of integrators
- Global evolution operators
- Global monitoring & guidance

- Set $S_2$ of partial solvers
- Population $P_2$ of partial solutions

- Set $S_3$ of partial solvers
- Population $P_3$ of partial solutions
Zoom on Partial Solver Organization

GA\(_1\) partial solver

GA\(_2\) partial solver

TS\(_1\) partial solver

TS\(_2\) partial solver

Legend

- Selective migration
- Evolution

Guiding migration
MDPVRP and MDPVRPTW

☀ Multiple depots
  ✷ Given number of homogeneous vehicles at each depot

☀ Periodic problem
  ✷ Planning horizon of $t$ “days” (periods)
  ✷ For each customer: a list of acceptable visit day “patterns”

☀ Each customer must be assigned to a single depot and a single pattern and routes must be constructed for each depot & day, in such a way that the total cost of all the resulting routes is minimized
An Important Property → Decomposition

任何形式 MDVRP(TW) or MDPVRP(TW) 实例可以被转换为一个较大 PVRP(TW) 实例

可以使用相同的解决方案来解决这些问题

自然分解 MDPVRP(TW) 为两个子问题:
- PVRP(TW) → 固定 depot 分配
- MDVRP(TW) → 固定模式分配

我们可以通过相同的 solver 作为 Partial Solvers 来解决两个子问题，以及作为 Global Solver
Current Solvers

- A neighbourhood-based search based on the Unified Tabu Search (UTS) procedure of Cordeau et al. (2001)
- A new hybrid population-based solver
  - Giant tour with no delimiters representation + split algorithm (Prins 2002) + pattern chromosome
  - Admits infeasible solutions with respect to capacity and route length constraint violations, with self-adjusting penalties (UTS)
  - Offspring education: local search (route, patterns)
  - Population management; in parent selection as well
An Evolutionary ICS MDPVRP Illustration

- 3-population scheme
  - P0: “Global” population
  - P1: Fixed patterns
  - P2: Fixed depots
- 3 solvers for each population: 2 GA, 1 UTS
- GA as Integrator
- GA as Partial Solver: education does not change the patterns corresponding to the “fixed” attributes
ICS MDPVRP Illustration – Integrators

8 2 Integrators: Random select in 25% best
   8 2 parents → extract depot and period patterns →
   generate population → evolve → send best valid
   8 100 couples: crossover + educate & repair → send
   best (valid)
ICS MDPVRP Illustration – Guidance

Information collection on (customer/depot/pattern)
- No. occurrences
- Frequencies in best/average/bad sub-populations
- Best fitness and average rank, …

Build good “guiding” individuals, with promising attributes in relation with previous measures

Guiding migration on partial population stalling
Send guiding individuals + a good complete solution + some random ones
Preliminary Results

❖ Gaps with respect to the best known solutions

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ICS without Guidance

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❖ Guidance improved average results
❖ 5 min of ICS competitive with 1h of sequential solving
Conclusions & Perspectives

- Rich (combinatorial optimization) problems present interesting challenges and opportunities
- Parallel cooperation performs very well
- ICS appears promising when complexity grows
- Still a lot of work on all aspects of the approach and applications