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ANALYSIS OF LAMINAR LIQUID FILM FLOWING DOWN A VERTICAL SURFACE

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ABSTRACT

The model for the prediction of hydromechanical parameters of laminar liquid film flowing down a vertical surface is presented in the study. It was explored laminar plane film flow and the film flowing on surface of vertical tube with different curvature. The calculations evaluating velocity distribution across water and transformer oil film and correspondingly the thickness of the film were performed in the paper. Equations for the calculation of local velocities in the film with respect to cross curvature of wetted surface have been established. Evaluation of cross curvature influence on the film thickness is presented. Calculations showed that the influence of cross curvature on high viscosity film is considerable. Applied prediction limits of cross curvature are presented in the paper as well.

Keywords: laminar film, cross curvature, thickness

NOMENCLATURE

D tube diameter (m)
f cross sectional area of film flow (m^2)
G liquid mass flow rate (kg/s)
g acceleration of gravity (m/s^2)
Ga_R Galileo number (gR^3 / ν^2)
R tube external radius (m)
r variable radius (m)
Re Reynolds number of liquid film [$4\,\Gamma/(\rho\,\nu)$]
v^* dynamic velocity $(\tau_w/\rho)^{1/2}$ (m/s)
w local velocities of stabilized film (m/s)
$\overline{w}$ average velocities of stabilized film (m/s)
y distance from wetted surface (m)

Greek Letters

Γ wetting density [kg/(m·s)]
δ film thickness on surface of vertical tube (m)
δ_{pl} film thickness on vertical plane surface (m)
ε_R relative cross curvature of the film (δ/R)
η dimensionless distance from the wetted surface (y/δ)
η_δ dimensionless thickness of the film ($v^*\delta/\nu$)
ν kinematic viscosity (m^2/s)
ρ liquid density (kg/m^3)
φ dimensionless film velocity (w/v^*)
τ_w shear stress at the wall (Pa)

Subscripts

g gas or vapour
pl plane surface
w wetted surface

INTRODUCTION

The determination of hydromechanical parameters of liquids in falling films emerging from a slit is an interest in many applications of chemical engineering including the treatment of highly viscous fluids such as polymer, food, paint, lacquer and many others. Liquid coolant in the form of thin film has a significant potential utility in high heat absorbing capacity that includes the heat of vaporization (Gantchev, 1987). Typical falling film saline water evaporators produce water vapour from a thin film of saline water flowing down. Falling films are also used for safety in potentially hazardous industries. Special spray systems on the surface of vessels containing inflammables liquids can be used to create the fallings films, which may reduce the increase of temperature inside the vessels in case of nearby external sources of fire.

The most of the film apparatus for the thermal treatment of liquid products consists of vertical tubes with falling films on their external surfaces (Alekseenko et al., 2001; Tananayko and Vorontszov, 1975). In the case when liquid film flows down a vertical tube the curvature of its surface and the film itself effects heat transfer characteristics on the surface, shear stress distribution and correspondingly thickness of the liquid film (Gimbutis, 1988). A film of large cross curvature exists when its thickness is of the same range as the radius of a tube. The cross curvature of wetted surface and the film leads to alteration of hydromechanical parameters of the liquid film. Simultaneously the intensity of the heat exchange between a wetted surface and liquid film is altered.

Many papers concerning the film flow have been published (Rifert et al., 2000; Vlachogiannis and Bontozoglou, 2001; Bertani and De Salve, 2001; Chinnov et al. 2001), however it mostly deals with the film flow on horizontal tubes or plane surfaces. Sinkunas and Kiela (2003) have showed that hydromechanical parameters of gravitational film are affected by initial velocity of the film. The precise prediction of internal or external films flow plays an important role in the design of heat exchangers. Surface of heat exchangers must be large enough in area to transfer sufficient heat at small temperature difference. The falling film evaporators typically perform to within 70% to 80% of the maximum thermodynamic performance at acceptable hydraulic losses. Therefore, a complete knowledge of liquid film flow on vertical surfaces from entrance to the exit should be of the interest industrially and academically. The methods that evaluate the behaviour of film on vertical convex surface are not completely finished despite of the fact that liquid film flow due to gravity is deeply analysed.

MODEL DESCRIPTION

Laminar Vertical Plane Film Flow

The major contribution to the analysis of falling down laminar film is Nusselt's solution of the motion equation

$$\frac{d^2w}{dy^2}+\frac{g}{\nu}\left(1-\frac{\rho_g}{\rho}\right)=0 \tag{1}$$

with boundary conditions

$$w=0,\text{ for } y=0\,;\ \frac{dw}{dy}=0,\text{ for } y=\delta \tag{2}$$

In the case of stabilized gravity film flow often $\rho_g << \rho$, so the member $1-\rho_g/\rho$ can be ignored. When the member $(1-\rho_g/\rho)$ << 1, one can substitute acceleration of gravity g to the following expression $g(1-\rho_g/\rho)$ in all further equations and relationships.

Solving Eq. (1) one can obtain parabolic equation of velocity distribution for laminar film

$$w=\frac{gy\delta}{\nu}\left(1-0.5\frac{y}{\delta}\right) \tag{3}$$

The dimensionless form of Eq.(3) can be expressed as follows

$$\varphi=\eta\left(1-0.5\frac{\eta}{\eta_\delta}\right) \tag{4}$$

Integration of equation with respect to the film thickness δ

$$\overline{w}=\frac{1}{\delta}\int_0^\delta w\,dy \tag{5}$$

yields the relationship between average velocity and thickness of laminar film

$$\overline{w}=\frac{g\delta^2}{3\nu} \tag{6}$$

Multiplying Eq.(6) by $\delta\rho$, we obtain the formula for thickness determination of laminar film

$$\delta=\left(\frac{3\Gamma\nu}{\rho g}\right)^{1/3}=\left(\frac{3}{4}\frac{\nu^2}{g}Re\right)^{1/3} \tag{7}$$

The dimensionless form of Eq.(7) is as follows

$$\eta_\delta=\left(\frac{3}{4}Re\right)^{1/2} \tag{8}$$

Solving a set of Eqs.(6) and (7), we obtain such formula for mean velocity of laminar film

$$\overline{w}=\left(\frac{g\nu}{48}Re^2\right)^{1/3} \tag{9}$$

Laminar Film Flow on the Outside Surface of Vertical Tube

Equation of motion for the liquid film in cylindrical coordinates is as follows

$$\frac{d^2w}{dr^2}+\frac{1}{r}\frac{dw}{dr}+\frac{g}{\nu}=0 \tag{10}$$

Working out Eq.(10) with the following boundary conditions

$$w=0,\text{ for } r=R;\ \frac{dw}{dr}=0,\text{ for } r=R+\delta \tag{11}$$

we obtain velocity distribution across the film

$$w=\frac{g(R+\delta)^2}{2\nu}ln\frac{r}{R}-\frac{g}{4\nu}\left(r^2-R^2\right) \tag{12}$$

or

$$w=\frac{gR^2}{2\nu}\left[(1+\varepsilon_R)^2 ln\frac{r}{R}-0.5\left(\frac{r^2}{R^2}-1\right)\right] \tag{13}$$

The mean velocity for film flow on the external tube surface we can express as

$$\overline{w}=\frac{\int_R^{R+\delta} wrdr}{\int_R^{R+\delta} rdr} \tag{14}$$

and obtain the following relation

$$\overline{w}=\frac{gR^3}{4\nu(1+0.5\varepsilon_R)\delta}\{(1+\varepsilon_R)^4\left[ln(1+\varepsilon_R)-0.75\right]+ +(1+\varepsilon_R)^2-0.25\} \tag{15}$$

Mass flow rate of liquid film is defined as

$$G=\overline{w}\rho f=2\pi R\overline{w}\rho\delta\left(1+0.5\varepsilon_R\right) \tag{16}$$

and substitution of Eq.(15) into Eq.(16) leads to

$$G = \frac{\pi \rho g R^4}{2\nu} \{(1+\varepsilon_R)^4 [ln(1+\varepsilon_R) - 0.75] +$$
$$+(1+\varepsilon_R)^2 - 0.25\} \quad (17)$$

Defining Reynolds number for film flow as

$$Re = \frac{4G}{2\pi R \rho \nu} \quad (18)$$

and substituting Eq.(17) into Eq.(18), the resultant equation is

$$Re = \frac{g\delta^3}{\varepsilon_R^3 \nu^2} \{(1+\varepsilon_R)^4 [ln(1+\varepsilon_R) - 0.75] +$$
$$+(1+\varepsilon_R)^2 - 0.25\} \quad (19)$$

Thickness of the film may be expressed from Eq.(19)

$$\delta = \varepsilon_R \left(\frac{\nu^2}{g} Re\right)^{1/3} \{(1+\varepsilon_R)^4 [ln(1+\varepsilon_R) - 0.75] +$$
$$+(1+\varepsilon_R)^2 - 0.25\}^{-1/3} \quad (20)$$

Thickness of the film on vertical plane surface according Tananayko and Vorontszov (1975)

$$\delta_{pl} = \left(\frac{3}{4}\frac{\nu^2}{g} Re\right)^{1/3} \quad (21)$$

Practically the case with values of relative cross curvature exceeding 1 is not significant. Therefore, with sufficient accuracy we have

$$\delta = \frac{\delta_{pl}}{1+0.3\varepsilon_R} \quad (22)$$

In engineering calculations the cross curvature radius of the wetted surface usually is known and the thickness of the film flowing down a vertical plane surface we can estimate from Eq.(21). Then, the equation describing the thickness of the film flowing down a convex surface results from Eq.(22)

$$\delta = 1.67R\left(\sqrt{1+1.2\frac{\delta_{pl}}{R}} - 1\right) \quad (23)$$

Substituting Eq.(21) into Eq.(23) we obtain

$$\delta = 1.67R\left(\sqrt{1+1.09(Re/Ga_R)^{1/3}} - 1\right) \quad (24)$$

As it was mentioned above, in engineering calculations relative cross curvature of the film $\varepsilon_R < 1$, so it is better to estimate relative cross curvature from the following equation

$$\varepsilon_R = 1.67\left(\sqrt{1+1.09(Re/Ga_R)^{1/3}} - 1\right) \quad (25)$$

and then the film thickness

$$\delta = \varepsilon_R R \quad (25)$$

The function $\varepsilon_R = f(Re/Ga_R)$ graphically is presented in Figure 1. The results show that for relative cross curvature of the film in interval from 0 to 1, the variation of dimensionless ratio values Re/Ga_R is from 0 to 3 approximately.

Three values of Reynolds numbers were chosen for the computations of velocity distribution in water and transformer oil films. The calculations were performed for the liquid film flowing down a plane and convex vertical surfaces correspondingly with different outside diameters of vertical tubes (3 mm and 30 mm diameters).

The results of calculations are shown in Figures 2, 3 and 4. As we can see from Figure 2, the film thickness for small viscosity liquid is not so large and relative cross curvature of the film is rather small and therefore velocity distribution within the falling film on the surface of 30 mm diameters tubes are similar to plane surface. The influence of cross curvature for the liquid film of high viscosity is noticeable (Figure 3 and 4).

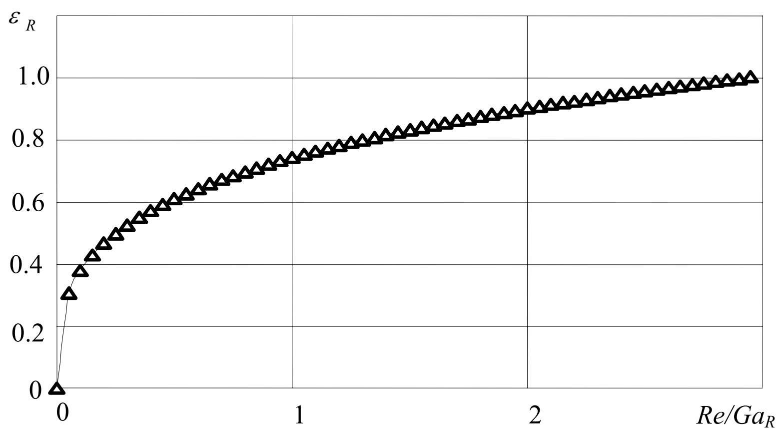

Figure 1: Dependence of relative cross curvature ε_R as ratio Re/Ga_R.

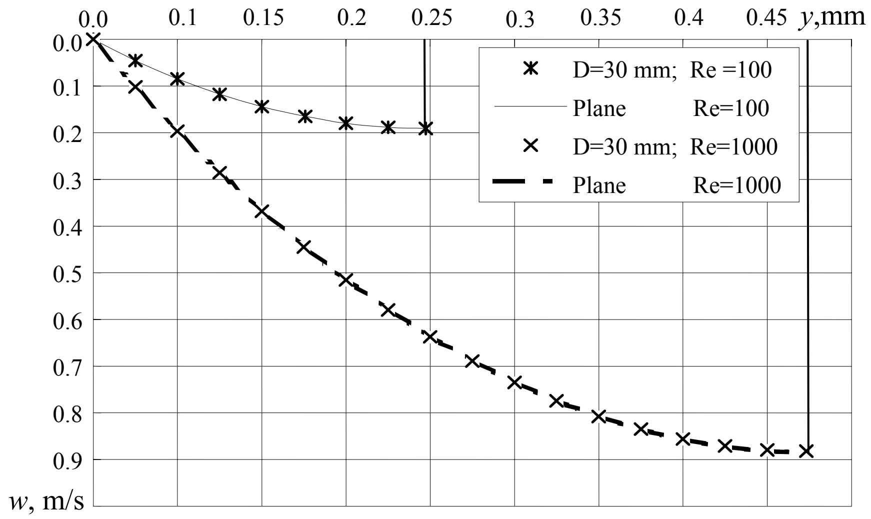

Figure 2: Local velocity profiles of water film flowing down a plane and convex surfaces.

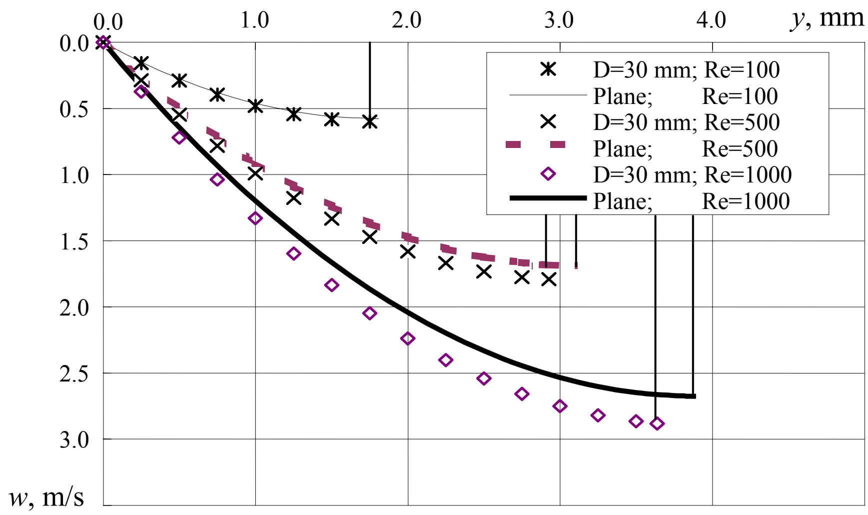

Figure 3: Local velocity profiles of transformer oil film flowing down a plane and convex surfaces.

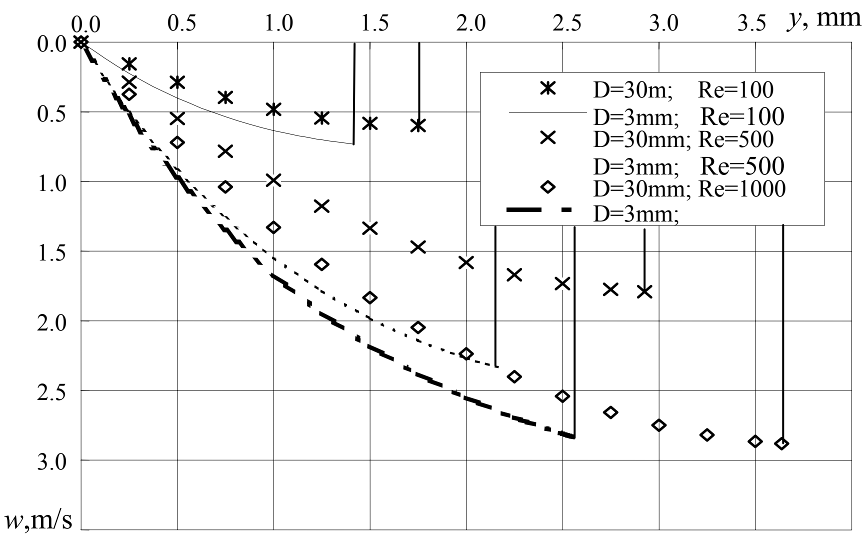

Figure 4: Comparison of local velocity profiles of transformer oil film with different curvature.

CONCLUSION

A simple model evaluating the influence of surface cross curvature on hydromechanical parameters of the liquid film was proposed. It has been considered a stabilized laminar film flow on the external surface of a vertical tube and on vertical plane surface respectively. We assume a stabilized gravity film flow when the film thickness and average velocity are not changeable in flow direction. In this case thickness of hydrodynamic boundary layer is equal to the film thickness. When the ratio of the film thickness and the radius of the tube $(\varepsilon_R = \delta/R)$ is negligible, the regularities of momentum transfer for the film flow on the surface of vertical tube are in small difference with regularities for vertical plane surface $(R = \infty, \varepsilon_R = 0)$. In the case of values ε_R >> 0 regularities of momentum transfer for the film flow on the surface of vertical tube significantly depends upon the cross curvature of wetted surface. The model can be useful for the calculations of liquid film hydromechanical parameters in apparatus of vertical tubes and other installations. The following conclusions can be drawn from this study:

1. It is obtained that under the influence of convex surface curvature the film thickness decreases changing local velocity distribution simultaneously. An equation for the calculation of local velocities in the film with respect to wetted surface curvature has been established.

2. The influence of cross curvature of water film flowing down on external surface of vertical tube with the diameter of 30 mm is negligible due to small relative cross curvature of the film.

3. Physical properties of the liquid, especially its viscosity, have greater influence on hydromechanical parameters of the film than cross curvature of the wetted surface.

4. The limits of surface cross curvature and Re/Ga_R ratio value are proposed from the practical point of view. In practice, value of surface cross curvature $\varepsilon_R \leq 1$ and $Re/Ga_R \leq 3$.

5. Velocity gradient arises in the liquid film of large curvature, while it can be one of the ways to increase hear transfer intensity in laminar film flow.

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