Visualization method for fracture flow problems

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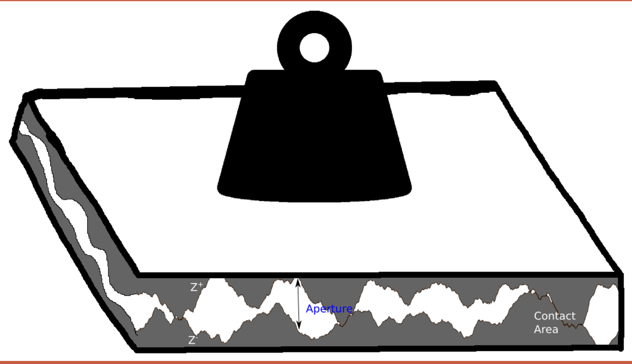
Introduction

Fracture flow may dominate in rocks with low porosity and it can accompany both industrial and natural processes. Typical examples of such processes are natural flows in crystalline rocks and industrial flows in geothermal systems or hydraulic fracturing. Fracture flow provides an important mechanism for transporting mass and energy. For example, geothermal energy is primarily transported by the flow of the heated water or steam rather than by the thermal diffusion. The geometry of the fracture network and the distribution of the mean apertures of individual fractures are the key parameters with regard to the fracture network transmissivity. Transport in fractures can occur through the combination of advection and diffusion processes like in the case of dissolved chemical components. The local distribution of the fracture aperture may play an important role for both flow and transport processes.

Fracture definition



A fracture is volume limited by two surfaces which may coincide over what is called a contact area. Surfaces can by represented by function $z^{\pm}(\mathbf{x})$.

$$z^{\pm}(x) = h_0^{\pm} + h^{\pm}(x) \tag{1}$$

where: $h_0^{\pm}$ - mean value, $h^{\pm}$ - fluctuation of random character.

Fracture geometry properties

A statistical description of random fluctuations $h^{\pm}$ is given by autocorrelation function.

$$C_{h^{\pm}}(u) = \frac{1}{L}\int_0^L h^{\pm}(x)h^{\pm}(x+u) \tag{2}$$

$$S(k) = \int h(x)exp\,(2i\pi kx)\,dx = F(h(x)) \tag{3}$$

$$F(C_h) = F(h)F^*(h) = S^2(k) \tag{4}$$

where S(k) - power spectrum. Surface is self-affine structure, so spectrum has power low distribution with α index.

$$S(k) \sim k^{-\alpha} \tag{5}$$

$$C_{h^{\pm}}(u) = \sigma_h^2 exp\left(-(\frac{u}{l_c})^{2H}\right) \tag{6}$$

The two surfaces of a given fracture may be correlated one to another, this intercorrelation is characterized by a dimensionless parameters:

$$\theta = \frac{1}{L}\int_0^L h^+(x)h^-(x) \tag{7}$$

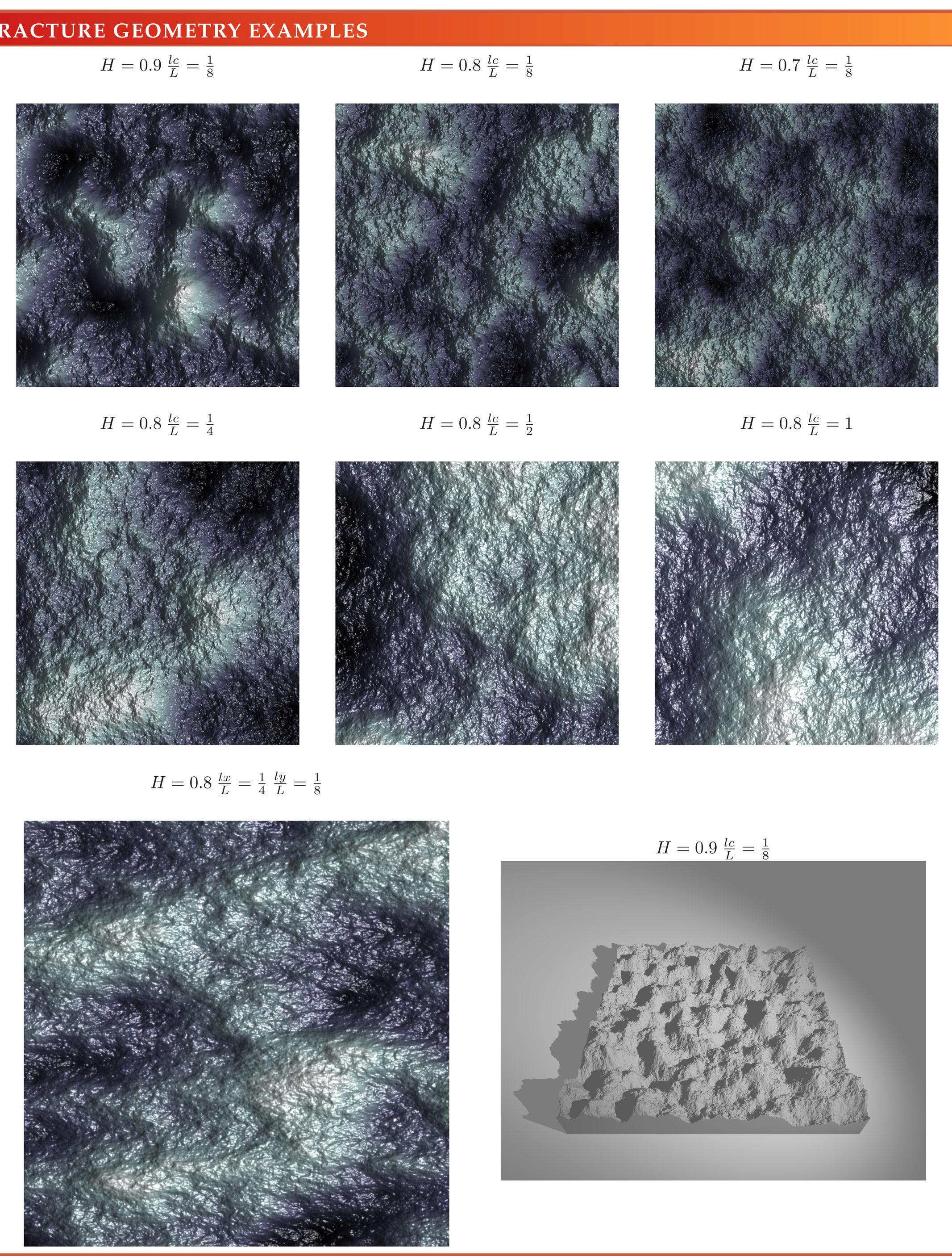
θ ranges from -1 (asymmetric) to 1 (symmetric) surfaces. The easiest way to correlate surfaces is copy one and shift by u, then for Gaussian correlated fracture $\theta = C_h(u)$
Aperture:

$$\begin{aligned} w(x) &= z^+(x) - z^-(x), \\ b(x) &= w(x), \quad w(x) \geq 0 \\ &\quad\;\; 0, \quad w(x) \leq 0 \end{aligned} \tag{8}$$

We create fracture by generating single surface next we copy, separate and shift surfaces.
Hence this model contains four independent dimensionless parameters:

- $\frac{b_m}{\sigma_h}$, b_m - mean separation of surfaces,
- $\frac{l_c}{\sigma_h}$, l_c - correlation length,
- H or α index - information about spatial correlation,
- θ - intercorrelation.

Fracture geometry examples



Fracture geometry reconstruction

Correlated field can be created by a linear combination of random field X.

$$Y(i) = \sum_{m=0}^{N_L} a(m)X(i+m) \tag{9}$$

Then, coefficient a(m) can be determined from the correlation function:

$$C(k) = \sum_{m=k}^{N_L} a(m)a(m-k), \quad k \leq L_c \tag{10}$$

Correlated field can be obtained by solving the equation 10. In practice is better to use the Fourier transform.

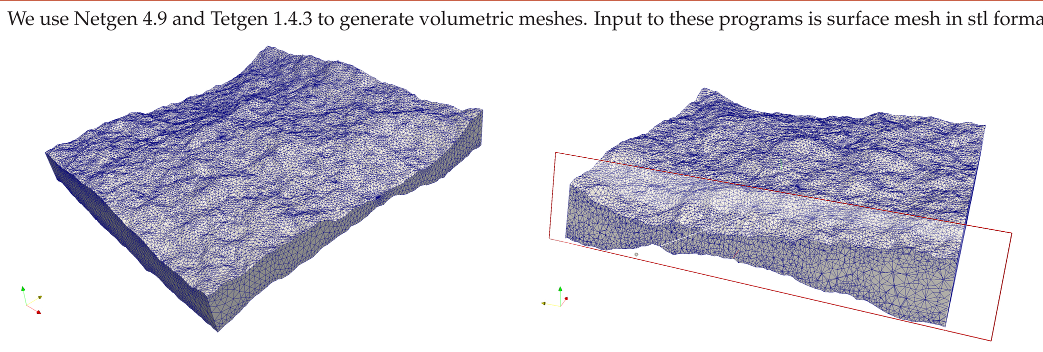
$$Y_m = \sum_{s=0}^{N_L-1} \sqrt{\hat{C}_s}\hat{X}_s \exp\left(-\frac{2\pi ims}{N_L}\right) \tag{11}$$

$$\hat{X}_s = \frac{1}{N_L}\sum_{m=0}^{N_L-1} X_m \exp\left(\frac{2\pi ims}{N_L}\right) \tag{12}$$

$$\hat{C}_s = \frac{1}{N_L}\sum_{m=0}^{N_L-1} C_m \exp\left(\frac{2\pi ims}{N_L}\right) \tag{13}$$

3D mesh generating

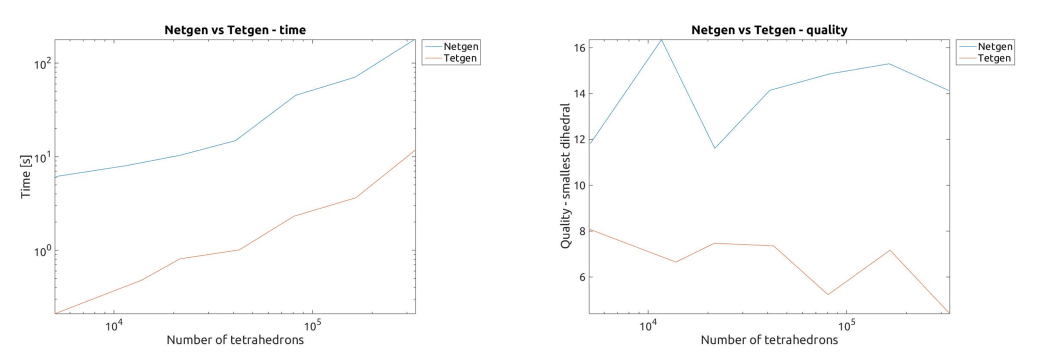
We use Netgen 4.9 and Tetgen 1.4.3 to generate volumetric meshes. Input to these programs is surface mesh in stl format.



Surface mesh & Volumetric mesh.

Netgen vs Tetgen

We have performed the following tests to compare these mesh generators.



Remarks:
For netgen we try generate as good mesh as possible by changing distribution of elements per angle and curve. For tetgen generating mesh is simpler, because in tetgen is options -qqX which ensure that the smallest dihedral is greater than X.

Stokes equation

We use the Stokes equation for incompressible flow for low Reynolds number.

$$-\nabla p + \mu\nabla^2\mathbf{v} + \mathbf{F} = 0 \tag{14}$$

$$\nabla\cdot\mathbf{v} = 0 \tag{15}$$

where: p - pressure, **v** - velocity, μ - viscosity factor, **F** - external force.

Reynolds equation

$$Q = \int\int udxdy \tag{16}$$

$$J = \frac{Q}{W} = -\frac{\sigma}{\mu}\frac{\Delta p}{L} \tag{17}$$

where: Q - flow rate, u - velocity component in x direction, W - width, L - length, Δp - pressure change in x direction.
Stokes equation for *Poiseuille* flow:

$$\mu\frac{\partial u}{\partial z^2} = \frac{dp}{dx}, \quad \frac{\partial u}{\partial x} = 0 \tag{18}$$

$$u = -\frac{1}{2\mu}\frac{dp}{dx}z(b_m - z) \tag{19}$$

$$J = \int_0^b udz = -\frac{b^3}{12\mu}\frac{dp}{dx} \tag{20}$$

Continuity equation:

$$\nabla\cdot\mathbf{v} = 0 \quad \rightarrow \nabla\cdot\mathbf{J} = 0 \tag{21}$$

operator ∇ in **J** is two-dimensional . Substituting the expression for J, we obtain Reynolds equation:

$$\nabla\cdot(b^3\nabla p) = 0 \tag{22}$$

This equation is much simpler than the Stokes equation , because the only unknown is the pressure.

Transmissivity

$$J = -\frac{\sigma}{\mu}\frac{p2-p1}{L} \qquad J = \int v_x dy \tag{23}$$

Poiseuille flow:

$$v_x = -\frac{1}{2\mu}\frac{dp}{dx}z(b-z) \qquad J = -\frac{b^3}{12\mu}\frac{p2-p1}{L} \tag{24}$$

Reynolds:

$$J = -\frac{L}{\int_0^L \frac{1}{b^3}dx}\frac{p2-p1}{L\mu} \tag{25}$$

where: J - flow rate, σ - transmissivity, μ - viscosity.

Simple Test

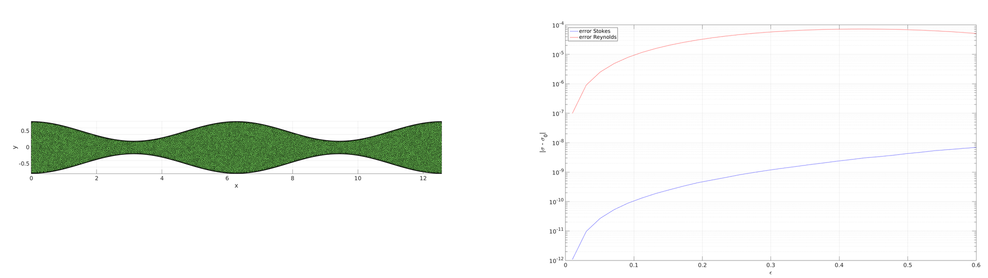
Malevich et al. (2006) solved Stokes flow through channels enclosed by two wavy walls. An example is provided by two - dimensional channel bounded by the surfaces:

$$z^+ = \frac{b_m}{2}(1+\epsilon cos(x)), \quad z^- = -\frac{b_m}{2}(1+\epsilon cos(x)) \tag{26}$$

They calculate transmissivity:

$$\sigma(\epsilon) = \frac{b_m^3}{12} * (1 - 3.14963\epsilon^2 + 4.08109\epsilon^4 - 3.48479\epsilon^6 + 2.93797\epsilon^8 - 2.56771\epsilon^{10} + 2.21983\epsilon^{12} - 1.93018\epsilon^{14} + 1.67294\epsilon^{16} - 1.45302\epsilon^{18} + 1.26017\epsilon^{20} - 1.09411\epsilon^{22} + 0.949113\epsilon^{24} - 0.823912\epsilon^{26} + 0.714804\epsilon^{28} - 0.620463\epsilon^{30}) \tag{27}$$

We can compare σ obtained from Stokes and Reynolds solvers with analytic solution.



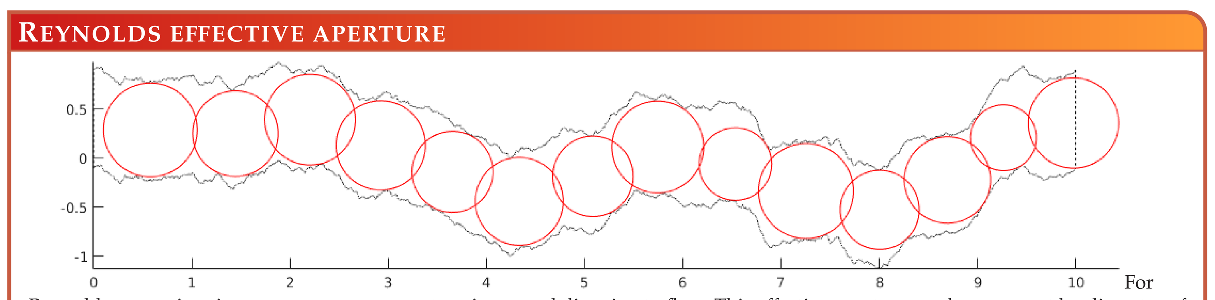
Stokes vs Reynolds general discussion

We can use Reynolds approach when two condition are fulfilled:

- $\phi_1 = \frac{<b>}{\sigma_h} >> 1$,
- $\phi_2 = \frac{lc}{<b>} >> 1$,

Transmissivity is first properties, which we can study to rate differences between Stokes and Reynolds flow. We introduce the following notation: σ_S - transmissivity calculated from Stokes Flow, σ_R - transmissivity in Reynolds approach and σ_{RT} - transmissivity in Reynolds approach with effective aperture.

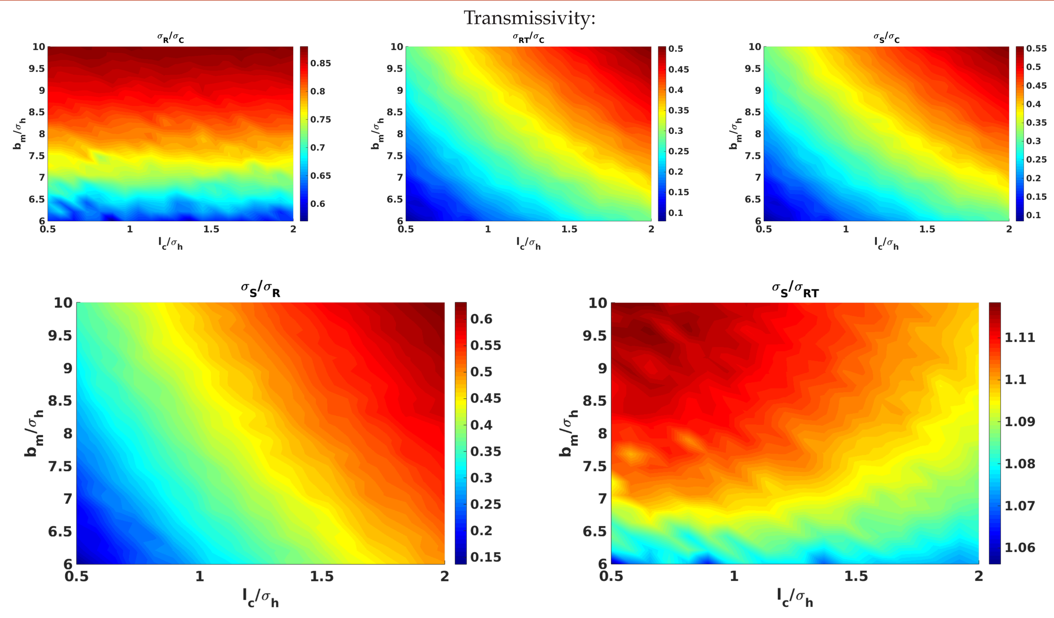
Reynolds effective aperture



For Reynolds approximation we can measure aperture in normal direction to flow. This effective aperture can be represent by diameter of inscribed circles.

Stokes vs Reynolds in 2D

Transmissivity:

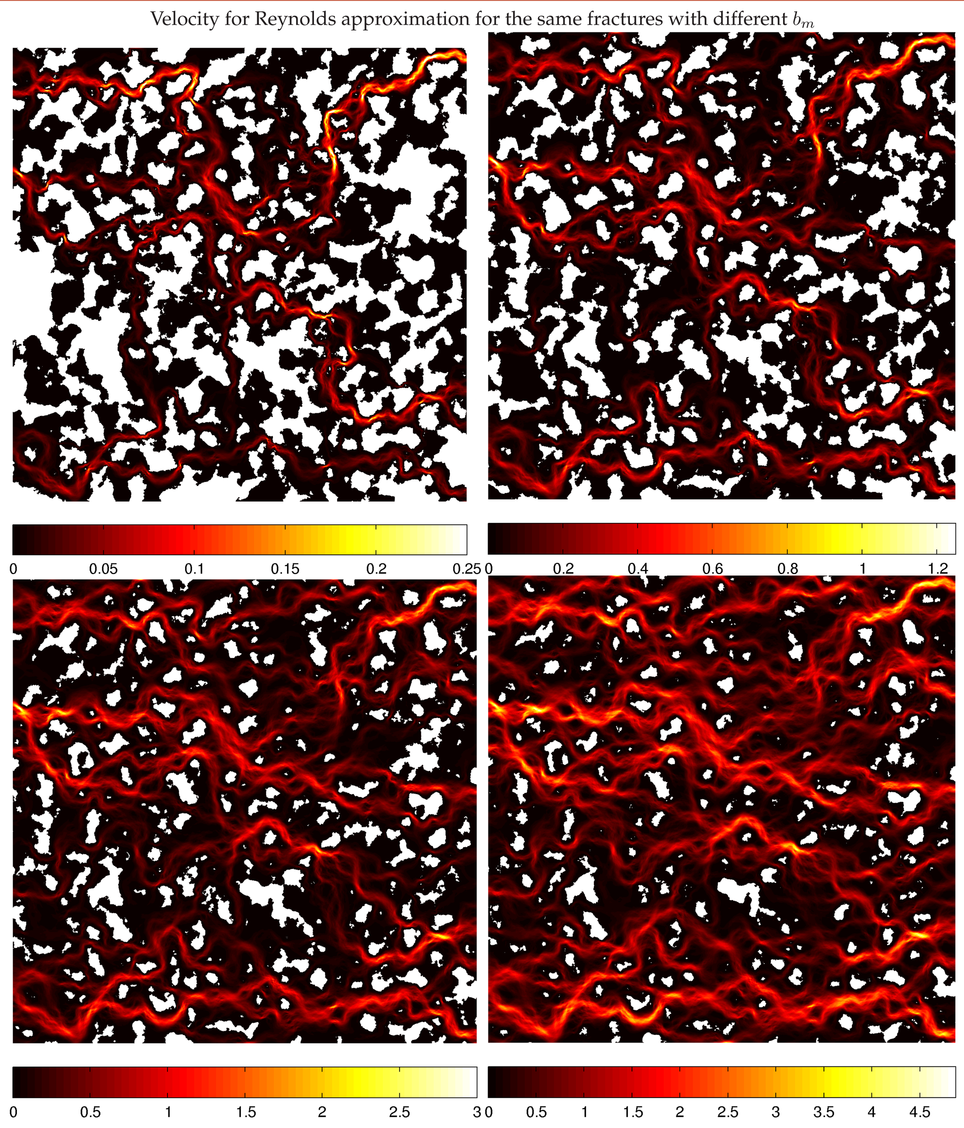


Remarks:

- Reynolds approximation does not depend on correlation length and overestimate transmissivity,
- Reynolds approximation with effective aperture depends on correlation length and underestimate transmissivity,
- Reynolds approximation with effective aperture gives much better approximation.

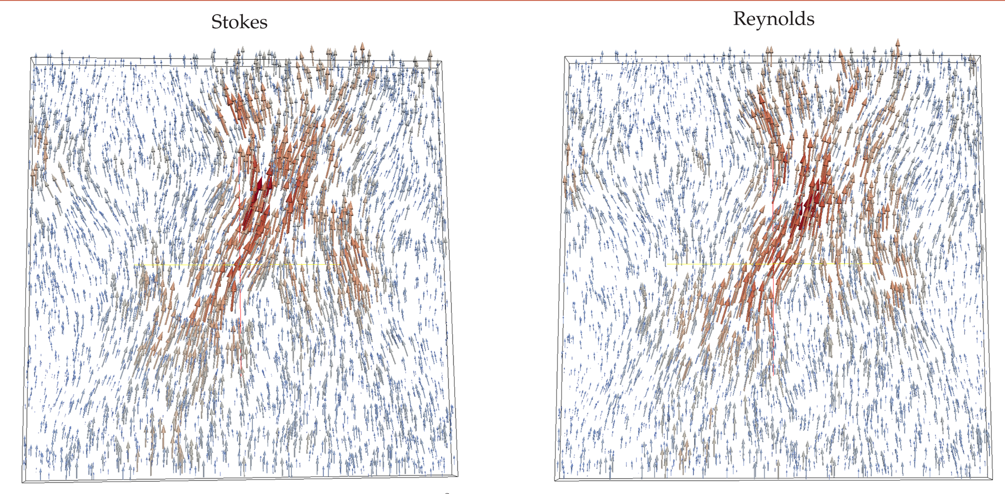
Reynolds 3D examples

Velocity for Reynolds approximation for the same fractures with different b_m



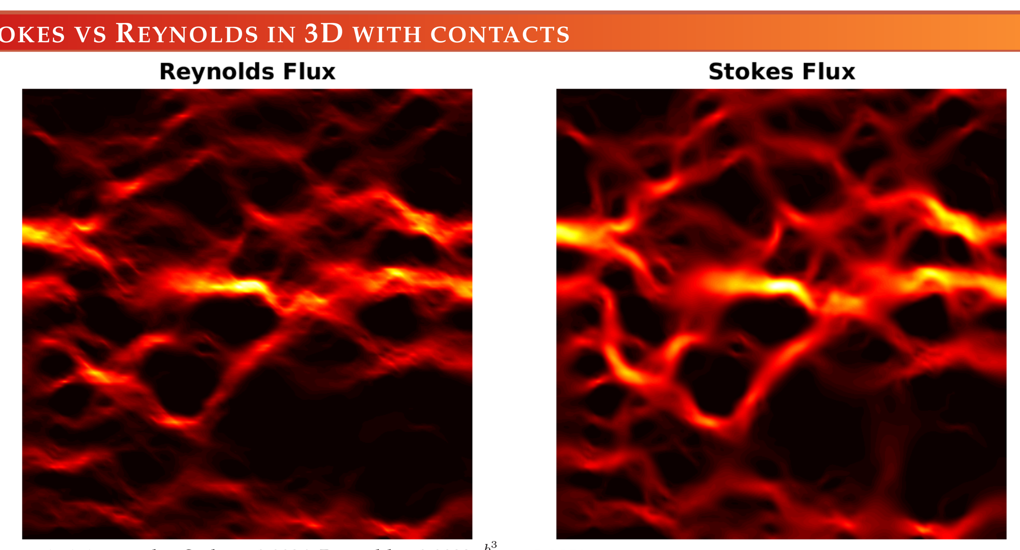
We can see that by b_m we can control fracture closure.

Stokes vs Reynolds in 3D without contacts



Transmissivity results: Reynolds - 9.47, Stokes - 9.33, $\frac{b_m^3}{12} = 10.41$

Stokes vs Reynolds in 3D with contacts



Transmissivity results: Stokes - 0.0024, Reynolds - 0.0032, $\frac{b_m^3}{12} = 0.0071$

Advection & Diffusion

We examine how the differences between velocity fields obtained from the Stokes and Reynolds equations impact on transport process. We use following methods for advection processes: FVM, Characteristics, Markers and Marker chain.

Advection & Diffusion equation:

$$\frac{\partial c}{\partial t} + \mathbf{v}\cdot\nabla c - D\nabla^2 c = 0 \tag{28}$$

where: c - concentration, **v** - velocity, D - diffusion coefficient.

Up scaled Advection & Diffusion equation:

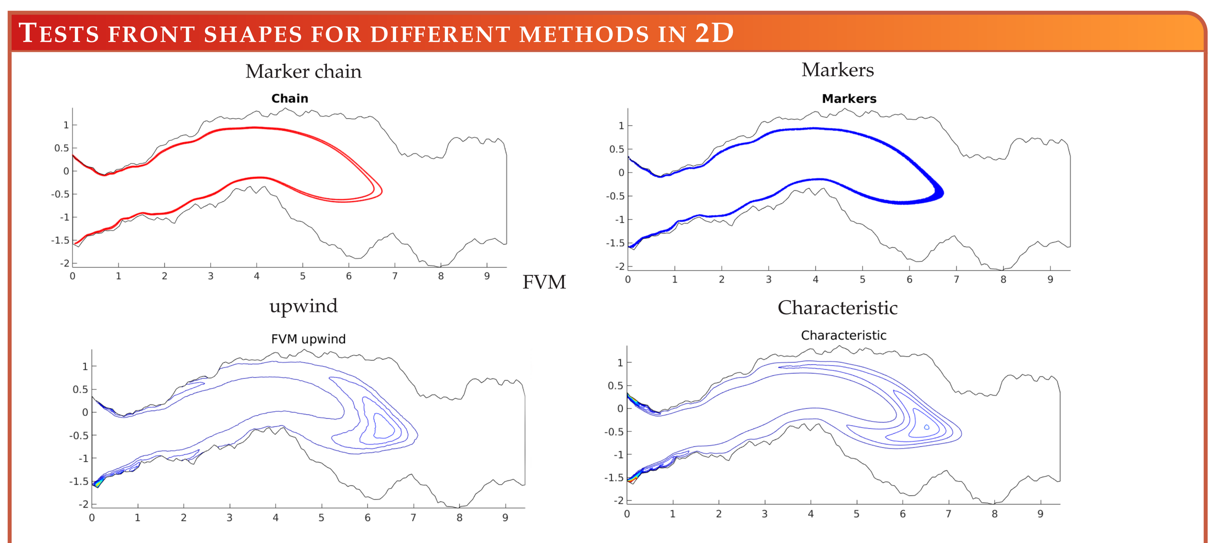
$$\bar{b}\frac{\partial\bar{c}}{\partial t} + \nabla\cdot\left(\bar{b}\bar{\mathbf{v}}\bar{c} - D^*\nabla\bar{c}\right) = 0 \tag{29}$$

where: $\bar{c}$ - local average concentration, $\bar{b}$ - local average aperture, D^* - effective dispersion coefficient.

Mass moments:

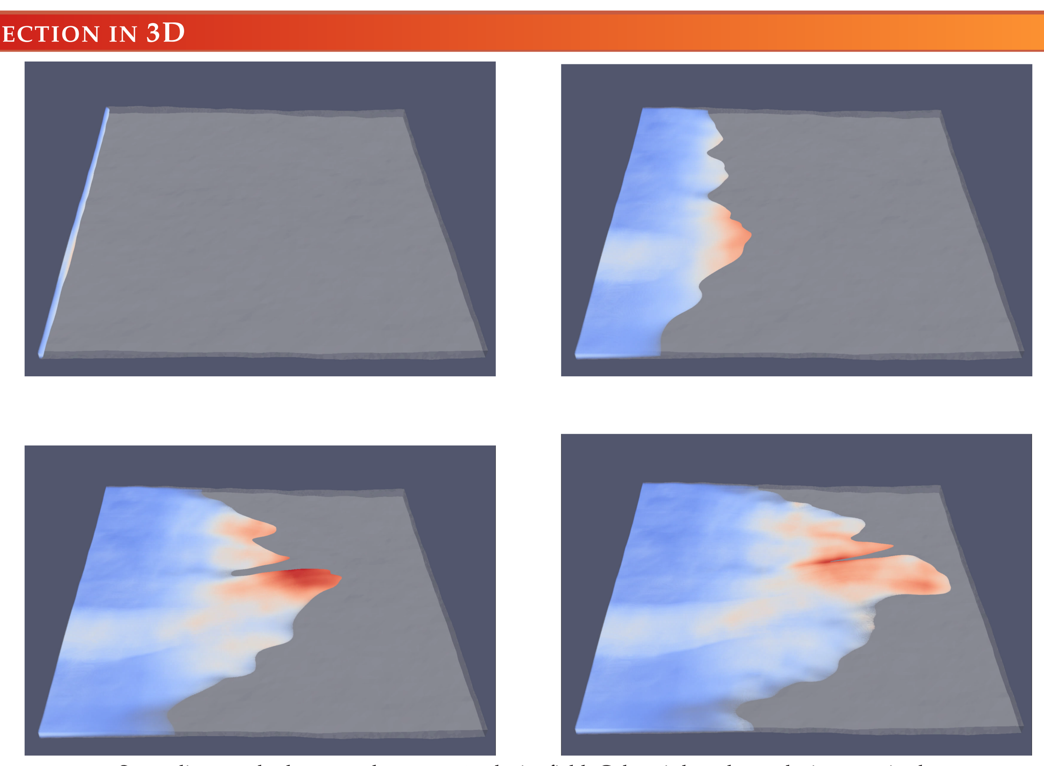
$$M = \sum_i m_i, \quad \mathbf{r}_m = \frac{\sum_i m_i\mathbf{r}_i}{M}, \quad I = \frac{\sum_i m_i\mathbf{r}_i^2}{M}, \quad D^* = \frac{1}{2}\lim_{t\to\infty}\frac{d}{dt}(I - \mathbf{r}_m^2), \quad \mathbf{v}_c = \lim_{t\to\infty}\frac{d\mathbf{r}_m}{dt}.$$

Tests front shapes for different methods in 2D



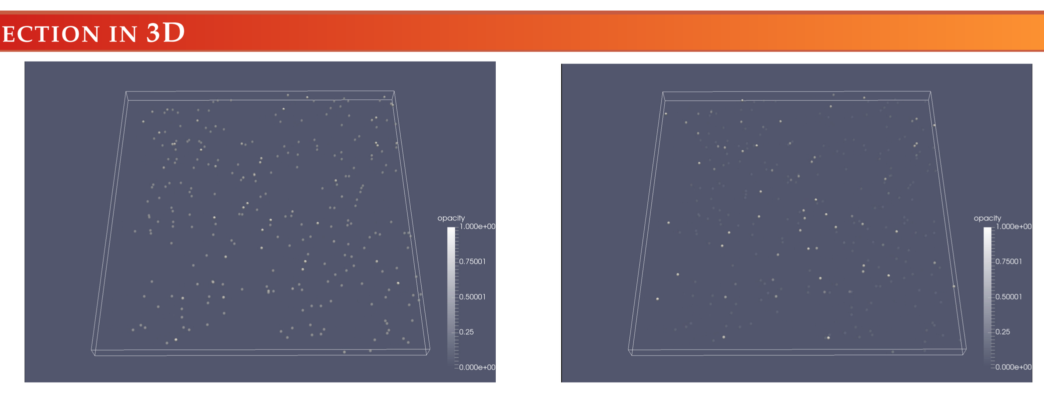
We can visualise velocity field by solving advection equation for concentration of chemical species.

Advection in 3D



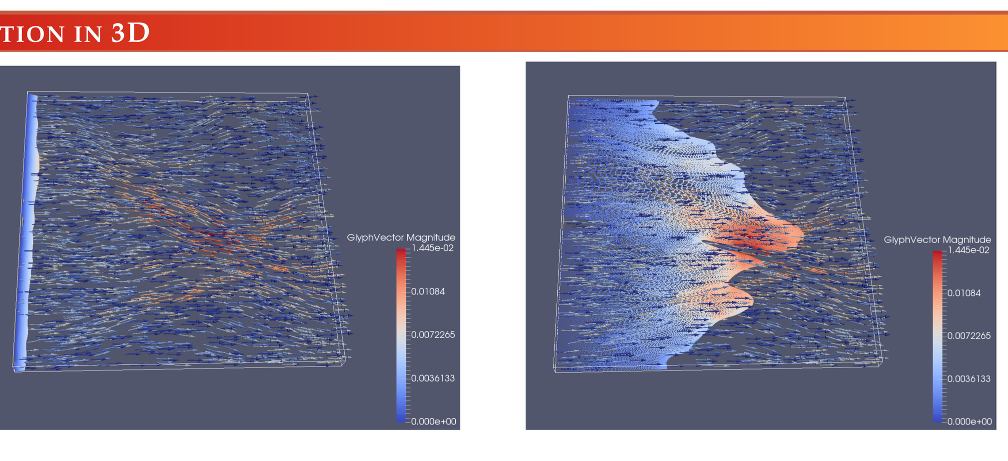
Streamlines method was used to present velocity field. Colour is based on velocity magnitude.

Advection in 3D



Markers method was used to present velocity field. Opacity is based on time life of markers.

Advection in 3D



Markers method was used to present velocity field. Colour is based on velocity magnitude.

Conclusion

- We can describe fracture by statistical properties.
- Tetgen is faster mesh generator, but generate worse mesh quality then netgen. Flag -qq for tetgen sometimes gives unpredictable results.
- Reynolds approach is much easier than Stokes, but we can use it only under certain conditions.
- We can visualise velocity field obtain from Stokes and Reynolds equation by following methods:
  - drawing velocity magnitude or flux,
  - integrating velocity field over time - streamlines, markers
  - studding transport process.

Literature

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- A.E. Malevich, V.V. Mityushev, et al. Stokes flow through a channel with wavy walls, 2006
- G.I. Barenblatt, V.M. Entov - Theory of fluid flows through natural rocks, 1990