UNIVERSITÀ DEGLI STUDI DI BRESCIA

# Matheuristics for routing problems

Claudia Archetti

University of Brescia

VeRoLog 2014, Oslo, June 22-25 2014

# Outline

- Routing problems: evolution of exact and heuristic algorithms
- Matheuristics: why are they so successful?
- Matheuristics for routing problems
  - Decomposition approaches
  - Improvement heuristics
  - B&P/Column generation based heuristics
- Evolution of matheuristics for routing problems
- Conclusions and hints

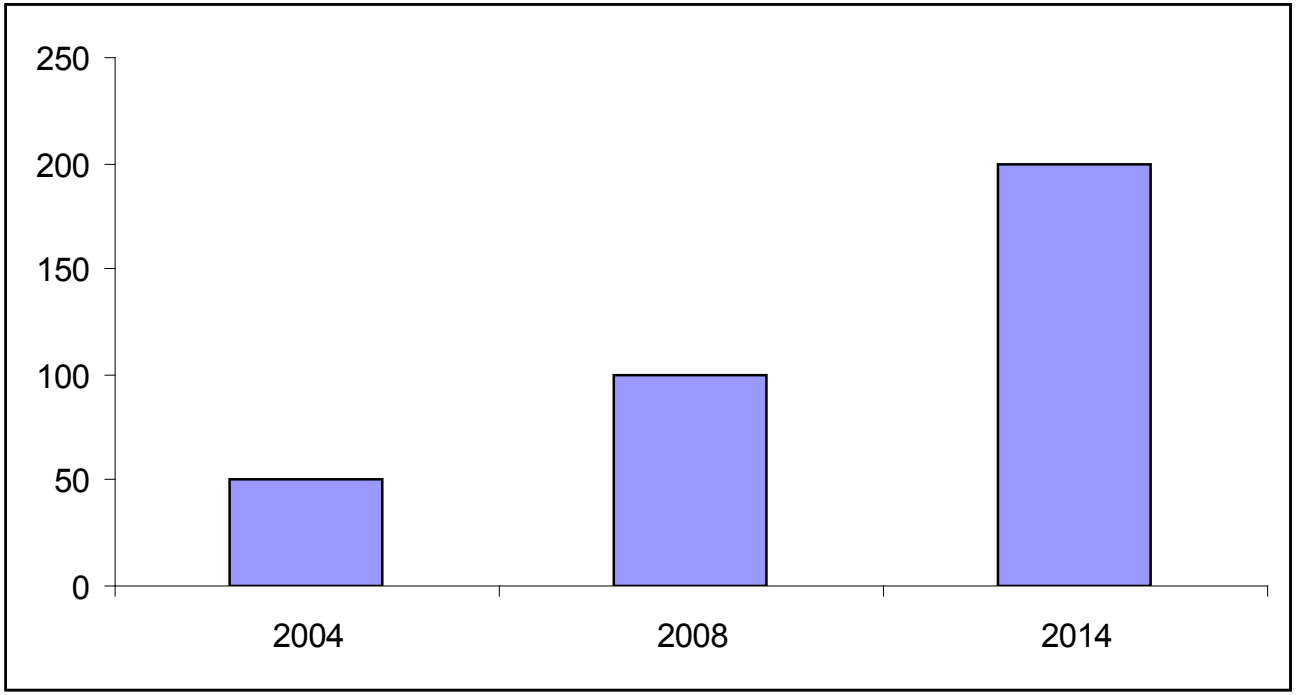
# Vehicle routing problem: exact algorithms

- Branch-and-bound/Branch-and-cut algorithms
  - Vehicle flow formulation
  - Commodity flow formulation
- Branch-and-price algorithms
  - Set partitioning formulation

The current leading methodology is a branch-and-price-and-cut

# Vehicle routing problem: exact algorithms

## Size of instances solved systematically



# Vehicle routing problem: heuristic algorithms

- Classical heuristics
  - Savings algorithm
  - Cluster-first route-second
  - Improvement heuristics
- Metaheuristics
  - Neighborhood based
  - Population based
  - Learning methodologies

# Vehicle routing problem: heuristic algorithms

Metaheuristics provide excellent results, mainly neighborhood and population based

They are typically tested on large instances and can be used to handle practical problems

**BUT**

They are often over engineered

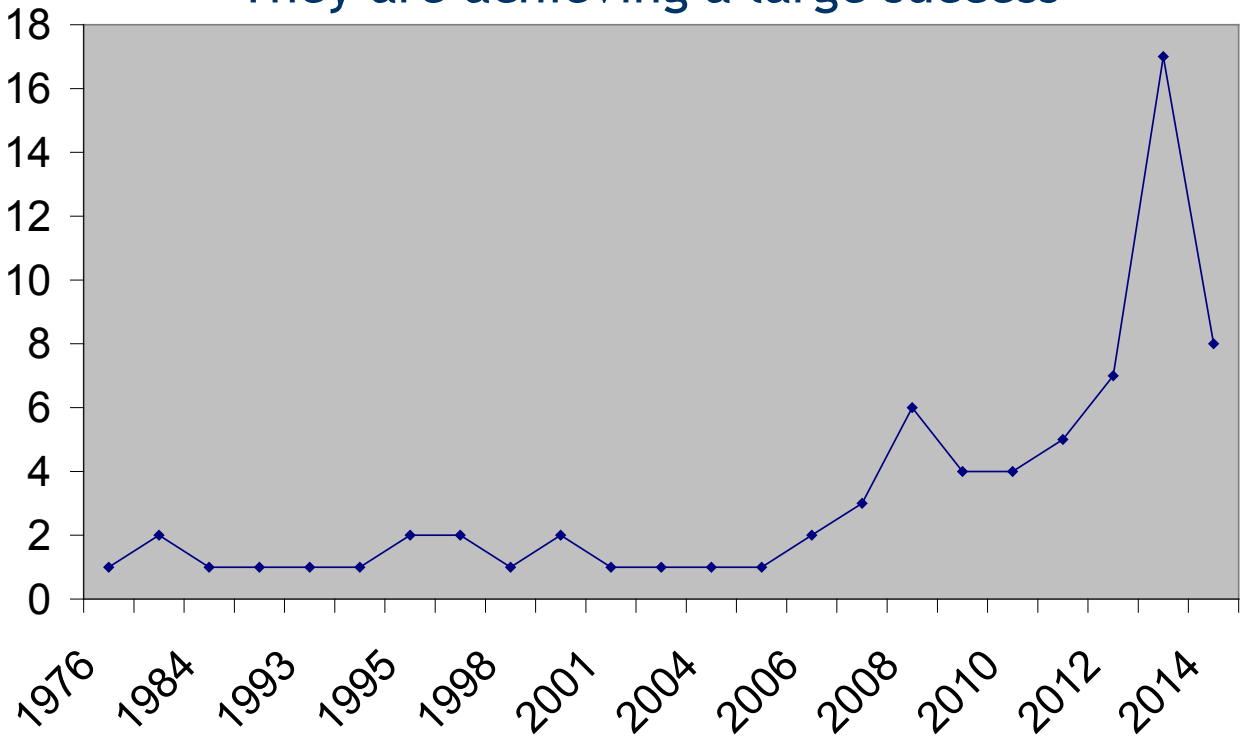
This reduces their flexibility

# Matheuristics

Heuristic algorithms which embed the optimal solution of mathematical programming models, typically MILPs

(Boschetti et al. – in "Hybrid Metaheuristics", 2010)

They are achieving a large success

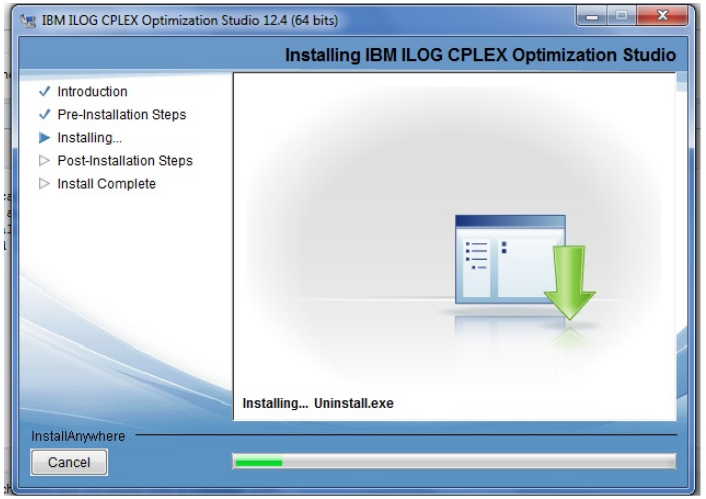



Number of papers devoted to matheuristics for routing problems per year

## Why?

Computational effectiveness of commercial exact solvers

- CPLEX 11 (2007) is almost 30.000 times faster than CPLEX 1 (1991)!
- Gurobi: first version in 2009. Version 4.6 is 16 times faster!



# Matheuristics

**Why?**

Easiness of embedding MILPs in user codes: availability of user-friendly libraries

CPLEX Callable Library

VS.

CPLEX Concert Technology

# Matheuristics

## Why?

Machine improvements: speedup of 2000 times from 1991!


# Matheuristics for routing problems

## CLASSIFICATION

(Archetti and Speranza, 2013)

- Decomposition approaches

  Problems are divided into smaller and simpler subproblems. At least one subproblem is solved through a MILP

- Improvement heuristics

  A MILP is used to improve a solution found by a heuristic

- B&P/Column generation based approaches

  Use the B&P and/or the column generation technique to obtain a heuristic solution

# Decomposition approaches

Problems are divided into smaller and simpler subproblems. At least one subproblem is solved through a MILP

Due to their nature, they are particularly suitable for the solution of complex and integrated problems

Classification:

- Cluster-first route-second
- Two phase approaches
- Partial optimization approaches
- Rolling horizon approaches

# Decomposition approaches: cluster-first route-second

Divide the two main decisions:

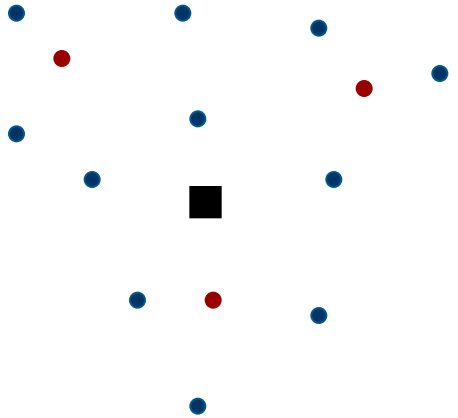
- Assignment of customers to vehicles
- Sequencing of customers for each vehicle

Typically the sequencing of customers is made heuristically

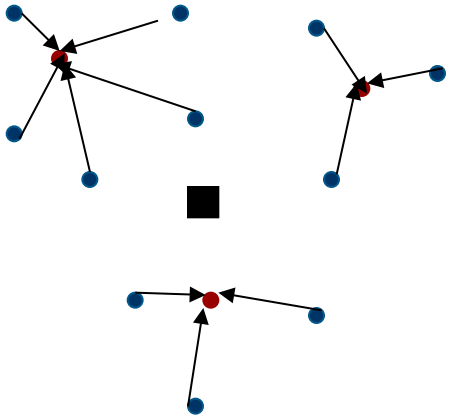
The idea dates back to 1981 (Fisher and Jaikumar) for the solution of the CVRP:

1. Identify seed customers
2. Solve an assignment problem with a MILP by approximating routing costs
3. Solve heuristically a TSP on each cluster

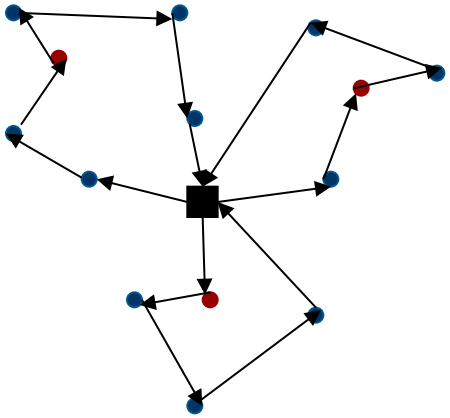
# Decomposition approaches: cluster-first route-second



Seed customers



Assignment



TSP heuristic

# Decomposition approaches: cluster-first route-second

## Most recent approaches are iterative approaches

- The solution obtained in the second phase is used to better approximate the routing costs in the first phase
- The procedure is repeated

# Decomposition approaches: cluster-first route-second

## Applications

- VRP (Fisher and Jaikumar, 1981, Bramel and Simchi-levi, 1995)
- VRPTW (Koskosidis et al., 1992, Bramel and Simchi-Levi, 1996)
- IRP (Federgruen and Zipkin, 1984, Campbell and Savelsbergh, 2004)
- PRP (Absi et al., 2014)
- LRP (Prins et al., 2007)
- ILRP (Guerrero et al., 2013)

# Decomposition approaches: two phase approaches

They decompose the problem into two phases and solve them separately but are not cluster-first route-second

Applications to routing problems involve real-case problems complicated by the presence of many side constraints

# Decomposition approaches: two phase approaches

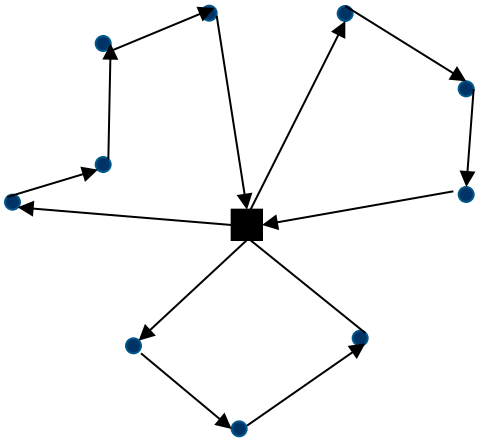
## Applications

- LRP arising in disaster response activities (Yi and Özdamar, 2007)
- IRP with split deliveries (Yu et al., 2008)
- PDP in forestry (Flisberga et al., 2009)
- Routing and scheduling in maritime logistics (Halvorsen-Weare and Fagerholt, 2013)
- Waste collection (Hemmelmayr et al., 2014)
- Bus routing problem (Schittekat et al. 2013, Walteros et al., 2014)
- Home health care (Allaoua et al., 2013)

# Decomposition approaches: partial optimization approaches

One or more MILPs are used to handle a part of the problem, typically the one that does not include routing decisions

## Heuristics



Routing

## MILP/MILPs

Quantities → IRP


Speed → Pollution Routing Problem

Schedule → Routing and Scheduling

# Decomposition approaches: partial optimization approaches

## Applications

- IRP (Coelho et al. 2011 and 2012)
- PRP (Adulyasak et al. 2012)
- Pollution routing problem (Demir et al. 2012)
- Routing and scheduling (Goel and Meisel, 2013)

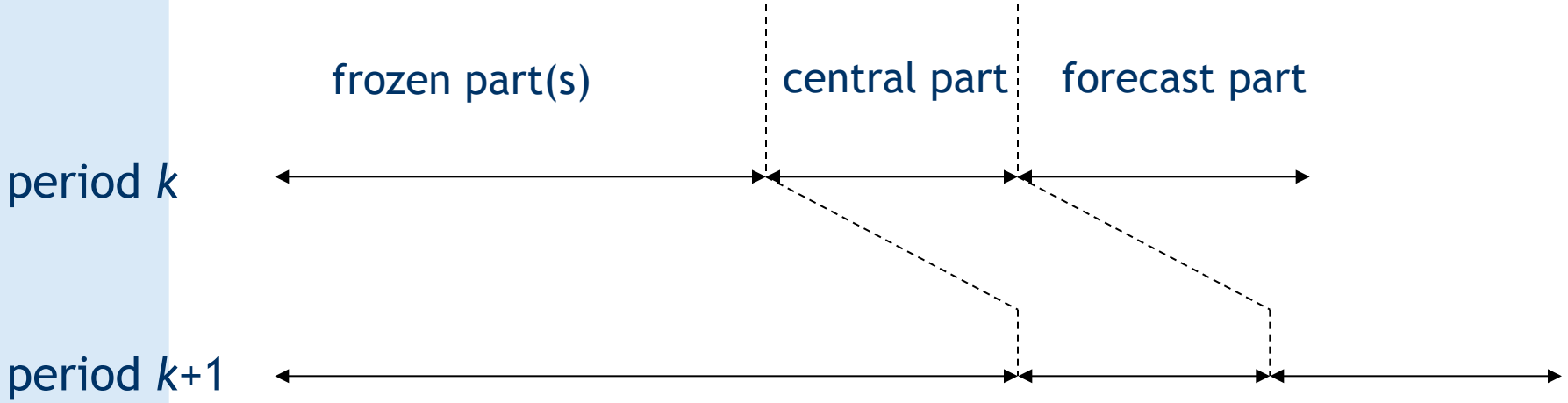
# Decomposition approaches: rolling horizon approaches

Applied to problems where decisions must be taken over time. Subproblems are considered on a rolling horizon basis

Example: Rakke et al., 2011: IRP arising in maritime logistics

- The planning period is divided into smaller periods
- For each sub-period, three partitions are considered:
  - Frozen part
  - Central part
  - Forecast part
- The central part of period *k* becomes the frozen part of period *k+1*

# Decomposition approaches: rolling horizon approaches




For each period $k$, a MILP is solved:

- The values of the variables of the frozen part are fixed
- Integrality is required for the variables of the central part
- Integrality is relaxed for the variables of the forecast part

Agra et al. 2014, similar problem

Combined approach including rolling horizon, local branching, feasibility pump and rounding

# Decomposition approaches: remarks

- The decomposition principle is highly flexible and can be applied to a wide variety of applications
- Decomposition approaches are mainly used to deal with very difficult problems, like IRPs or PRPs
- The choice between a one-shot approach and an iterated approach highly depends on the difficulty of the MILP that must be solved
- The most recent approaches are iterated approaches

# Improvement heuristics

A MILP is used to improve a solution obtained from a heuristic

Different ways to use the MILP:

- As a final improvement procedure
- Integrated in the heuristic scheme

Classification

1. One-shot approaches
2. Iterated approaches

# Improvement heuristics: one-shot approaches

One or more MILPs are solved once to improve a solution obtained from a heuristic.

- There is no feedback to the heuristic
- Given that MILPs are called only once, a higher computing time is typically allowed

# Improvement heuristics: one-shot approaches

## Applications

- IRP (Savelsbergh and Song, 2008, Song and Furman, 2013)
- Maritime logistics (Stalhåne et al., 2012)
- Directed profitable rural postman problem (Archetti et al. 2014)
- Free newspaper delivery problem (Archetti et al. 2013)

# Improvement heuristics: iterated approaches

Use MILPs as operators inside a searching procedure

Different ways:

- As tools to explore the neighborhood
- As intensification tools
- As operators to complete a partial solution

The majority of the approaches proposed in the literature use MILPs as intensification tools

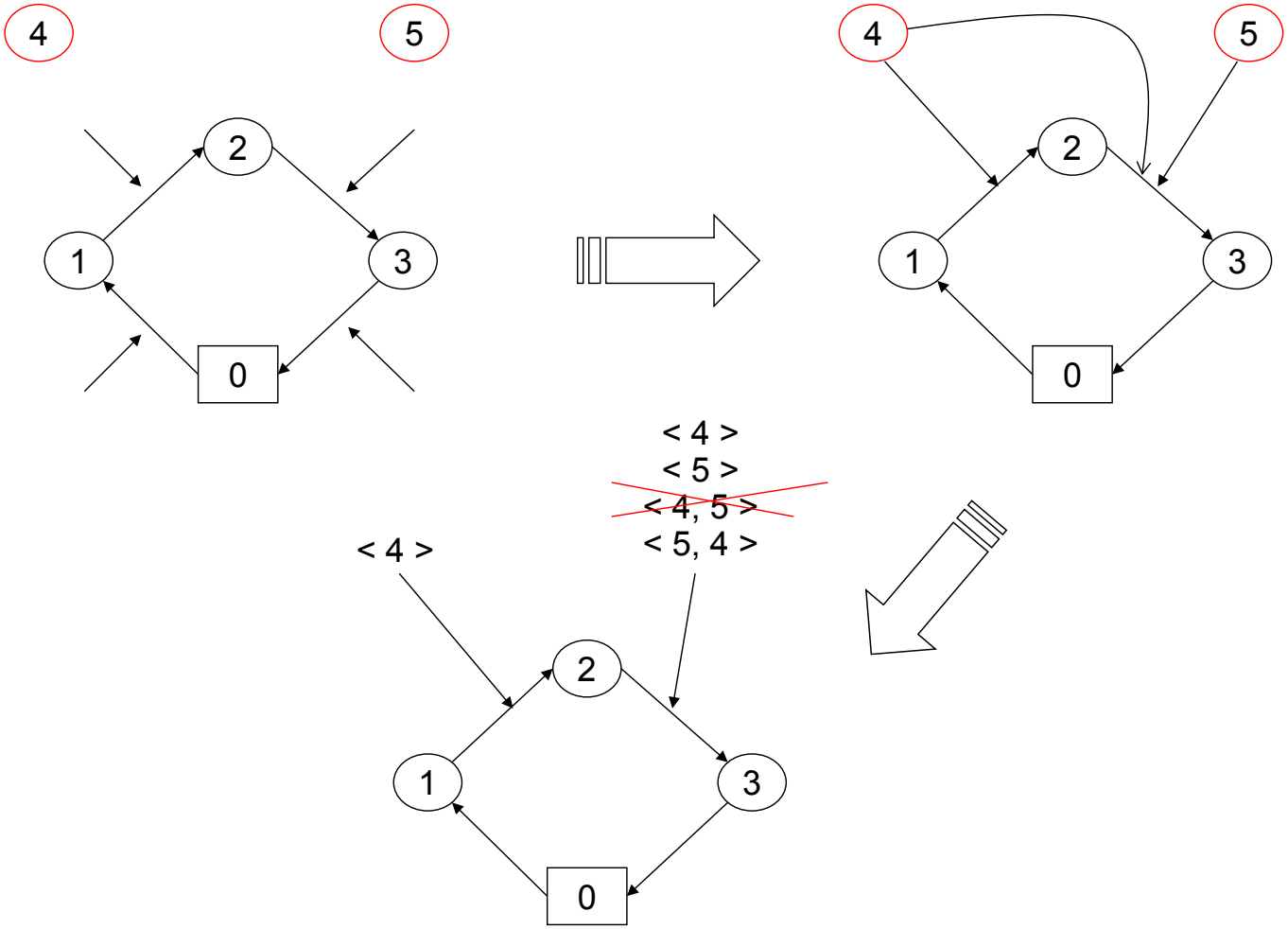
# Improvement heuristics: iterated approaches

1. MILPs as tools to explore the neighborhood

SERR algorithm for the VRP (De Franceschi et al., 2006)

- Start from a heuristic solution
- Remove a subset of customers and shortcut the routes
- Create a large number of sequences visiting the removed customers
- Solve a MILP to optimally reinsert the removed customers
- Repeat until there is an improvement in the last *n* iterations

# Improvement heuristics: iterated approaches



# Improvement heuristics: iterated approaches

## 2. MILPs as intensification tools

Matheuristic for the IRP (Archetti et al., 2012)

IRP:

- Planning horizon
- A single supplier
- A set of customers with daily demands and maximum inventory capacity
- A fleet of capacitated vehicles

Objective: minimize the total cost (transportation cost + inventory cost)

# Improvement heuristics: iterated approaches

## Matheuristic for the IRP (Archetti et al. 2012)

- Tabu Search
- Different MILPs are called whenever the incumbent solution is improved

The idea is to fix routes as they are built in the incumbent solution and use the MILPs to optimize the remaining decisions

1. MILP1: route assignment
2. MILP2: quantity improvement

# Improvement heuristics: iterated approaches

## Route Assignment

Objective: find an optimal assignment of routes to days optimizing the quantities delivered at the same time. Removal of customers is allowed

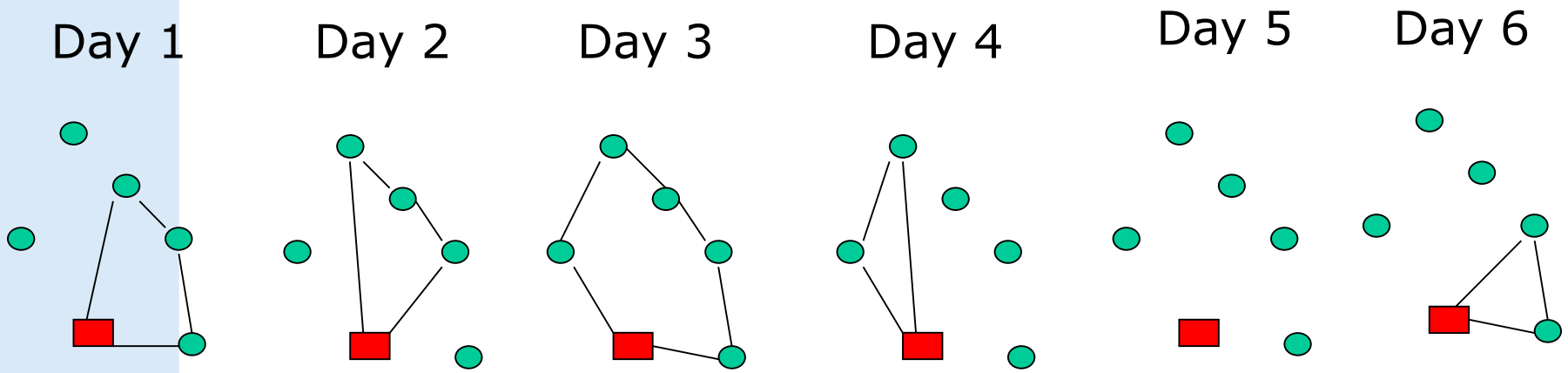
## Formulation:

Min inventory costs – saving for removals

s.t.

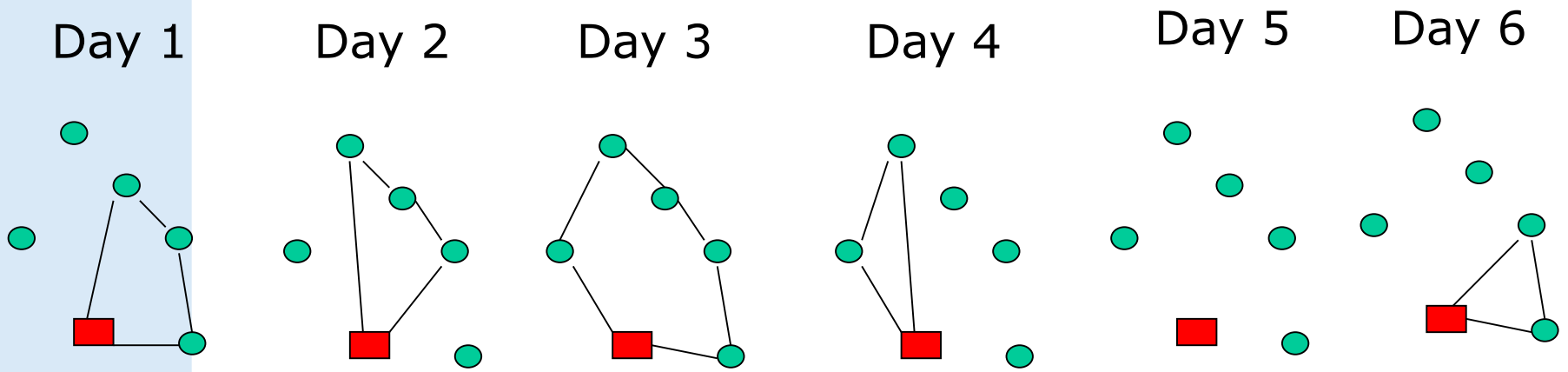
- Stock-out constraints
- Vehicle capacity constraints
- Each route can be assigned to one day at most
- Technical constraints on possibility to serve or remove a customer

# Improvement heuristics: iterated approaches

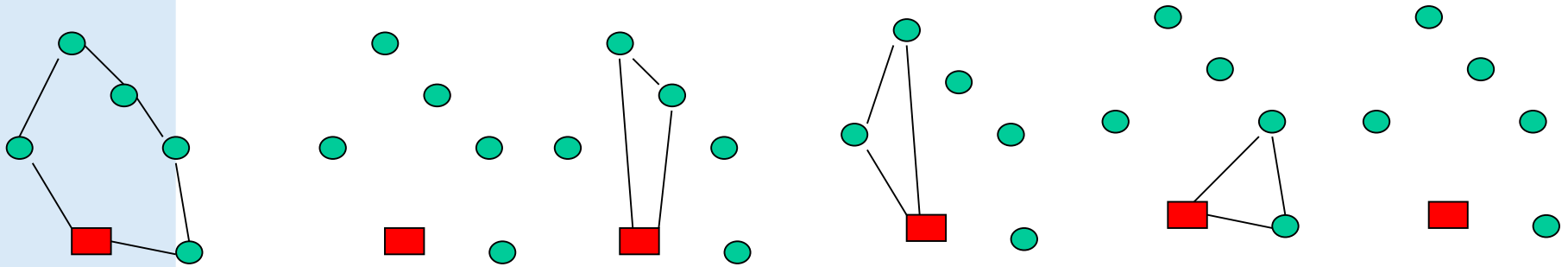



Incumbent solution

# Improvement heuristics: iterated approaches

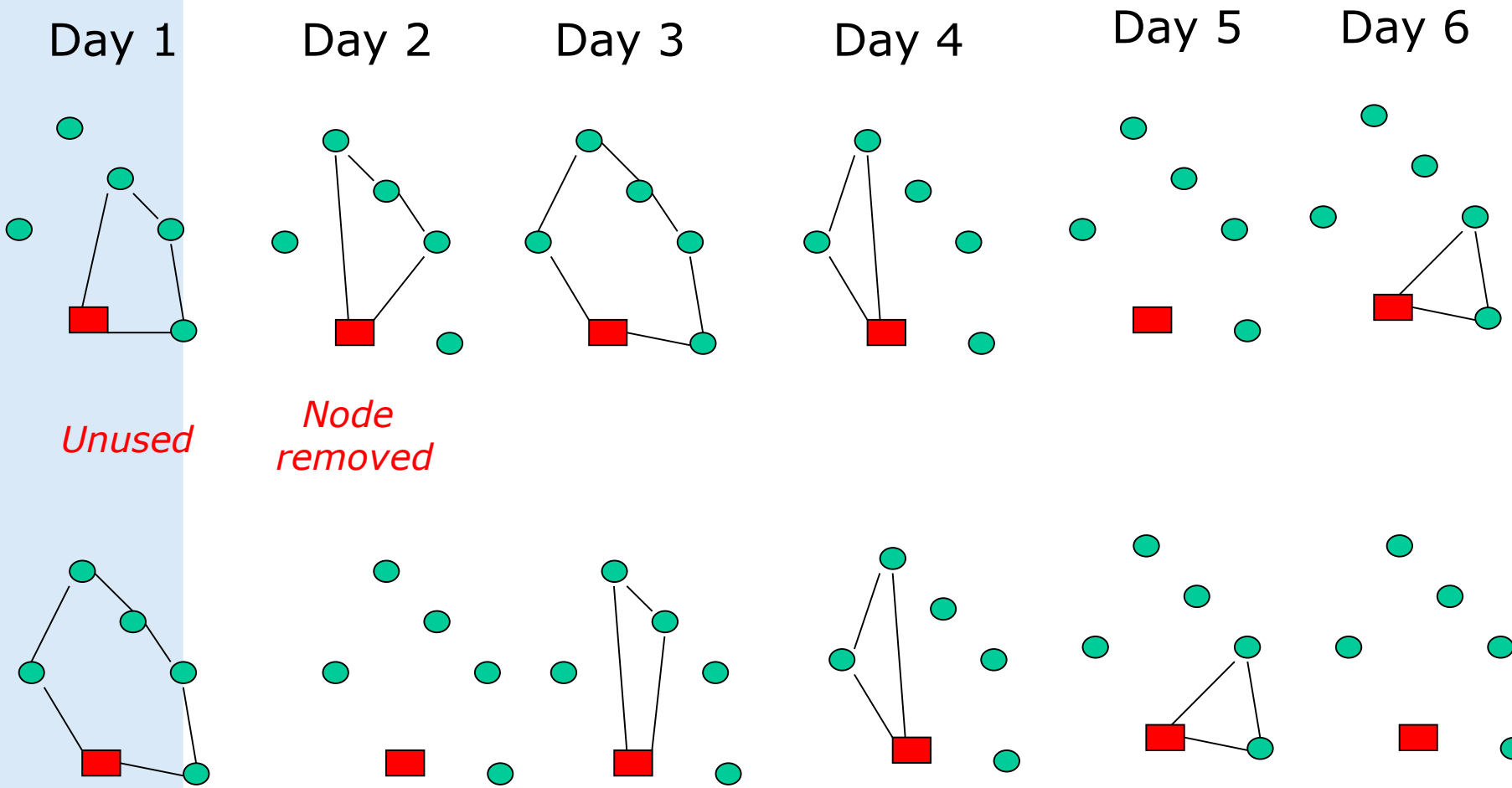



Incumbent solution



The optimal route assignment

# Improvement heuristics: iterated approaches



# Improvement heuristics: iterated approaches

## Quantity Improvement

Objective: improve the incumbent solution by removing customers from routes or inserting customers into routes. Quantities delivered are optimized

- + w.r.t. Route Assignment: we can insert customers into routes
- - w.r.t. Route Assignment: routes can not be assigned to a different day

Why not using a single MILP combing both decisions?

It becomes too cumbersome

# Improvement heuristics: iterated approaches

## Applications

- VRP (Xu and Kelly, 1996, De Franceschi et al., 2006, Yildirim and Çatay, 2014)
- SDVRP (Chen et al., 2007, Gulczynski et al., 2010)
- VRPTW (Prescott-Gagnon et al., 2009)
- Open VRP (Salari et al. 2010)
- *m*-peripathetic VRP (Ngueven et al., 2010)
- VRP with stochastic demands (Rei et al., 2010)
- Multi-commodity one-to-one PDTSP (Rodríguez-Martín and Salazar-González, 2012)
- Routing of barge container ships (Maraš et al., 2013)
- Team orienteering arc routing problem (Archetti et al. 2013)
- Directed profitable rural postman problem (Colombi and Mansini, 2014)

# Improvement heuristics: remarks

- The complexity of the MILP/MILPs to be solved highly influences the nature of the approach: one-shot approaches are more suitable when MILPs are difficult
- The success of these approaches is related to the fact that MILPs typically allow for a more exhaustive search of the neighborhood w.r.t. classical heuristic operators
- Thus, they are typically used as intensification tools

# B&P/Column generation based approaches

Use branch-and-price and/or column generation to build heuristic solutions

- Restricted master heuristics
- Heuristic column generation
- Heuristic branching
- Relaxation based approaches

# B&P/Column generation based approaches: restricted master heuristics

Choose a subset of columns generated by the column generation algorithm and solve a set partitioning formulation

They are typically embedded in exact branch-and-price algorithms

**Applications:**

- VRPTW (Danna and Pape, 2005)
- Routing problems with profits (Archetti et al., 2013 and 2014)

# B&P/Column generation based approaches: heuristic column generation

Generate columns heuristically and solve a set partitioning formulation

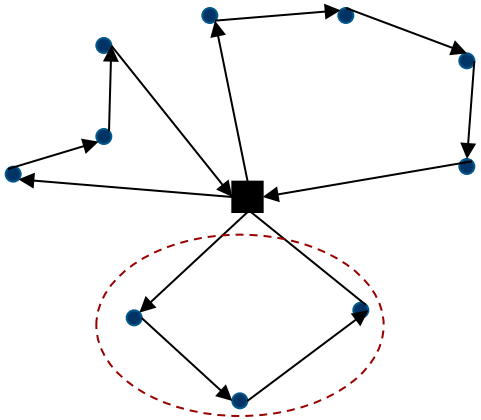
Example: Matheuristic for the VRP by Kelly and Xu (1999)

- Different “simple” heuristics are used to build a set of solutions
- A set partitioning formulation is solved to choose the best routes

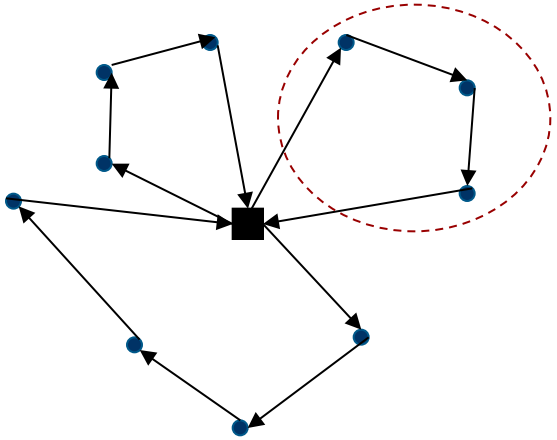
The idea is that a poor solution produced by a simple heuristic may contain a subset of “good” routes

# B&P/Column generation based approaches: heuristic column generation

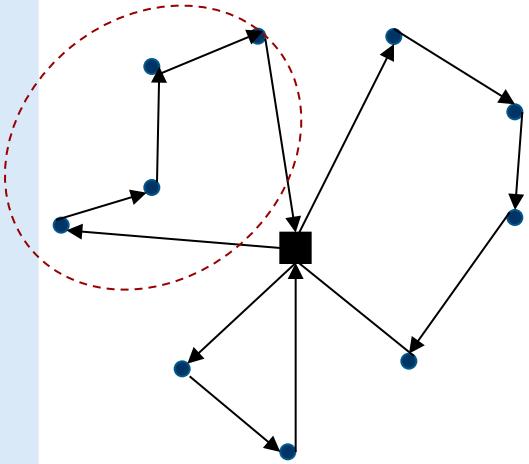
Matheuristic for the VRP by Kelly and Xu (1999)



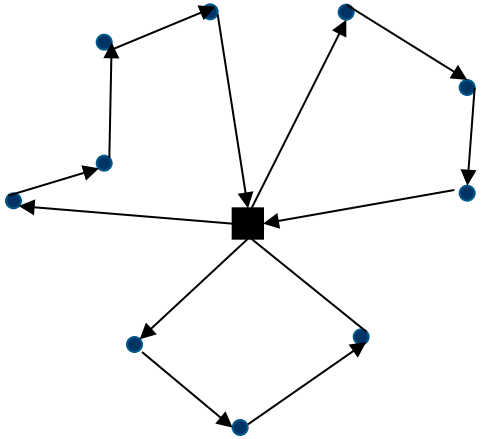
Solution 1



Solution 2



Solution 3



Consolidated solution

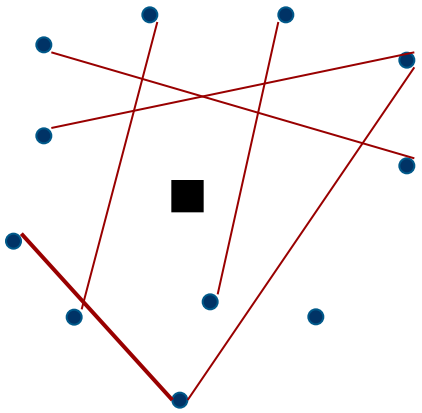
# B&P/Column generation based approaches: heuristic column generation

Different approach: use the information provided by a heuristic to generate a subset of columns

Example: Matheuristic for the Split Delivery VRP by Archetti et al. (2008)

- Run a TS
- Collect information on
  - Traversal of each arc
  - Number of splits to each customer
- Reduce the graph
- Build all possible routes on the reduced graph
- Solve a generalized set covering formulation

# B&P/Column generation based approaches: heuristic column generation



Reduce the graph

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 9 | 3 | 1 | 3 | 2 | 9 |
| 5 | 2 | 10 | 6 | 0 | 5 | 10 |
| 7 | 0 | 11 | 11 | | 6 | 7 |
| 4 | | 8 | 9 | | 0 | 6 |
| 0 | | 2 | 0 | | | 3 |
| | | 0 | | | | 0 |

Generate routes

MILP

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 9 | 3 | 1 | 3 | 2 | 9 |
| 5 | 2 | 10 | 6 | 0 | 5 | 10 |
| 7 | 0 | 11 | 11 | | 6 | 7 |
| 4 | | 8 | 9 | | 0 | 6 |
| 0 | | 2 | 0 | | | 3 |
| | | 0 | | | | 0 |

# B&P/Column generation based approaches: heuristic column generation

## Applications

- VRP (Foster and Ryan, 1976, Cullen et al., 1981, Rochat and Taillard, 1995, Kelly and Xu, 1999, Subramanian et al., 2013, Villegas et al., 2013)
- IRP (Aghezzaf et al., 2006, 2008 and 2009)
- SDVRP (Archetti et al., 2008)
- Rich VRP (Schmid et al., 2009)
- Dial-a-ride problem (Parragh et al., 2012, Parragh and Schmid, 2013)
- VRP with stochastic demands (Mendoza and Villegas, 2013)
- Routing and scheduling (Cohn and Barnhart, 2003, Pillac et al., 2013)
- Periodic VRP (Cacchiani et al., 2014)

# B&P/Column generation based approaches

## Heuristic branching

Branch-and-price algorithms where branching is performed heuristically so to prune a large number of B&B nodes

**Applications**: Routing and scheduling (Cordeau et al., 2001)

## Relaxation based approaches

Construct a feasible solution from a solution obtained by a relaxation of the problem

**Applications:**

SDVRP (Sierksma and Tijssen, 1998, Jin et al., 2008)

# B&P/Column generation based approaches: remarks

- They are becoming more and more popular, thanks to the success of branch-and-price exact algorithms
- Most of the approaches rely on heuristic schemes for the generation of columns
- Set covering/set partitioning formulations are easily adaptable to problem variants

# Matheuristics for routing problems: a look from above

- Many approaches belong to different classes

  Example: the matheuristic for the SDVRP by Archetti et al., 2008, belongs to the class of improvement heuristics and B&P/column generation base approaches

- A basic procedure can be used in different ways

  Example:

1. Run a heuristic to obtain a solution and gather information

   2.a Build a large number of solutions and choose the best one through a set partitioning/covering model

   2.b Improve the heuristic solution through a MILP which makes an extensive search in the neighborhood

# Matheuristics for routing problems: a look from above

Many different ways of integrating MILPs with heuristics

How to find the best scheme?

Difficult task

It depends on:

- Problem characteristics
- Computational burden
- Underlying heuristic scheme:
  - Some algorithms heavily rely on a good heuristic and the MILPs are used to only slightly improve the solution
  - The MILPs can be called only once or several times

## Number of papers appeared in the literature for each class

| | Decomposition approaches | Improvement heuristics | B&P/Column generation based approaches |
|---|---|---|---|
| Within 2000 | 6 | 1 | 8 |
| After 2000 | 19 | 20 | 18 |

# Matheuristics for routing problems

## Number of papers appeared in the literature for problem classes

| | Decomposition approaches | Improvement heuristics | B&P/Column generation based approaches |
|---|---|---|---|
| VRP | 7 | 8 | 13 |
| SDVRP | -- | 2 | 5 |
| PDVRP | -- | 3 | 1 |
| IRP | 9 | 6 | 3 |
| PRP | 3 | -- | -- |
| LRP | 4 | -- | -- |
| RPP | -- | 2 | 4 |
| RSP | 2 | -- | 3 |

# Conclusions

- The largest number of contributions are proposed for the VRP
- Recent contributions are mainly improvement heuristics or B&P/column generation based approaches
- Decomposition approaches are still popular to solve complex problems like IRP, PRP and LRP

# Future?

Can matheuristics become the leading heuristic methodology for routing problems?

- The potentiality of commercial solvers can be further exploited
- It's becoming easier and easier to embed them in user codes

HOWEVER

Computational time is certainly an issue

# Challenge

Build heuristic algorithms which are flexible enough to handle many problem variants

Probably MILPs can help to achieve this goal

# THANK YOU FOR YOUR ATTENTION!