DaCS
Durable advanced Concrete Solutions

Report No. 02

Early age crack assessment
Codes, guidelines and calculation methods

WP 1.2 Calculation of crack spacing and crack widths

Author(s)
Anja Birgitta Estensen Klausen


Shrinkage crack in the wing dike of the St. Francis Dam, 1928
www.scvhistory.com

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AUTHOR(S)
Anja Birgitta Estensen Klausen

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ABSTRACT

Concrete in the hardening phase is subjected to volume changes caused by thermal dilation (TD) and autogenous deformation (AD). If these volume changes are restrained, they may lead to cracking. The major concern when it comes to such early age cracking is "through-cracking", which may go through the whole thickness of the concrete member and further lead to functionality-, durability- and esthetical problems. The current memo describes and compares a selection of prevailing regulations and requirements regarding cracking and design in the serviceability limit state (SLS) with respect to early age volume changes. In addition, different guidelines and calculation methods to evaluate early age cracking have been discussed and compared. This comparison is further illustrated by an example, where the restrained strain and the succeeding crack widths have been calculated and evaluated by the different guidelines in question.

PREPARED BY
Anja Klausen

CHECKED BY
Terje Kanstad

APPROVED BY
Kjell Tore Fosså

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Preface

This report and the related work have been carried out within the research project “Durable advanced Concrete Solutions” (DaCS). The project started in 2015, and is a 4-years’ research program with focus on concrete structures for severe conditions. The main R&D objective is to enable production of sustainable and durable concrete structures for coastal and offshore arctic applications, considering both production and service life phases.

Multiple researchers from the Norwegian University of Science and Technology, SINTEF and industry partners, together with 3 PhD-students and a number of MSc-students, work on four focus areas:

- WP 1: Early age cracking and crack calculation in design
- WP 2: Production and documentation of frost resistant concrete
- WP 3: Concrete ice abrasion
- WP 4: Ductile, durable Lightweight Aggregate Concrete

The industry partners are leading multinational companies in the cement and building industry, together with Norwegian engineering companies and offshore industry. Together our aim is to improve the concrete material quality to produce environmentally friendly and durable concrete structures for future arctic offshore and coastal applications. Combining the existing knowledge and experience cross industries with the recognised research capabilities of NTNU and SINTEF, provides a good basis for both high quality and industry relevant research. Achieving the overall research objectives, will strengthen the Norwegian industry’s relevance, attractiveness and competitiveness.

The DaCS project partners are: Kværner AS (project owner), Axion AS (representing Stalite), AF Gruppen Norge AS, Concrete Structures AS, Mapei AS, Multiconsult AS, NorBetong AS, Norcem AS, NPRA (Statens Vegvesen), Norges Teknisk-Naturvitenskapelige Universitet (NTNU), SINTEF Byggforsk, Skanska Norge AS, Unicon AS and Veidekke Entreprenør AS. The project has received financial contribution from the Norwegian Research Council.
For more information, see https://www.sintef.no/projectweb/dacs/.

Summary

Concrete in the hardening phase is subjected to volume changes caused by thermal dilation (TD) and autogenous deformation (AD). If these volume changes are restrained, they may lead to cracking. The major concern when it comes to such early age cracking is "through-cracking", which may go through the whole thickness of the concrete member and further lead to functionality-, durability- and esthetical problems. The current memo describes and compares a selection of prevailing regulations and requirements regarding cracking and design in the serviceability limit state (SLS) with respect to early age volume changes. In addition, different guidelines and calculation methods to evaluate early age cracking have been discussed and compared. This comparison is further illustrated by an example, where the restrained strain and the succeeding crack widths have been calculated and evaluated by the different guidelines in question.

The currently investigated standards and guidelines are Eurocode 2, CIRIA C660, the upcoming revised Eurocode 2 and the appurtenant Annex D, Model Code 2010, CEOS.fr, NS3473, JCI Guideline and the BAW Guideline. The model codes Eurocode 2, Model Code 2010 and NS3473 all state that imposed deformations, temperature, creep and shrinkage should be included when it comes to crack risk assessment and crack width calculations. It could thus be strongly argued that also early age effects, i.e. thermal dilation and shrinkage, should be taken into consideration during design. However, in general, limited information is provided on how to include such early age effects, and hence several assumptions and adjustments had to be made during the currently performed calculations. These interpretations must be considered subjective, and the calculations and appurtenant results are thus to some degree affected by the background and experience of the author.

All the investigated calculation approaches predicted that cracking would occur for the given wall example. Several of the determined parameters varied considerably between the different approaches, e.g. reduction parameters, restraint, crack-inducing strain and transfer length. However, for some of the guidelines, these different parameters seemed to neutralize each other, resulting in quite similar calculated crack widths. The calculation approaches found in Eurocode 2, BAW, and NS3473, provided crack widths that were somewhat larger than for the other methods. For the given example, the calculated crack width at 28 days determined by seven different calculation approaches varied between 0.109 mm and 0.254 mm. In addition, the calculations based on the revised Eurocode 2 decreased the calculated 28-day crack width by approximately 38% when compared with the existing Eurocode 2. The crack widths found from the revised Eurocode 2 also gave very good agreement with the crack widths found based on CIRIA, CEOS.fr (MC2010) as well as the JCI guideline. However, it should be noticed that the current calculation approaches should be interpreted as simplified and rough estimation methods, meaning that the user should be encouraged to take the step towards more advanced methods if necessary.

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1 Introduction

Concrete in the hardening phase is subjected to volume changes caused by thermal dilation (TD) and autogenous deformation (AD). If these volume changes are restrained, they may lead to cracking. The major concern when it comes to early age cracking is "through-cracking", which, as the name describes, may go through the whole thickness of the concrete member and further lead to functionality-, durability- and esthetical problems.

One aim of the current memo is to identify and compare a selection of prevailing regulations and requirements regarding cracking and design in the serviceability limit state (SLS) with respect to early age volume changes. In addition, different guidelines and calculation methods to evaluate cracking induced by restrained early age volume changes have been discussed and compared. This comparison is further illustrated by an example, where the restrained strain and the succeeding crack widths have been calculated and evaluated by the different guidelines in question.

2 Codes and Guidelines

When it comes to design in the SLS, regulations and requirements are provided by Eurocode 2 [1] and NS3473 [2]. NS3473 is actually withdrawn and superseded by Eurocode 2, but it is still used for offshore concrete installations in Norway, as Eurocode 2 is defined not to cover such structures. In the following, these two standards are reviewed with respect to cracking caused by imposed early age deformations.

Eurocode 2 is an abbreviation for *NS-EN 1992, Eurocode 2: Design of concrete structures*. The document consists of four parts, and two of them are relevant for and referred to in the current memo:

- *NS-EN 1992-1-1:2004 - Design of concrete structures. Part 1-1: General rules and rules for buildings,* [3]
- *NS-EN 1992-3:2006 - Design of concrete structures. Part 3: Liquid retaining and containment structures,* [4]

Eurocode 2 Part 1-1 *Paragraph 2.3.3 Concrete volume changes* states that consequences of volume changes caused by temperature, creep and shrinkage should be taken into consideration during design. Further, *Paragraph 7.3 Crack width limitation* states that "*Cracking is common in reinforced concrete structures exposed to flexural, shear, torsion or direct tensile loading caused by either direct loading, restraint or imposed deformations*" [3]. Early age volume changes are by definition "imposed deformations", and cracking induced by such volume changes are thus covered by the general crack width limitations presented in Eurocode 2 Part 1-1 table 7.1N.

Eurocode 2 Part 3 provide additional elaboration of early age effects when it comes to the SLS and cracking. *Paragraph 7.3.4 Minimising cracking due to restrained imposed deformations (116)* underlines that *"Special care should be taken where members are subjected to tensile stresses due to the restraint of shrinkage or thermal movements"* [4]. *Paragraph 7.3.5 Minimising cracking due to restrained imposed deformations* describes measures on how to minimise the formation of cracks caused by restrained imposed deformations, e.g. limiting the temperature rise due to cement hydration, removing or reducing restraints, etc. The paragraph further referrers to Annex L, which provides a simplified method to assess strains and stresses in restrained concrete members [4].

A new Eurocode 2 annex regarding early age cracking is currently being prepared: *Annex D, Guidance to restrict early age cracking*. Annex D is meant to give guidance in cases where Eurocode 2 is used to limit early age cracking, and it is planned implemented together with the revised Eurocode 2 which is to be published in 2025. In the current report, the calculations by Annex D are based on the versions dated June 2017 (Annex D) and May 2017 (Eurocode 2).

As previously described, the withdrawn standard *NS3473: Concrete structures – Design and detailing rules* is still used for offshore concrete structure design in Norway. NS3473 addresses imposed deformation, temperature, creep and shrinkage in several paragraphs as cited in the following [2]:

- *Paragraph 9.1.2: "The effect of temperature changes, creep and shrinkage should be taken into consideration when these effects influence the functionality and capacity of the concrete structure"*
- *Paragraph 15.1.3: "Temperature, shrinkage and imposed deformations should be included and combined with structural loading"*
- *Paragraph 15.6.1: "Stress from temperature, creep, shrinkage, imposed deformations should be taken into consideration if these affect the crack development"*

In addition, the characteristic crack width expression in NS3473 *Paragraph 15.6.2* includes a parameter representing the free shrinkage strain. Early age stress development arises due to restrained volume changes caused by temperature and shrinkage, and it could thus be strongly argued that early age effects should be taken into consideration during design according to NS3473. Consequently, the prevailing crack width limits defined in NS3473 could be interpreted to also include cracking caused by early age volume changes.

The publication ***fib*** Model Code 2010 [5] presents new developments and ideas with regard to concrete structures and structural materials, and it aims to serve as a basis for codes for concrete structures. When it comes to cracking, ***fib*** Model Code 2010 *Paragraph 7.6.4.4.2* states: *"The effect of cracking may be considered for the analysis of combined effects of loads and imposed deformations. Hence, where cracking is due to imposed deformation and loads, the steel stresses at the cracks due to loads as well as imposed deformations should be taken into account"* [5].

Summarized; NS3473, Eurocode 2 and ***fib*** Model Code 2010 all suggest that early age effects should be taken into consideration when it comes to cracking and design in the serviceability limit state. Both NS3473 and ***fib*** Model Code 2010 also describe that temperature and shrinkage effects should be combined with structural loading, while such a combination is more vaguely formulated in Eurocode 2. It has however not been common practice to combine early age crack widths with those arising from structural loading, and there seems to be an overall perception that stresses due to early age volume changes are self-equilibrating over time, with creep being a significant factor.

For massive concrete structures, the above described codes do neither reflect the complete structural behaviour with respect to early age effects nor describe techniques and methods on how to include such effects. Therefore, several research institutes as well as research projects have proposed dedicated guidelines to address these issues. The below listed guidelines contain more precise calculation methods and applications when it comes to early age cracking in massive concrete structures:

- CIRIA (Construction Industry Research and Information Association) C660:
  Early age thermal crack control in concrete, [6]
- Research Project CEOS.fr[1]:
  Control of Cracking in Reinforced Concrete Structures, [7]
- JCI (Japanese Concrete Institution):
  JCI Guidelines for Control of Cracking of Mass Concrete, [8]
- BAW (Bundesanstalt für Wasserbau):
  Rissbreitenbegrenzung für frühen Zwang in massiven Wasserbauwerken (MFZ) [9]
  (In English: Crack width limitation for early age restraint in massive retaining structures)

The English guideline CIRIA C660: *Early age thermal crack control in concrete* provides clearly described methods to estimate the magnitude of crack-inducing strain and the risk of cracking. The guideline further provides guidance on the design of reinforcement to control crack widths where cracking is predicted. CIRIA C660 states that the designer should consider whether cracking due to subsequent deformations will add to early age effects, and thus decide if early age effects should be combined with structural loads or not. If so, the guideline further provides guidance on how to deal with long-term thermal strains and drying shrinkage. CIRIA C660 intends to be Eurocode 2-compliant throughout.

CEOS.fr was a French research programme (2008-2015) that aimed to *"make a significant step forward in the engineering capabilities for predicting the expected crack pattern of special structures under anticipated in-service or extreme conditions"* [7]. One of the project deliveries was the guideline CEOS.fr: *Control of Cracking in Reinforced Concrete Structures.* The guideline is mainly dedicated to the proposed rules (i.e. engineering

[1] CEOS (Comportement et l'Evaluation des Ouvrages Spéciaux– fissuration, retrait. In English: Behaviour and evaluation of special structures – cracking and shrinkage)

rules for crack width and space assessment of possible crack patterns in massive structures) based on the outcome from the CEOS.fr project, and aims to supplement the rules presented in Eurocode 2 and ***fib*** Model Code 2010.

The Japanese guideline *JCI: JCI Guidelines for Control of Cracking of Mass Concrete* was originally published in 1986 to encounter the increasing need for methods to predict and control early age thermal cracking of massive concrete structures. In 2008, the guideline was thoroughly revised to provide the latest control and analysis technologies for thermal cracking. The guideline contains a performance-based verification system, and it provides the basic principles of control of thermal cracking, design values for concretes with different types of cement, as well as simplified equations for both crack widths and the thermal cracking index.

The German guideline *Rissbreitenbegrenzung für frühen Zwang in massiven Wasserbauwerken (MFZ)* was published by BAW (Bundesanstalt für Wasserbau) in 2011, and it provides calculation methods to determine the required reinforcement in order to control early age cracking and crack widths. The BAW guideline was made for hydraulic structures, which are subjected to strict regulations. The permitted concretes have a relatively high w/c-ratio, hence the autogenous deformation is low compared to thermal dilation, and drying shrinkage is merely a surface problem before the structure is in service. Therefore, the BAW calculation method depends primarily on the adiabatic temperature development and the geometry of the structure.

Relevant parameters for calculations are thermal dilation, autogenous deformation, restraint (internal and/or external), tensile strength, E-modulus development and creep. Some of these parameters are not described in the codes, for instance, Eurocode 2 provides no indication of the likely magnitude of early age thermal strains. The above listed guidelines provide supplementary information and approaches on how to estimate all the necessary input parameters.

3 Calculation example: Wall on a rigid foundation

3.1 General

One aim of the present investigation was to discuss and compare different guidelines and calculation methods to evaluate cracking induced by restrained early age volume changes. The current section presents a calculation example, where the different guidelines in question were used to calculate and evaluate the crack risk and the succeeding crack widths for a wall on a rigid foundation.

3.2 Calculation basis

3.2.1 Geometry

The calculation example was defined to be a 4.2-meter-high and 0.8-meter-thick concrete wall, cast onto a 1.0 meter deep and 4.8 meter wide rigid foundation, see Figure 3-1. The wall was cast in 15-meter lengths (i.e. the L/H-ratio of the wall was 3.6), rotation was assumed prevented and the reinforcement cover was set to be 40 mm. The most unfortunate combination of temperature and restraint was assumed to be approximately one wall thickness away from the casting joint [10], as illustrated by the hatched area in Figure 3-1.

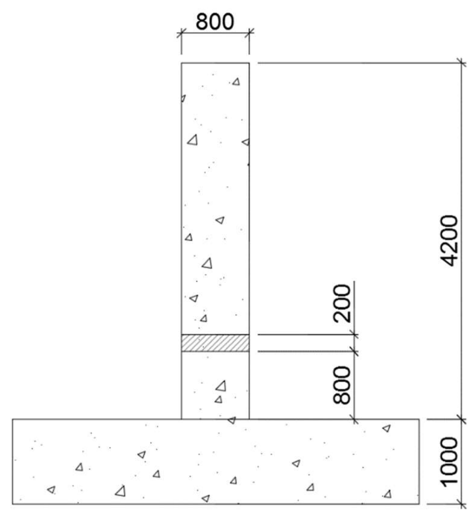

Figure 3-1; Concrete wall structure used as calculation example [mm]

The given example (both the wall geometry and the appurtenant concrete) has previously been thoroughly investigated at NTNU, both by analyses and various experiments in the laboratory [11], [12], [13].

3.2.2 Mix design and mechanical properties

The wall was assumed cast with a concrete denoted ANL FA. This concrete is made with 365 kg/m^3 Portland-fly ash cement Norcem Anlegg FA (CEM II/A-V42.5 N), and it has a fly ash content of 17% (by weight of cement and fly ash content). Previously performed compressive strength tests showed a mean compressive cube strength of 71.2 MPa for the given concrete at 28 days. When adjusted to cylinder strength (multiplied by 0.8), this strength corresponds to strength class C50/60 in Eurocode 2. In the following calculation example, the mechanical properties listed in Eurocode 2 for strength class C50/60 have been applied, Table 3.1.

Table 3.1: Mechanical properties for ANL FA, i.e. strength class C50/60 in Eurocode 2

	f_{ck}	f_{cm}	f_{ctm}	$f_{ctk,\,0.05}$	E_{cm}	ε_{ctu} *
3 days	-	34.7**	2.5**	-	31.7**	77
5 days	-	41.2**	2.9**	-	33.4**	87
8 days	-	46.7**	3.3**	-	34.7**	95
28 days	50	58.0	4.1	2.9	37.0	111

*) *The tensile strain capacity is set to* $\varepsilon_{ctu} = f_{ctm}/E_{cm}$

**) *Calculated from 28-day values and Eurocode 2, paragraph 3.1*

3.2.3 Ambient conditions

The concrete wall was presumed cast during summer conditions. The concrete formwork was assumed to be plywood which was to be removed at approximately 7 days after casting. The fresh concrete temperature, the initial temperature of the foundation and the mean ambient temperature were all assumed to be 20 °C. A mean ambient temperature of 20 ℃ is probably somewhat high, considering both day and night temperatures. Even so, a mean ambient temperature of 20 ℃ was chosen in order to match corresponding experiments previously performed in the laboratory.

4 Calculations based on Eurocode 2 and CIRIA C660

4.1 General

Eurocode 2 states that consequences of volume changes caused by temperature, creep and shrinkage should be taken into consideration during design, and also that special care should be taken where members are subjected to tensile stresses due to the restraint of shrinkage or thermal movements. However, techniques and calculation methods on how to include such early age effects are not described. Originally, the intention of the current memo was to perform crack calculations according to Eurocode 2. However, due to the lack of information and calculation approaches, it was decided to include supplementary information in the means of the guideline CIRIA C660, which claims to be Eurocode 2-compliant throughout.

The first aim of the calculation example was to estimate the risk of cracking due to early age volume changes. This was done by calculating and comparing the restrained strain (i.e. the restrained component of the free strain) with the corresponding strain capacity: Risk of cracking was confirmed when the restrained strain exceeded the strain capacity. Further, crack widths were calculated based on the crack-inducing strain and the maximum crack spacing.

CIRIA distinguishes between early age calculations and long-term calculations of the restrained strain and the corresponding cracking risk. Early age calculations are defined to comprise the total early age thermal contraction as well as the mechanical properties and autogenous deformation at 3 days. The long-term calculations are to be performed beyond the period of the early age temperature cycle, typically beyond 28 days. The long-term calculations include early age effects and also drying shrinkage and the possible long-term variation in temperature due to seasonal temperature variations. In the current example, the long-term calculations were decided conducted at 28 days and 90 days.

When studying the curing temperature development of the given concrete wall, Figure 5-1, it can be seen that after 3 days, only 24% of the early age thermal contraction has actually occurred. It could seem overly conservative to combine mechanical properties at 3 days with the total amount of early age thermal contraction, however, beyond 3 days, the strain capacity of the concrete has already reached a substantial value and develops at a much lower rate than the hydration-induced thermal contraction. The proper time for early age crack risk estimations is further discussed in Section 4.7.

4.2 Imposed deformations

The volume changes in a concrete structure during the hardening phase include thermal dilation, autogenous deformation and drying shrinkage. For massive concrete structures, and in a short-term perspective, drying shrinkage will be small and may generally be ignored. However, in the given example, drying shrinkage was decided included for comparison reasons.

Thermal dilation

Eurocode 2 does not indicate the likely magnitude of early age thermal strains. In CIRIA, the thermal dilation consists of two contributions: short-term and long-term, see Equation 4.1. The short-term contribution T_1 only includes the contraction phase, whereas the initial expansion due to temperature increase is ignored.

$$\varepsilon_T = \alpha_c \cdot (T_1 + T_2) \qquad \text{Equation 4.1}$$

where α_c is the coefficient of thermal expansion (in the current calculations, α_c was set to 10 με/°C as recommended by Eurocode 2), T_1 is the difference between peak temperature and mean ambient temperature and T_2 is the long-term variation in temperature which takes into account the time of year at which the concrete was cast.

T_1 in Equation 4.1 is defined as the difference between the peak temperature and the mean ambient temperature. T_1 can be estimated from tables and charts in CIRIA, and it is based on the thickness of the structural element, the cement content and the type of formwork. From this, T_1 was estimated to be 39 *°C* for the given example*,* i.e. the short-term thermal dilation component was found to be ε_{T1} = 390 $\mu\varepsilon$. T_2 represents the long-term variation in temperature which takes into account the time of year at which the concrete was cast. Based on CIRIA, T_2 was estimated to be 20 *°C* for the given case, i.e. the long-term thermal dilation component was found to be ε_{T2} = 200 $\mu\varepsilon$.

Autogenous deformation

The autogenous deformation development for the given wall case was estimated according to Eurocode 2, Equation 4.2.

$$\varepsilon_{as}(t) = (1 - \exp(-0.2\sqrt{t})) \cdot 2.5(f_{ck} - 10) \cdot 10^{-6} \qquad \text{Equation 4.2}$$

where t is the time in days and f_{ck} is the characteristic cylinder strength at 28 days

With a compressive cylinder strength f_{ck} of 50 MPa, the estimated autogenous deformation values for the given concrete at 3, 28 and 90 days, were 29, 65 and 85 $\mu\varepsilon$ respectively. However, CIRIA strongly argues that the 28 days value of autogenous deformation is to be used beyond 28 days, as the autogenous deformation is assumed included in the drying shrinkage estimations. I.e. CIRIA argues that autogenous deformation will have been included in the measurements from which the drying shrinkage model was derived beyond 28 days.

Drying shrinkage

Drying shrinkage development was estimated according to Eurocode 2, Equation 4.3.

$$\varepsilon_{ds}(t) = \frac{(t - t_s)}{(t - t_s) + 0.04\sqrt{h_0^3}} \cdot k_h \cdot \varepsilon_{cd,0} \qquad \text{Equation 4.3}$$

where t is time after casting (in days), t_s is the age of the concrete at the beginning of drying, k_h is a parameter depending on the notional size of the cross-section h_0 and $\varepsilon_{cd,0}$ is the nominal unrestrained drying shrinkage derived from equations given in Eurocode 2 Appendix 2

The formwork was assumed removed after approximately one week; hence, the age of the concrete at the beginning of drying is 7 days. The notional size of a wall drying from both faces is equal its thickness, i.e. 800 mm for the current wall, and from this follows that the parameter k_h is 0.7. The nominal unrestrained drying shrinkage for the given concrete was found to be 379 $\mu\varepsilon$. Based on these parameters, the drying shrinkage for the given concrete wall at 3, 28 and 90 days, were estimated to be 0, 6 and 22 $\mu\varepsilon$, respectively.

The estimated free strain caused by thermal dilation, autogenous deformation and drying shrinkage at 3 and 28 days are summarized in Table 4.1 and illustrated in Figure 4-1.

Table 4.1: Free strain estimated from Eurocode 2 and CIRIA [$\mu\varepsilon$]

	3 days	28 days
Thermal dilation	390	590
Autogenous deformation	29	65
Drying shrinkage	-	6
Total	419	661

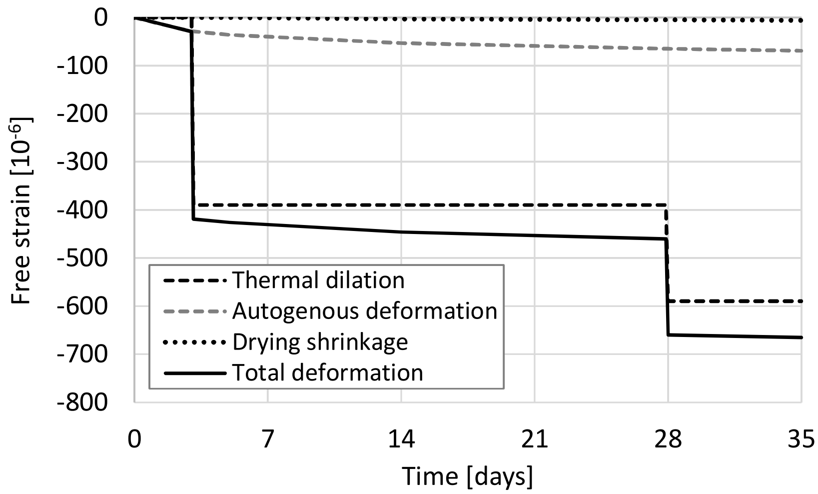

Figure 4-1; Free strain estimated from Eurocode 2 and CIRIA [με], where the total hydration-induced thermal contraction is applied at 3 days.

4.3 Restraint

The restrained strain is the restrained component of the free strain, hence the occurring restraint in the investigated wall had to be determined. Several documents provide guidance on restraint with reviews of values for different restraint conditions. However, as seen in the following, such restraint values are presented in different ways and should be adopted with caution.

CIRIA provides an estimation of edge restraint as given by Equation 4.4. The restraint calculated by this equation represents the true restraint at the joint. This restraint value can further be adjusted to take account of the reduction in restraint with distance from the joint, using a chart and the length/height ratio of the new concrete. The obtained restraint is also to be multiplied by a factor K_1 = 0.65, which takes into account the effect of stress relaxation due to creep under sustained loading. In Eurocode 2, on the other hand, the presented restraint values already include a modification factor to take account of creep under sustained loading. Consequently, where the Eurocode 2 restraint values are applied, the K_1 factor for creep should be set to 1.0.

$$R_j = \frac{1}{1 + \frac{A_n}{A_0} \cdot \frac{E_n}{E_0}} \qquad \text{Equation 4.4}$$

Where A_n is the cross-section of the new (restrained) concrete, A_0 is the cross-section of the old (restraining) concrete, E_n is the modulus of elasticity of the new concrete and E_0 is the modulus of elasticity of the old concrete, the E_n/E_0-ratio is recommended set to 0.7-0.8

For the given wall structure, Eurocode 2 provides a restraint of R = 0.5 and a factor for stress relaxation of K_1 = 1.0. CIRIA, on the other hand, suggests a restraint as obtained from Equation 4.4 and an appurtenant factor for stress relaxation of K_1 = 0.65, see Table 4.2. This means that the restraint R should be seen in combination with the K_1 factor. From the current evaluation follows that the Eurocode 2 approach is a simpler, but more conservative, approach.

Table 4.2: Restraint, Eurocode 2 and CIRIA

Eurocode 2,	R	0.5
$K_1 = 1.0$	**$K_1 \cdot R$**	**0.5**
CIRIA,	R^*	0.57
$K_1 = 0.65$	**$K_1 \cdot R$**	**0.37**

**) The restraint factor obtained from CIRIA includes a reduction factor of 0.87 which accounts for the reduction in restraint at the distance 800 mm from the joint*

4.4 Risk of cracking

Cracking occurs when the restrained strain exceeds the strain capacity, Equation 4.5. According to CIRIA, the risk of cracking should be assessed at both early age (defined to be 3 days) and at longer terms (evaluated at 28 days and 90 days). CIRIA further states that the tensile strain capacity found in Eurocode 2 should be increased by 23% to account for the effects of sustained loading. For the given concrete, this results in a tensile strain capacity of 95 $\mu\varepsilon$ and 137 $\mu\varepsilon$ at 3 and 28 days, respectively.

$$\varepsilon_r > \varepsilon_{ctu} \qquad \text{Equation 4.5}$$

where ε_r is the restrained strain according to Equation 4.6, and ε_{ctu} is the tensile strain capacity of the given concrete under sustained loading

In CIRIA, the restrained strain is determined according to Equation 4.6.

$$\varepsilon_r = K_1\{[\alpha_c T_1 + \varepsilon_{ca}]R_1 + \alpha_c T_2 R_2 + \varepsilon_{cd} R_3\} \qquad \text{Equation 4.6}$$

where K_1 takes into account the effect of stress relaxation due to creep under sustained loading ($K_1 = 0.65$), α_c is the coefficient of thermal expansion, ε_{ca} is autogenous deformation, R_1 is the restraint during early thermal cycle, T_2 is the long-term variation in temperature, R_2 and R_3 are restraint values representing long-term thermal movement ($R_1 = R_2 = R_3$ in the current example) and drying shrinkage and ε_{cd} is the drying strain.

The restrained strain for the given wall example was calculated by Equation 4.6, and further compared with the tensile strain capacity as presented in Table 3.1. The restrained strain was found to exceed the tensile strain capacity at both 3, 28 and 90 days, i.e. according to CIRIA cracking was predicted both at early age as well as at longer terms, Table 4.3. As described in the previous section, the restrained strain calculated from Eurocode 2 is somewhat different as the presented restraint values already include a modification factor K_1. Also Eurocode 2 predicts cracking at both at early age and at longer terms, Table 4.4.

Table 4.3: Risk of cracking [με], CIRIA

	3 days	28 days	90 days
ε_r	155	245	251
ε_{ctu}	95	137	137
Cracking predicted	Yes	Yes	Yes

Table 4.4: Risk of cracking [με], Eurocode 2

	3 days	28 days	90 days
ε_r	210	331	339
ε_{ctu}	77	111	111
Cracking predicted	Yes	Yes	Yes

4.5 Minimum reinforcement area for crack control and crack spacing

Eurocode 2 defines the minimum reinforcement area for crack control $A_{s,min}$ according to Equation 4.7.

$$A_{s,min} = k_c \cdot k \cdot A_{ct} \cdot f_{ct,eff}(t)/\sigma_s \qquad \text{Equation 4.7}$$

where $A_{s,min}$ is the minimum reinforcement area, k_c is a coefficient which takes into account the stress distribution within the cross-section (=1.0), k is a coefficient which accounts for a reduction in restraint due to non-uniform and self-equilibrium effects (=0.75 as defined in CIRIA), A_{ct} is the area of the concrete within the tensile zone, $f_{ct,eff}(t)$ is the mean value of the concrete tensile strength at the time t the cracks may first be expected to occur and σ_s is the absolute value of the maximum stress permitted in the reinforcement after formation of a crack (usually taken as the yield strength of the steel)

From Equation 4.7 follows that the minimum reinforcement is time-dependent. Consequently, the minimum reinforcement was calculated for cracking at 3 days, see Table 4.5. The maximum crack spacing was calculated from Eurocode 2, Equation 4.8. Assuming a concrete cover of 40 mm and a bar diameter 20 mm, the maximum crack spacing was found to be 946 mm when applying the thermal contraction at 3 days, see Table 4.5.

$$S_{r,max} = 3.4c + 0.425 \frac{k_1 \varphi}{\rho_{p,eff}} \qquad \text{Equation 4.8}$$

where $S_{r,max}$ is the maximum crack spacing, c is the concrete cover, k_1 is a coefficient which considers the stress distribution within the cross-section, φ is the bar diameter and $\rho_{p,eff}$ is the ratio of the area of reinforcement to the effective area of concrete

Table 4.5: Minimum reinforcement [mm²] and maximum crack spacing [mm], Eurocode 2 and CIRIA

Application of thermal contraction	3 days
$A_{s,min}$ (per face)	1495
$A_{s,min}$ chosen (per face)	ϕ20c210
$S_{r,max}$	946

4.6 Crack width

For a member subjected to restraint along one edge, the crack width is calculated according to Equation 4.9.

$$w_r = S_{r,max} \cdot \varepsilon_{cr} \qquad \text{Equation 4.9}$$

where w_r is the crack width, $S_{r,max}$ is the maximum crack spacing and ε_{cr} is the crack-inducing strain

When a crack occurs, not all of the restrained strain is relieved. Hence, the crack-inducing strain is less than the restrained strain by the amount of residual tensile strain in the concrete after cracking. Therefore, CIRIA defines the crack-inducing strain used in the derivation of crack widths as the expression given in Equation 4.10. Such a reduction in crack-inducing strain is not included in Eurocode 2, and will thus lead to slightly higher crack width estimations by Eurocode 2 when compared to CIRIA.

$$\varepsilon_{cr} = \varepsilon_r - 0.5 \cdot \varepsilon_{ctu} \qquad \text{Equation 4.10}$$

where ε_{cr} is the crack-inducing strain for derivation of crack widths, ε_r is the restrained strain according to Equation 4.6, and ε_{ctu} is the tensile strain capacity of the concrete under sustained loading

Similarly as for crack risk assessment, also the crack width calculations should be performed considering both early age and long-term strains, the latter including drying shrinkage and long-term effects of temperature. The crack-inducing strain, the crack spacing and the crack widths were calculated as defined in CIRIA, and the results are presented in Table 4.6. The given wall is subjected to continuous edge restraint and the crack pattern is assumed to form at early age. The long-term restrained contraction will thus cause the crack widths to increase, and not affect the crack spacing $S_{r,max}$, Table 4.6. As previously described, both the crack-inducing strain and the restraint described in Eurocode 2 is far more conservative than in CIRIA. By applying the Eurocode 2 approach, the crack widths at 3 and 28 days will increase with as much as 94% and 87% to 0.198 mm and 0.313 mm, respectively, see Table 4.6.

Table 4.6: Crack width calculations, CIRIA and Eurocode 2, A_{smin}(3 days)

		CIRIA		Eurocode 2	
		Early age (3 days)	Long-term (28 days)	Early age (3 days)	Long-term (28 days)
ε_{cr}		108	177	210	331
$S_{r,\,max}$	[mm]	946	946	946	946
$\boldsymbol{w_r}$	[mm]	**0.102**	**0.167**	**0.198**	**0.313**

The described standard calculation approach is to increase the reinforcement area if the estimated crack widths do not satisfy the prevailing crack width limits. In the current example however, no specific crack width limits were defined. For comparison reasons, the crack-inducing strain and the corresponding crack widths were also calculated for a minimum reinforcement area based on the 28-day value of the tensile strength, see Table 4.7.

Table 4.7: Crack width calculations, CIRIA and Eurocode 2, A_{smin}(28 days)

		CIRIA			Eurocode 2		
		Early age (3 days)	Long-term (28 days)	Long-term (90 days)	Early age (3 days)	Long-term (28 days)	Long-term (90 days)
ε_{cr}		108	177	183	210	331	349
$S_{r,\,max}$	[mm]	618	618	618	618	618	618
$\boldsymbol{w_r}$	[mm]	**0.067**	**0.109**	**0.113**	**0.130**	**0.204**	**0.215**

4.7 Discussion

The given crack assessment and succeeding crack width estimations represent a simplified calculation, and in addition, several of the chosen approaches, parameters and interpretations include uncertainties. This is illustrated by the difference in calculated crack width provided by the two approaches, see Table 4.6 and Table 4.7. Consequently, the early age crack calculations presented in both Eurocode 2 and CIRIA must be considered as simplified and quite rough estimations.

The differences between the CIRIA and Eurocode 2 calculation approaches were that 1) CIRIA defined that the 28-day-value of autogenous deformation should be used beyond 28 days, 2) CIRIA provided a lower reduction coefficient due to relaxation of stresses, 3) CIRIA introduced a crack-inducing strain, i.e. a reduction in restrained strain prior to the crack width calculations and 4) CIRIA defined an increase in tensile strain capacity due to sustained loading. Each of these four differences led to a reduced calculated crack width when calculated by the CIRIA approach.

Prior to the calculations, it seemed relatively conservative to apply the entire hydration-induced thermal contraction at 3 days (as defined by CIRIA). However, it was seen that beyond 3 days, the strain capacity of the concrete had already reached a substantial value and developed at a much lower rate than the hydration-induced thermal contraction. In addition, it would be difficult to propose a "time critical" which is valid for all structures, geometries and conditions. For comparison reasons, the strain capacity over time for the given concrete was calculated according to Eurocode 2, Figure 4-2. The figure also presents restrained strain versus time calculated from CIRIA, where the entire hydration-induced thermal contraction was applied at 3 days. The actual temperature history for the given wall example was available from temperature- and stress development measurements and calculations performed at NTNU. Therefore, a comparison between restrained strain for the following cases was performed: 1) applying the entire hydration-induced thermal contraction at 3 days and 2) applying the thermal dilation according to the actual temperature history. The latter was implemented for two cases, demoulding at 3 days and demoulding at 7 days. Cracking occurs when the restrained strain exceeds the strain capacity. Figure 4-2 shows that for the given concrete wall example, cracking will occur at approximately 3.5 and 4.5 days when demoulding at 3 and 7 days, respectively. Consequently, applying the entire hydration-induced thermal contraction at 3 days seems appropriate for the given case.

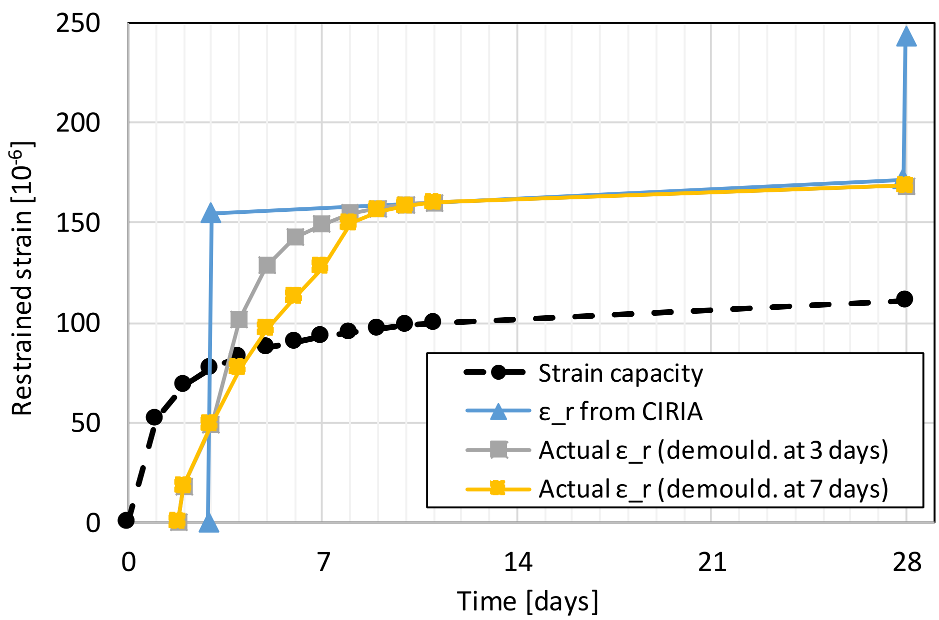

Figure 4-2; Restrained strain estimated from CIRIA vs measured restrained strain [με]

5 Calculations based on the new Annex D and the revised Eurocode 2

5.1 General

A new Eurocode 2 annex is currently being prepared. This annex is denoted "*Annex D, Guidance to restrict early age cracking*" and it aspires to give guidance in cases where the revised Eurocode 2 is used to limit early age cracking. The crack width calculations in the current section are based on Annex D dated June 2017 and the revised Eurocode 2 dated May 2017.

Annex D defines the critical time for cracking as the time when the hardening member is assumed to be in temperature equilibrium with the restraining structure. For the given example, the estimated temperature development indicates a critical time for cracking at 8 days, Figure 5-1. In addition, Annex D also includes long-term crack calculations which for the current example was set to both 28 days and 90 days for comparison reasons.

5.2 Maturity

Annex D includes temperature effects on the concrete properties by providing an equation which calculates the maturity (the temperature-adjusted age) of the concrete, Equation 5.1.

$$t_T = \sum_{i=1}^{n} \Delta t_i \cdot exp\left[\frac{E_T(T_i)}{R} \cdot \left\{\frac{1}{293} - \frac{1}{273 + T_i}\right\}\right] \qquad \text{Equation 5.1}$$

where t_T is the temperature-adjusted concrete age, R is the gas constant, T is the temperature and E_T is the activation energy: $E_T = A + B(20-T_i)$, where $B = 0$ for $T > 20$ °C and B has a given value for $T < 20$ °C, and A has a fixed value for all temperatures. The default value A = 33000 J/mol corresponds to $E_T/R = 4000$ K^{-1}

Input regarding an expected temperature development or magnitude is not provided by Annex D. Hence, it was decided to use the temperature history found from the previously referred laboratory experiments and appurtenant calculations, Figure 5-1. This temperature history combined with Equation 5.1 results in an effective concrete age of 18 days after 8 days, 38 days after 28 days and 100 days after 90 days.

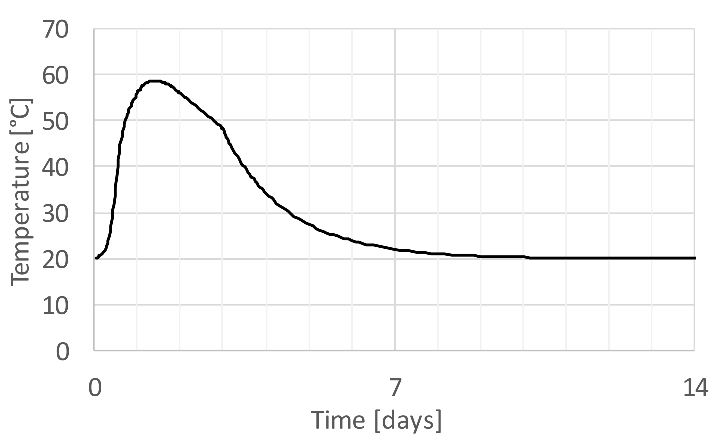

Figure 5-1; Temperature history

5.3 Imposed deformations

As presented in the previous sections, the volume changes in a concrete structure during the hardening phase include thermal dilation, autogenous deformation and drying shrinkage. For massive concrete structures, and in a short-term perspective, drying shrinkage will be small and may generally be ignored. However, in the given example, drying shrinkage was decided included in the calculations for comparison reasons.

Thermal dilation

Annex D does not indicate the likely magnitude of early age thermal strains. Instead, it refers to the use of computer programs, hand calculations and diagrams from guidelines and handbooks. For the current example, the temperature development curve from the previously referred laboratory experiments were used, see Figure 5-1. This results in a fresh concrete temperature T_{ci} = 20 °C, maximum temperature T_{max} = 59 °C, temperature of the restraining structure T_0 = 20 °C and a long-term maximum temperature drop ΔT_{min} which was assumed to be 20 °C. Some additional information was also needed: the start time for stress development t_{dor} was set to 0.4 days, and the time when the concrete starts to develop tensile stresses t_2 was set to 2 days (i.e. 7 days of maturity).

Autogenous deformation

The autogenous deformation development (which in the revised Eurocode 2 is denoted basic shrinkage) for the given wall case was estimated according to the revised Eurocode 2 as presented in Equation 5.2.

$$\varepsilon_{cbs}(t) = (1 - \exp(-0.2\sqrt{t})) \cdot \alpha_{bs} \left[\frac{(0.1 \cdot f_{cm})}{(6 + 0.1 \cdot f_{cm})}\right]^{2.5} \cdot 10^{-6} \qquad \text{Equation 5.2}$$

where ε_{cbs} is the autogenous deformation, t is the time in days, α_{bs} is a parameter describing the effect of cement type (=700), f_{cm} is the mean compressive strength at 28 days

With a compressive cylinder strength f_{ck} of 50 MPa, the estimated autogenous deformation values for the given concrete at 8 days (18 days of maturity), 28 days (38 days of maturity) and 90 days (100 days of maturity) were 68, 84 and 103 $\mu\varepsilon$ respectively. Autogenous deformation after 2 days (7 days of maturity) was found to be 49 $\mu\varepsilon$.

Drying shrinkage

Drying shrinkage development was estimated according to the revised Eurocode 2, see Equation 5.3.

$$\varepsilon_{cds}(t, t_s) = \varepsilon_{cds0}(f_{cm}) \cdot \beta_{RH}(RH) \cdot \beta_{ds}(t - t_s) \qquad \text{Equation 5.3}$$

where t is time after casting (in days), t_s is the age of the concrete at the beginning of drying, ε_{cds0} is the notional drying shrinkage coefficient derived from equations given in Eurocode 2, β_{RH} is a factor taking into account the ambient relative humidity and β_{ds} is the time function according to Eurocode 2

The formwork was assumed removed after approximately one week; hence, the age of the concrete at the beginning of drying is 7 days. The notional size of a wall drying from both faces h_0 is equal its thickness. The notional drying shrinkage coefficient for the given concrete was found to be 329 $\mu\varepsilon$. Based on these parameters, the drying shrinkage for the given concrete wall at 8 days (18 days maturity), 28 days (38 days maturity) and 90 days (100 days maturity) were estimated to be 4, 14 and 27 $\mu\varepsilon$, respectively.

5.4 Restraint

Annex D does not specify how to estimate the degree of restraint, but for uniaxial cases the degree of restraint may be defined as the stiffness of the restraining structure divided by the total stiffness. The revised Eurocode 2 states that the restraint at the base of a wall may be taken as 0.75. It was decided to use $R = 0.65$ in the given calculation, i.e. including an adjustment to take account of a reduction in restraint at a distance 800 mm from the joint.

5.5 Risk of cracking

Cracking occurs when the restrained strain exceeds the strain capacity. According to Annex D, the risk of cracking should be assessed at 8 days (18 days of maturity) and at longer terms (defined to be 28 days, i.e. 38 days of maturity and 90 days, i.e. 100 days of maturity). The restrained strain can be determined according to Equation 5.4.

$$\varepsilon_r = R_1[k_1\alpha_T(T_{\max} - T_0) + \varepsilon_{ca}(t_{crit} - t_2)] + R_2\alpha_T\Delta T_{\min} + R_3\varepsilon_{cd} \qquad \text{Equation 5.4}$$

where $k_1 = 0.9$ takes into account that the tensile stress development starts slightly after the maximum temperature (due to the initial thermal expansion), α_T is the coefficient of thermal expansion = 10 $\mu\varepsilon$/°C, ε_{ca} is autogenous deformation (=ε_{cbs}), t_{crit} and t_2 are as described previously in the current section, $\varepsilon_{ca}(t_{crit} - t_2)$ is the development of ε_{ca} between t_2 and t_{crit}, R_1 is the restraint during early thermal cycle, ΔT_{min} is the long-term variation in temperature, R_2 and R_3 are restraint values representing long-term thermal movement and drying shrinkage ($R_1 = R_2 = R_3$ in the current example) and ε_{cd} is the drying strain (=ε_{cds})

The restrained strain for the given wall example was calculated by Equation 5.4: 241 $\mu\varepsilon$ after 8 days, 392 $\mu\varepsilon$ after 28 days and 412 after 90 days. The restrained strain was found to exceed the tensile strain capacity (Table 3.1) at both 8, 28 and 90 days, also when considering increased tensile strain capacity due to maturity. Hence, cracking was predicted both at early age as well as at longer terms.

5.6 Minimum reinforcement area for crack control and crack spacing

The minimum reinforcement area for crack control and the corresponding crack spacing estimations are altered in the revised Eurocode 2 which is the basis for Annex D. For comparison reasons, it was decided to use the minimum reinforcement area based on 28 days properties as calculated from the prevailing Eurocode 2 in the previous section, i.e. 2460 mm^2 per face. According to the revised Eurocode 2, the calculated maximum crack spacing is defined as presented by Equation 5.5.

$$S_{r,max} = 2c + 0.35\frac{k_b\varphi}{\rho_{p,eff}} \qquad \text{Equation 5.5}$$

where $S_{r,max}$ is the calculated maximum crack spacing, c is the concrete cover, k_b is a coefficient which takes into account of the bond properties of the bonded reinforcement (=0.8), φ is the bar diameter and $\rho_{p,eff}$ is the ratio of the area of reinforcement to the effective area of concrete

Assuming a concrete cover of 40 mm and a 20-mm bar diameter, the maximum crack spacing was found to be 359 mm.

5.7 Crack width

According to the revised Eurocode 2, the surface crack width may be calculated by Equation 5.6.

$$w_r = S_{r,max} \cdot (\varepsilon_{sm} - \varepsilon_{cm} + \eta_r \varepsilon_{cs}) \qquad \text{Equation 5.6}$$

where w_r is the surface crack width, $S_{r,max}$ is the maximum crack spacing, ε_{sm} is the mean strain in the reinforcement (incl. imposed deformations and tension stiffening effects), ε_{cm} is the mean strain in the concrete between cracks at the same level of ε_{sm}, η_r is 0 for short-term loading and long-term loading in the crack formation phase and equal to R_{ax} in other cases and ε_{cs} is the shrinkage strain

For elements subjected to restrained imposed strains and restrained at the edges, (ε_{sm} - ε_{cm} + $\eta_r\varepsilon_{cs}$) in Equation 5.6 may be calculated by Equation 5.7.

$$\varepsilon_{sm} - \varepsilon_{cm} + \eta_r \varepsilon_{cs} = R_{ax}\varepsilon_{free} - k_t \cdot \frac{f_{ct,ef}}{E_{cm}} \qquad \text{Equation 5.7}$$

where R_{ax} is the restraint factor (= R_1 = R_2 = R_3 in the current example), ε_{free} is the imposed strain, k_t is a coefficient dependent on the nature and duration of the load (k_t = 0.4), $f_{ct,ef}$ is the mean value of the tensile strength of the concrete effective at the time when the cracks are first expected to occur and E_{cm} is the secant modulus of elasticity

For elements restrained at the edges, Annex D states that ($R_{ax}\varepsilon_{free}$) in Equation 5.7 can be taken as ε_r as presented in Equation 5.4. From this follows that the crack width can be calculated according to Equation 5.8.

$$w_r = S_{r,max} \cdot \left(\varepsilon_r - k_t \cdot \frac{f_{ct,ef}}{E_{cm}}\right) = S_{r,max} \cdot \varepsilon_{cr} \qquad \text{Equation 5.8}$$

where ε_r is the restrained strain and ε_{cr} is the crack-inducing strain

The crack width calculations should be performed considering both early age and long-term strains, the latter including drying shrinkage and long-term effects of temperature caused by seasonal variations. The restrained strain, the crack-inducing strain, the crack spacing and the crack widths were calculated according to Annex D and the revised Eurocode 2 as defined in the previous sections, and the results are presented in Table 5.1. The calculations were performed at short terms (8 days) and at longer terms (28 days and 90 days). For comparison reasons, the values for minimum reinforcement ratio and crack spacing were based on the 28 days properties of the concrete.

Table 5.1: Crack width calculations, Annex D

		Early age (8 days)	Long-term (28 days)	Long-term (90 days)
ε_r		241	390	411
ε_{cr}		203	352	373
$S_{r,max}$	[mm]	359	359	359
$\boldsymbol{w_r}$	[mm]	**0.073**	**0.126**	**0.134**

5.8 Discussion

The calculations and corresponding results in the given section must be considered as preliminary as Annex D and the revised Eurocode 2 are not yet finalized and there could be some uncertainties regarding the chosen parameters.

Both the autogenous deformation and the drying shrinkage models are altered in the revised Eurocode 2 when compared with the prevailing Eurocode 2. The revised Eurocode 2 adopts the modelling approaches found in Model Code 2010. In addition, the revised version of the Eurocode 2 includes shrinkage strains in the crack width calculation formulae, and a tension stiffening approach for elements subjected to restrained imposed strains, i.e. the crack-inducing strain is lower than the restrained strain.

When compared to the existing Eurocode 2, the revised Eurocode 2 together with Annex D give lower crack widths at both early ages and longer terms. The main reason for this is that the calculated transfer length is lower in the revised Eurocode 2, which adopts the Model Code 2010 approach.

The revised Eurocode 2 calculation method is quite similar to the CIRIA approach, for instance the differentiation between restrained strains and crack-inducing strains. However, while both the short-term and long-term strains are calculated with a factor K_1 (= 0.65) compensating for the relaxation of stresses in CIRIA, this is not the case in Annex D. In Annex D, only the short-term thermal load is calculated with a parameter k_1 (= 0.9) which accounts for the initial thermal expansion, which leads to a considerable lower crack-inducing strain according to the CIRIA approach. On the other hand, the transfer length is higher in CIRIA than in the revised Eurocode. As these two effects equalizes each other, both approaches give quite similar crack widths.

6 Calculations based on Model Code 2010 and CEOS.fr

6.1 General

The ***fib*** Model Code 2010 states that early age effects should be taken into account during design. However, techniques and calculation methods on how to include such early age effects are not described. Due to the lack of information and calculation approaches provided in Model Code 2010, it was decided to include supplementary information in the means of the guideline CEOS.fr, which claims to follow Model Code 2010 throughout.

The CEOS.fr guideline is based on the outcome of the French national research project CEOS.fr which aimed to improve the prediction of crack pattern of special structures, mainly massive structures. The guideline states that Eurocode 2 rules do not fully reflect the complete behaviour of massive concrete structures such as thick slabs or thick walls throughout time.

CEOS.fr defines an appropriate time for estimations of cracking caused by bulk heating and cooling of concrete. This time is dependent on h_0 (mean radius of cross-section), and for the given example, h_0 = 730 mm corresponds to an appropriate crack estimation time of t = 90 days.

6.2 Maturity

Both Model Code 2010 and CEOS.fr include temperature effects on concrete properties. CEOS.fr introduce the term "maturity" and provides an equation which calculates the maturity (the temperature-adjusted age) of the concrete, Equation 6.1.

$$t_T = \sum_{i=1}^{n} \Delta t_i \cdot exp\left[13.65 - \frac{4000}{273 + T(\Delta t_i)}\right] \qquad \text{Equation 6.1}$$

where t_T is the temperature-adjusted concrete age (or effective concrete age) which replaces t in the corresponding equations in [days]

Input regarding an expected temperature development or magnitude was not provided by the current guidelines. Hence, it was decided to use the temperature history found from the previously referred laboratory experiments and appurtenant calculations, see Figure 5-1 in the previous section. This temperature history combined with Equation 6.1 results in an effective concrete age of 38 days after 28 days, and 100 days after 90 days.

6.3 Imposed deformations

The volume changes in a concrete structure during the hardening phase include thermal dilation, autogenous deformation and drying shrinkage.

Thermal dilation

Model Code 2010 does not provide information regarding the likely magnitude of early age thermal strains. CEOS.fr proposes an equation to determine the thermal dilation strains, Equation 6.2. The guideline does however not indicate the magnitude or progress of the temperature history the concrete is exposed to. In Equation 6.2, the short-term contribution to the thermal dilation (T_{max} – T_{ini}) only includes the contraction, whereas the initial expansion due to temperature increase is accounted for by multiplying the dilation with the reduction factor 0.6. In addition, the 0.6-coefficient also accounts for the relaxation of stresses.

$$\varepsilon_{cT} = \alpha_c \cdot [0.6(T_{max} - T_{ini}) + T_{ini} - T_{min}(t)] \qquad \text{Equation 6.2}$$

where α_c is the coefficient of thermal expansion (in the current calculations, α_c was set to 10 $\mu\varepsilon$/°C as recommended by Model Code 2010), T_{max} is the maximum temperature reached, T_{ini} is the initial temperature of the concrete at the time of casting and T_{min} is the minimum temperature (average) to which the structure was exposed during the period up to time t.

For the given example, T_{max} is set to 59 ºC, T_{ini} is set to 20 ºC and $T_{min}(t)$ is set to 0 ºC to match with the CIRIA calculations. This temperature input results in a thermal dilation strain of 434 $\mu\varepsilon$ at 90 days.

<u>Autogenous deformation</u>

The autogenous deformation development for the given wall case was estimated according to Model Code 2010, Equation 6.3.

$$\varepsilon_{cbs0}(f_{cm}) = -\alpha_{bs}\left[\frac{(0.1 \cdot f_{cm})}{(6 + 0.1 \cdot f_{cm})}\right]^{2.5} \cdot 10^{-6} \cdot \beta_{bs}(t) \qquad \text{Equation 6.3}$$

where t is the time in days, α_{bs} is a parameter describing the effect of cement type (=700), f_{cm} is the mean compressive strength at 28 days and β_{bs} is the time function according to Model Code 2010

With a compressive strength f_{cm} of 58 MPa, the estimated autogenous deformation values for the given concrete at 28 days (i.e. 38 days of maturity) and 90 days (i.e. 100 days maturity), was 84 and 103 $\mu\varepsilon$, respectively.

<u>Drying shrinkage</u>

Drying shrinkage development was estimated according to Model Code 2010, Equation 6.4.

$$\varepsilon_{cds}(t, t_s) = \varepsilon_{cds0}(f_{cm}) \cdot \beta_{RH}(RH) \cdot \beta_{ds}(t - t_s) \qquad \text{Equation 6.4}$$

where t is time after casting (in days), t_s is the age of the concrete at the beginning of drying, ε_{cds0} is the notional drying shrinkage coefficient derived from equations given in Model Code 2010, β_{RH} is a factor which accounts for the ambient relative humidity and β_{ds} is the time function according to Model Code 2010

The formwork was assumed removed after approximately one week; hence, the age of the concrete at the beginning of drying is 7 days. The notional size of a wall drying from both faces h_0 is equal its thickness. The notional drying shrinkage coefficient for the given concrete was found to be 329 $\mu\varepsilon$. Based on these parameters, the drying shrinkage for the given concrete wall at 28 days and 90 days was estimated to be 14 $\mu\varepsilon$ and 27 $\mu\varepsilon$ respectively.

6.4 **Restraint**

CEOS.fr proposes a specific calculation method to determine the restraint on a wall continuously restrained along its lower edge, see Equation 6.5, Equation 6.6 and Equation 6.7.

$$R_{effective} = K_{R0}^{i} \cdot R \qquad \text{Equation 6.5}$$

where $R_{effective}$ is the effective restraint at a given point, K^{i}_{R0} is the restraint coefficient and R is the elastic restraint coefficient on a perfectly rigid base

$$K_{Ro}^{i} = \frac{1}{1 + 1.05 \cdot \frac{A_n}{L_i B} \cdot \frac{E_n}{E_0}} \qquad \text{Equation 6.6}$$

where A_n is the cross-section of the new (restrained) concrete, L_iB is the area of the wall in contact with the surface of the base, E_n is the modulus of elasticity of the new concrete and E_0 is the modulus of elasticity of the old concrete, the E_n/E_0-ratio is recommended set to 0.7-0.8

$$R = \left[1.372\left(\frac{h}{L}\right)^2 - 2.543\left(\frac{h}{L}\right) + 1\right] + 0.044\left[\left(\frac{L}{H}\right) - 1.969\right]\left(\frac{h}{H}\right)^{1.349} \qquad \text{Equation 6.7}$$

where R is the elastic restraint coefficient on a perfectly rigid base, h is the distance from the considered point to the base, L is the wall length and H is the wall height

Based on the geometry and concrete properties of the given example, the restraint was found to be 0.73 (73%) at a point in the wall 800 mm over the base.

6.5 Risk of cracking

The estimated total amount of strain is given by Equation 6.8. Drying shrinkage was not included in the strain equation given in CEOS.fr. However, in the current example, it was decided to include drying shrinkage in the same way as autogenous deformation. I.e. drying shrinkage was multiplied with a factor of 0.5 and included in the total strain estimation.

$$\varepsilon_{cs} = 0.5 \cdot \varepsilon_{cas}(t) + \varepsilon_{cT} \qquad \text{Equation 6.8}$$

where ε_{cs} is the total shrinkage, ε_{cas} is the autogenous deformation and ε_{cT} is the thermal dilation as defined in Equation 6.2. The coefficient 0.5 reflects the relaxation of stresses when the shrinkage is restrained

The restrained strain for the given wall example was further found by multiplying the total strain in Equation 6.8 with the restraint determined in the previous section. The restrained strain was found to be 353 $\mu\varepsilon$ at 28 days and 364 $\mu\varepsilon$ at 90 days. Cracking occurs when the restrained strain exceeds the strain capacity, which in Table 3.1 was defined to be 111 $\mu\varepsilon$. Consequently, cracking was predicted at both 28 days and 90 days.

6.6 Minimum reinforcement area for crack control and crack spacing

The minimum reinforcement area $A_{s,min}$ was calculated by Eurocode 2, Equation 6.9.

$$A_{s,min} = k_c \cdot k \cdot A_{ct} \cdot f_{ct,eff}(t)/\sigma_s \qquad \text{Equation 6.9}$$

where $A_{s,min}$ is the minimum reinforcement area, k_c is a coefficient which takes into account the stress distribution within the cross-section, k is a coefficient which accounts for a reduction in restraint due to non-uniform and self-equilibrium effects, A_{ct} is the area of the concrete within the tensile zone, $f_{ct,eff}(t)$ is the mean value of the concrete tensile strength at the time t the cracks may first be expected to occur and σ_s is set to the yield strength of the steel

The minimum reinforcement was calculated based on a crack risk assessment at 90 days, i.e. the 28 day value for $f_{ct,eff}(t)$ was used. The force transition length between the reinforcement and the concrete was calculated from Model Code 10, Equation 6.10. Assuming a concrete cover of 40 mm and a bar diameter 20 mm, the force transition length $l_{s,max}$ was found to be 178 mm.

$$l_{s,max} = c + \frac{1}{4} \cdot \frac{f_{ctm}}{\tau_{bms}} \frac{\varphi}{\rho_{s,ef}} \qquad \text{Equation 6.10}$$

where $l_{s,max}$ is the length over which the force in the reinforcement bars is transmitted to the concrete rather than the crack spacing, the τ_{bms}/f_{ctm} ratio is assumed to be constant and equal to 1.8, ϕ is the bar diameter and $\rho_{p,eff}$ is the ratio of the area of reinforcement to the effective area of concrete

Table 6.1: Minimum reinforcement [mm²] and maximum crack spacing [mm], Model Code 2010 and CEOS.fr

Application of thermal contraction	90 days
$A_{s,min}$ (per face)	2460
$A_{s,min}$ chosen (per face)	$\phi 20c125$
$l_{s,max}$	178

6.7 Crack width

The crack width was calculated according to Equation 6.11 and the results are presented in Table 6.2.

$$w_k = 2 \cdot l_{s,max} \cdot (\varepsilon_{sm} - \varepsilon_{cm} - \varepsilon_{cs}) \qquad \text{Equation 6.11}$$

where $l_{s,max}$ is the length over which the force in the reinforcement bars is transmitted to the concrete rather than the crack spacing and (ε_{sm} - ε_{cm} - ε_{cs}) is the restrained strain (applies to edge restraint on a long wall) as found in Section 6.5

Table 6.2: Crack width calculations, CEOS.fr

		28 days	90 days
ε_{cr}		353	364
$l_{s,max}$	[mm]	178	178
$\boldsymbol{w_k}$	[mm]	**0.126**	**0.130**

6.8 Discussion

According to CEOS.fr, the appropriate time for crack risk estimations due to bulk heating and cooling is 90 maturity days for the given example. This is at a much later place in time than proposed by CIRIA, and it does not capture early age effects as for instance reduced tensile strength. The crack width calculated by CEOS.fr was not comparable to the CIRIA results due to different age for crack risk estimations. Hence, restrained strain and crack widths were also calculated at 28 days by the CEOS.fr approach.

7 Calculations based on NS3473

7.1 General

The Norwegian code NS3473 is withdrawn and superseded by Eurocode 2. However, the standard is still used for offshore concrete installations in Norway as Eurocode 2 is defined not to cover such structures. NS3473 states that imposed deformations, temperature, creep and shrinkage should be included and combined with structural loading when conducting crack width calculations. Early age stress development arises due to restrained volume changes caused by temperature and shrinkage, and it could thus be argued that early age effects should be taken into consideration during design according to NS3473. However, the standard provides limited information on how to include such early age effects, and hence several assumptions and adjustments had to be made during the following calculations.

The concrete material properties were defined from NS3473 based on a characteristic compressive cube strength of 60 MPa as described in Section 3.2.2. This provided a tensile strength of f_{tn} = 3.5 MPa as described in NS3473.

NS3473 does not specify an appropriate time for crack width calculations. Therefore, the given calculations were decided performed at both 28 days and 90 days for comparison reasons.

7.2 Imposed deformations

According to NS3473, imposed deformations caused by temperature, creep and/or shrinkage should be included when conducting crack width calculations. However, how to estimate the progress and magnitude of early age deformations (thermal dilation and autogenous deformation) are not described.

Thermal dilation

The thermal dilation was estimated by multiplying the expected temperature fall during the cooling phase with the coefficient of thermal expansion (10 $\mu\varepsilon/°C$). For the given example, the maximum temperature during curing was estimated to be T_{max} = 59 °C and the temperature of the restraining structure was assumed to be T_0 = 20 °C. Consequently, the thermal dilation during the cooling phase was calculated to be is 390 $\mu\varepsilon$. The long-term temperature fall due to seasonal variations was estimated to be approximately 20 °C, i.e. from 20 °C down to 0 °C, which gives a thermal dilation of 200 $\mu\varepsilon$.

Autogenous deformation

NS3473 does not provide any methods to estimate autogenous deformation. It was therefore decided to use the autogenous deformation measured in the previously referred experiments (see Figure 10-2 in Section 10.3). The autogenous deformation was estimated to be 64 and 160 $\mu\varepsilon$ at 28 and 90 days, respectively.

Drying shrinkage

Drying shrinkage development was estimated according to NS3473, Equation 7.1.

$$\varepsilon_{cs}(t, t_s) = \varepsilon_s \cdot \beta_s(t - t_s) \qquad \text{Equation 7.1}$$

where t is time after casting (in days), t_s is the age of the concrete at the beginning of drying, ε_s is the notional drying shrinkage coefficient derived from equations given in NS3473, β_s is the time function according to NS3473

The formwork was assumed removed after approximately one week; hence, the age of the concrete at the beginning of drying was 7 days. The notional size of a wall drying from both faces h_0 is equal its thickness. Relative Humidity was estimated to be 50%. The notional drying shrinkage coefficient for the given concrete was found to be 481 $\mu\varepsilon$. Based on these parameters, the drying shrinkage for the given concrete wall was estimated to be 15 and 29 $\mu\varepsilon$ at 28 and 90 days, respectively.

7.3 Restraint

The degree of restraint was not spesified in NS3473, and it was decided to use R = 0.6 in the given calculation.

7.4 Risk of cracking

NS3473 does not provide a description on how to include early age deformations. It was therefore decided to use the total thermal dilation of the curing cooling phase (390 $\mu\varepsilon$), and to add the possible long-term temperature fall due to seasonal variations (200 $\mu\varepsilon$), in addition to the estimated autogenous deformation and drying shrinkage. Also, the NS3473 calculation contains no reduction parameter accounting for the effect of stress-relaxation, as such a parameter was not properly described in NS3473.

The restrained strain was found by multiplying the restraint with the free deformation (thermal dilation, autogenous deformation and drying shrinkage). The restrained strain at 28 and 90 days were found to be 401 and 467 $\mu\varepsilon$, indicating that cracking will occur in the structure.

7.5 Minimum reinforcement area for crack control and crack spacing

The minimum reinforcement area $A_{s,min}$ was calculated according to NS3473, Equation 7.2. The standard specifies that the calculated minimum reinforcement area (Equation 7.2) should be doubled in cases were structural tightness and reduced crack widths are important. The results are presented in Table 7.1.

$$A_s = 0.6 \cdot A_c \cdot f_{tk}/f_{sk} \qquad \text{Equation 7.2}$$

where A_s is the minimum reinforcement area, A_c is the concrete area, f_{tk} is the mean value of the concrete tensile strength and f_{sk} is the yield strength of the steel

The force transition length was calculated by NS3473, Equation 7.3. This transition length is dependent on the reinforcement ratio, and it was calculated for both 1) a minimum reinforcement area according to NS3473, Table 7.1 and 2) a minimum reinforcement area according to Eurocode 2, Table 7.2. The latter was used for the succeeding comparison between the various calculation approaches.

$$l_{sk} = s_{rk} = 1.7 \cdot s_{rm} = 1.7\left\{s_{ro} + \frac{k_c \cdot A_{cef}}{\Sigma(\pi\phi/(f_{tk}k_b/\tau_{bk}))}\right\} \qquad \text{Equation 7.3}$$

where l_{sk} is the length over which the force in the reinforcement bars is transmitted to the concrete, s_{ro} is the concrete cover, k_c accounts for the tensile stress distribution, A_{cef} is the effective concrete area, ϕ is the bar diameter k_b accounts for bundled bars and (f_{tk}/τ_{bk}) is a parameter which accounts for the type of reinforcement bar

Table 7.1: Minimum reinforcement [mm^2] and maximum crack spacing [mm], NS3473

A_s (per face)	3360
A_s chosen (per face)	$\phi 20c90$
l_{sk}	478

Table 7.2: Minimum reinforcement [mm^2] after Eurocode 2 and maximum crack spacing [mm] according to NS3473

A_s (per face)	2460
A_s chosen (per face)	$\phi 20c125$
l_{sk}	632

7.6 Crack width

The crack width was calculated according to Equation 7.4 and the results are presented in Table 7.3.

$$w_k \quad = l_{sk} \cdot (\varepsilon_{sm} - \varepsilon_{cm} - \varepsilon_{cs}) \qquad \text{Equation 7.4}$$

where l_{sk} is the length over which the force in the reinforcement bars is transmitted to the concrete and (ε_{cs} - ε_{cas} - ε_{cs}) is the restrained strain (applies to edge restraint on a long wall) as found in Section 6.5

Table 7.3: Crack widths, NS3473

	$A_{s,min}$ from NS3473	$A_{s,min}$ from EC2	
Time	28 days	28 days	90 days
ε_{cr}	401	401	467
$l_{s,\,max}$ [mm]	478	632	632
$\boldsymbol{w_k}$ [mm]	**0.192**	**0.254**	**0.295**

7.7 Discussion

NS3473 provides very limited information regarding imposed deformations due to early age volume changes and on how to include these. Therefore, several parameters had to be assumed during the calculation (both thermal dilation, autogenous deformation and restraint). In addition, how to include the effect of stress-relaxation and/or the initial thermal expansion was not described and hence omitted in the current calculations. The currently used calculation approach must thus be considered quite conservative, and consequently the restrained strain was found to be quite high.

8 Calculations based on the JCI guideline

8.1 General

The calculation methods found in the JCI (Japanese Concrete Institution) guideline differs considerably from the other approaches evaluated in the current memo. The JCI guideline states that 3D-FEM analysis should be the standard analysis method for verification of thermal cracking. However, the document also provides a simplified evaluation for cases where a 3D-FEM method is not possible. This simplified evaluation comprises a comprehensive and complex equation with several interdependent parameters which calculates a thermal cracking index. The crack width can further be found from a chart, based on the thermal cracking index and the reinforcement ratio.

The appropriate time for crack width calculations was not found specified or defined. However, several of the input parameters in the simplified estimation were defined as concrete properties at 28 or 91 days. For the current calculation, the 28 days property values were used, and therefore the following calculation was assumed to represent crack widths beyond 28 days.

8.2 Imposed deformations

Thermal dilation

The JCI guideline does not provide a dedicated estimation of the thermal dilation. Instead, the guideline gives basis for calculating the thermal dilation from 3D-FEM analyses. For the simplified approach, the effect of thermal dilation seems to be included in the thermal cracking index equation by the adiabatic temperature rise of the concrete.

Autogenous deformation

The autogenous deformation development for the given wall case was estimated according to JCI, Equation 8.1.

$$\varepsilon_{as}(t_e) = \eta_c \varepsilon_{as,\infty} r_{as}(t_e) \qquad \text{Equation 8.1}$$

where t_e is equivalent time in days, η_c is a parameter describing the effect of cement type (=700), $\varepsilon_{as,\infty}$ is the ultimate value of the autogenous deformation and r_{as} is a function representing the development rate of the autogenous deformation

With a compressive strength f_{cm} of 58 MPa, the estimated autogenous deformation values for the given concrete at 90 days (i.e. 100 days of maturity), was found to be 111 $\mu\varepsilon$.

The simplified equation representing the thermal cracking index does not seem to include the effect of autogenous deformation. However, the equation consists of several interdependent parameters which could include the autogenous deformation without it being specified.

Drying shrinkage

The JCI guideline specifies that drying shrinkage may be neglected for early age crack estimations, and does not provide any calculation models.

8.3 Restraint

JCI does not provide dedicated restraint estimations. Instead, the guideline gives basis for 3D-FEM calculations which also include calculations of the restraint. In the simplified equation, the restraint seems to be included in form of an E-modulus ratio of restrained members by the restraining bearing ground.

8.4 Risk of cracking

The simplified thermal cracking index for a wall-type structure is given by Equation 8.2, where the cracking risk is in inverse ratio to the thermal cracking index.

$$\begin{aligned} I_{mra-WT} = & -1.93 \cdot 10^{-2} \cdot T_a - 2.8 \cdot 10^{-3} \cdot D - 1.17 \cdot 10^{-2} \cdot Q_\infty \\ & + 1.55 \cdot 10^{-2} \cdot {r_{AT}}^{SAT} + 8.72 \cdot 10^{-2} \cdot log_{10}(H_R) \\ & + 0.476 \cdot f_t - 0.165 \cdot log_{10}(L/H) + 0.224 \\ & \cdot log_{10}(E_c/E_R) + 0.015 \end{aligned}$$

Equation 8.2

where I_{mra-WT} is the thermal cracking index for a wall-type structure, T_a is the concrete temperature at casting, D is the minimum member thickness, Q_∞ is the ultimate adiabatic temperature rise, ${r_{AT}}^{SAT}$ is a constant representing the rate of the adiabatic temperature rise, H_R is a value denoting the effect of heat radiation, f_t is the splitting tensile strength of concrete at 28 days, L/H is the ratio of the length to the height of the wall and E_c/E_R is the ratio of modulus of elasticity of restrained member by restraining bearing ground

The cracking index calculated by Equation 8.2 is further adjusted by a reduction factor to keep the thermal cracking indices calculated by the simplified equation on the safe side when compared with those computed by the 3D-FEM approach, see Equation 8.3.

$$I_{cr} = I_{mra-WT} - I_b$$

Equation 8.3

Where I_{cr} is thermal cracking index calculated by the simplified equation, I_{mra-WT} is the thermal cracking index for a wall-type structure according to Equation 8.2 and I_b is a reduction factor equal to 0.3.

If the thermal cracking index obtained is equal to or greater than 1.85, the thermal cracking probability is equal to or less than 5%. For the current calculation, the thermal cracking index according to Equation 8.3 was found to be 0.95, i.e. cracking was predicted.

8.5 Crack width

The crack width was estimated from a chart based on the thermal cracking index and the reinforcement ratio. For comparison reasons, the minimum reinforcement area for crack control estimated in the previous sections was applied, i.e. 2515 mm^2/m. The corresponding crack width was found to be w_c = 0.130 mm.

8.6 Discussion

The JCI guideline basically provides models and a common basis for 3D-FEM analyses, i.e. estimations of autogenous deformation, hydration heat and expansive strains. The guideline also provides a "simplified method" which comprises a comprehensive and complex equation with several interdependent parameters which calculates the thermal cracking index which further provides a corresponding crack width. The nature and complexness of this equation makes it hard to decompose and analyse the approach. However, for the given case, the resulting crack width is in very good agreement with the crack widths found from the other approaches included in the current study.

9 Calculations based on the BAW guideline

9.1 General

The German guideline *Rissbreitenbegrenzung für frühen Zwang in massiven Wasserbauwerken (MFZ)* was published by BAW (Bundesanstalt für Wasserbau) in 2011. The guideline provides calculation methods to determine the reinforcement area necessary to control early age cracking and crack widths in massive hydraulic structures. Such hydraulic structures are subjected to strict regulations: The permitted concretes have a relatively high w/c-ratio, hence the autogenous deformation is small compared to thermal dilation, and drying shrinkage is merely a surface problem before the structure is in service.

The BAW calculation approach is based on deformation compatibility: The restrained deformation must be compensated for by strain- and crack development in the concrete. The BAW guideline differs between primary cracks, which go through the entire cross-section, and secondary cracks, which only occur in the effective tensile zone. The calculation approach in the BAW guideline can be described by the following steps:

1) Define the occurring restrained deformation and appurtenant stress caused by the hydration-induced temperature development
2) Determine the number of secondary cracks (in combination with primary cracks) necessary to achieve deformation compatibility
3) Calculate the reinforcement area necessary to limit the primary crack widths to a given threshold value

The guideline underlines that the given procedure is a calculation approach only, and that isolated crack widths above the threshold value may be found on-site.

The BAW guideline is defined valid for structures with a minimum thickness of 800 mm. The guideline states that due to this restriction, the concrete strength may be properly described by the 28-day strength values. From this follows that the relevant tensile strength for crack width calculations could be set to the 28 day strength, however, due to the maturity principle, the calculation could still represent a time prior to 28 days. However, as a simplification, the results were assumed comparable with the crack widths obtained from the other calculation approaches at 28 days.

9.2 Imposed deformations

The volume changes in a concrete structure during the hardening phase include thermal dilation, autogenous deformation and drying shrinkage. However, due to the high w/c-ratio of the concretes concerned, the calculation methods in the BAW guideline are only based on the adiabatic temperature development, and hence the autogenous deformation and the drying shrinkage was not determined.

Thermal dilation

The BAW guideline differs between axial and flexural restrained deformation when determining the equivalent temperature difference. In the given example, axial restraint was considered predominant, and therefore, the equivalent temperature difference was defined by Equation 9.1.

$$\Delta T_N = k_0^N \cdot k_{FK}^N \cdot k_{JZ}^N \cdot \Delta T_{adiab,7d} \qquad \text{Equation 9.1}$$

where ΔT_N is the equivalent temperature difference for axial restraint, k^N_0 is a parameter averaging the equivalent temperature difference (= $0.7 - 0.2/h^{0.3} \le 0.55$) where h is the thickness of the wall, k^N_{FK} is a parameter representing the strength class of the concrete (= 1.0), k^N_{JZ} is a parameter dependent on the time of the year (= 1.0) and ΔT_N is the adiabatic temperature rise after 7 days (= 45 K)

For the given example, the equivalent temperature difference with regards to axial restrained strain was found to be 21.9 K.

9.3 Minimum reinforcement area for crack control and crack spacing

The minimum reinforcement area for crack control by Eurocode 2 was applied the given calculations, i.e. $A_{s,min}$ = 2460 mm²/m per face, see Section 4.5, Equation 4.7.

The BAW guideline defines a crack spacing $l_{cr,W}$ for primary cracks as described by Equation 9.2. For the given example, the crack spacing for primary cracks was found to be 5.04 m.

$$l_{cr,W} = 1.2 \cdot h_{BA} \qquad \text{Equation 9.2}$$

where $l_{cr,W}$ is the crack spacing and h_{BA} is the wall height

The BAW guideline states that beyond the formation of primary cracks, an increase in sectional stresses will induce the formation of secondary cracks in the area around the primary cracks. The number of secondary cracks necessary to achieve deformation compatibility is found by Equation 9.3.

$$n \ge 1.1 \cdot \left(\frac{\Delta T_N \cdot \alpha_T \cdot l_{cr,W}}{w^P} - 1\right) \qquad \text{Equation 9.3}$$

where n is the number of secondary cracks, ΔT_N is the equivalent temperature difference, α_T is the thermal dilation coefficient (= 10 με/°C), $l_{cr,W}$ is the crack spacing and w^P is the primary crack width

9.4 Crack width

The BAW guideline presents an equation defining a minimum reinforcement area necessary to limit the primary crack width to a given threshold value, Equation 9.4. Equation 9.4 can further be rewritten as Equation 9.5.

$$a_{s,erf} = \sqrt{\left(\frac{d_s \cdot d_1^2 \cdot b^2 \cdot f_{ctm}}{w^P \cdot E_s} \cdot (0.69 + 0.34 \cdot n)\right)} \qquad \text{Equation 9.4}$$

$$w^P = \frac{d_s \cdot d_1^2 \cdot b^2 \cdot f_{ctm}}{a_{s,erf}^2 \cdot E_s} \cdot (0.69 + 0.34 \cdot n) \qquad \text{Equation 9.5}$$

where $a_{s,erf}$ is the reinforcement area per face (2460 mm²/m), d_s is the reinforcement diameter (20 mm), d_1 is the reinforcement cover (40 mm), b is running meter (1000 mm), f_{ctm} is the tensile strength of the concrete (4.1 MPa), w^P is the crack width of the primary cracks, E_s is the elasticity modulus of the reinforcement (200 GPa) and n is the number of secondary cracks

Equation 9.3 and Equation 9.5 constitute two equations with two unknown parameters: the number of secondary cracks n and primary crack width w^P. By combining these two equations, n was found to be 4.20, and w^P was found to be 0.229 mm.

9.5 Discussion

The BAW guideline presents a quite different calculation approach than the other investigated guidelines.

It should be noted that the BAW guideline provides a calculation method to determine the reinforcement area necessary (output) to keep the primary crack width within a given threshold value (input). In the given case, however, the calculation approach was used to determine the primary crack width (output) based on a given reinforcement area (input).

10 Comparisons and discussions

10.1 **General**

The current section contains a comparison between the calculation approaches and appurtenant parameters presented in the previous sections. Most of the presented calculation approaches are based on the principle of multiplying the restrained strain with the force transfer length between concrete and reinforcement. The JCI guidelines, however, contains a quite different approach. This guideline recommends and provides a basis for 3D-FEM calculations, but also a simplified approach in form of a complex crack index equation.

The main differences between the calculation approaches (with exception from the JCI guideline and the BAW guideline) are connected to the force transfer length and the restrained strain reduction parameters. The latter includes e.g. the effect of relaxation of stresses and/or a compensation for the initial expansion phase which is often excluded in restrained strain estimations.

There is no clear consensus between the guidelines regarding the appropriate time for crack risk and crack width calculations. CIRIA and Annex D request early age calculations at 3 and 8 days, respectively, CEOS.fr concludes that the crack risk should be calculated at 90 days, while the other guidelines do not provide an appropriate time. For comparison reasons, all calculation approaches where conducted at 28 days and 90 days.

The different calculation approaches suggested different minimum reinforcement areas, which further influenced the calculated crack width. To be able to compare the calculated crack widths, a common basis had to be chosen. A minimum reinforcement area based on the 28-day tensile strength was therefore decided applied all calculation approaches.

10.2 **Maturity**

The property development of hardening concrete is dependent on both time and temperature. This temperature effect on the concrete property development is included by the maturity principle in Model Code 2010, CEOS.fr, Annex D (i.e. the revised Eurocode 2) and the JCI guideline. CIRIA and Eurocode 2, on the other hand, do not clearly include the maturity principle when it comes to crack width estimations. The BAW guideline indirectly includes the maturity principle in the parametric study used to determine the equivalent temperature difference formula, and in addition, the maturity principle is also included by the fact that the 28 day tensile strength is permitted for the reinforcement design.

10.3 **Imposed deformations**

The driving forces for early age cracking are the volume changes occurring in the hardening phase. These volume changes include thermal dilation, autogenous deformation and drying shrinkage. For massive concrete structures, and in a short-term perspective, drying shrinkage will be small and may therefore generally be ignored. However, in the given example, drying shrinkage was decided included in the calculations for comparison reasons.

Thermal dilation

Thermal dilation is the temperature-induced volume change, which consists of two contributions: 1) An early age temperature increase followed by a temperature decrease caused by the exothermic hydration process and 2) Long-term changes in temperature caused by seasonal temperature variations.

CIRIA provides tables and charts where the short-term thermal dilation can be estimated based on type of cement, cement content and water-binder ratios. In the other investigated guidelines and standards, the thermal dilation was based on the known temperature development for the given concrete structure, see Figure 10-1. The short-term thermal dilation was found to be 390 $\mu\varepsilon$ for all current guidelines and standards. However, the given documents provide different reduction coefficients to account for stress relaxation and the initial expansion phase, and therefore the net short-term thermal dilation which contributes to the restrained strain varies between the guidelines, see Table 10.1.

Table 10.1: Short-term thermal dilation, i.e. induced by hydration heat [με]

	Thermal dilation (short-term)	Reduction factor	Net short-term thermal dilation
CIRIA	390*	0.65	254
Eurocode 2	390	-**	390
Annex D	390	0.90	351
MC2010 and CEOS.fr	390	0.60	234
NS3473	390	-	390

*) *Found from charts in CIRIA*

**) *The reduction factor is included in the succeedingly applied degree of restraint*

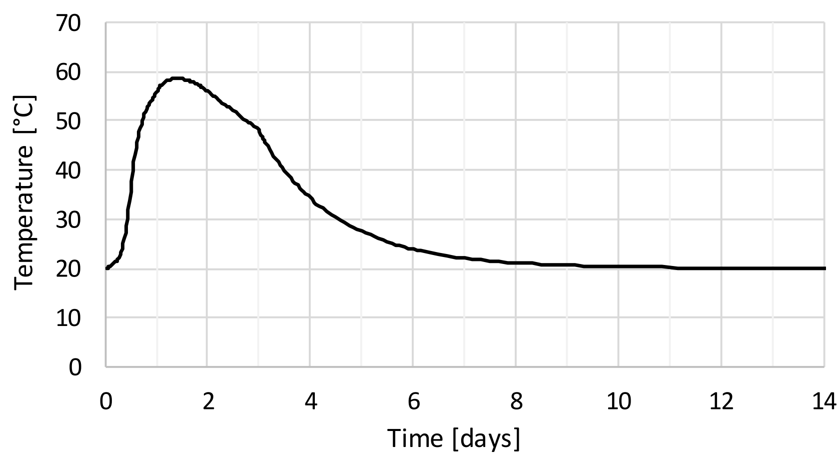

Figure 10-1; Temperature development, calculated

The long-term thermal dilation caused by seasonal temperature variations are quite similar in the investigated guidelines and standards. Only CIRIA includes a reduction factor to account for the relaxation of stresses over time, see Table 10.2.

Table 10.2: Thermal dilation long-term, i.e. seasonal temperature variations [με]

	Thermal dilation (long-term)	Reduction factor	Net long-term thermal dilation
CIRIA	200	0.65	130
Eurocode 2	200	-	200
Annex D	200	-	200
MC2010 and CEOS.fr	200	-	200
NS3473	200	-	200

Autogenous deformation
The autogenous deformation development of the given concrete was estimated based on the previously described codes and guidelines. The results are given in Table 10.3 and illustrated in Figure 10-2. The autogenous deformation for the given concrete has also been measured in the laboratory: both under 20 °C isothermal conditions and when subjected to a realistic temperature history during curing. Autogenous deformation estimated from the various guidelines gave quite good agreement, a small difference was however caused by the fact that Annex D, Model Code 2010, CEOS.fr and JCI include the maturity principle in the models. The use of an equivalent age (temperature adjusted age) caused a more rapid AD development in real time for a realistic temperature development. The measured autogenous deformation developments, on the other hand, differed from the modelled values, especially for the concrete subjected to a realistic temperature history.

While the autogenous deformation development model found in Model Code 2010 and CEOS.fr were dependent on the cement type and compressive strength, the corresponding model in Eurocode 2 and CIRIA was dependent on compressive strength only, and the model in the JCI guideline was dependent on w/c-ratio and maximum temperature during curing.

Table 10.3: Autogenous deformation, calculated and measured [με]

	3 days	7 days	14 days	28 days	90 days
CIRIA and Eurocode 2	29	41	53	65	85
Annex D (i.e. revised Eurocode 2)	56	67	74	84	103
MC2010 and CEOS.fr	56	67	74	84	103
JCI	63	72	79	89	111
Measured, isothermal	39	36	41	64	160
Measured, realistic temp	150	180	178	179	185

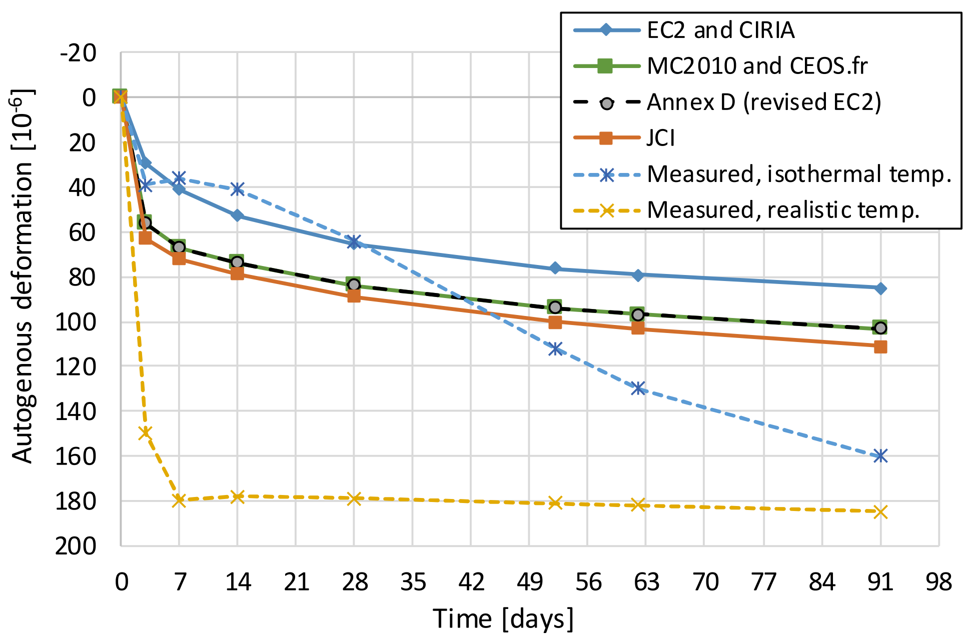

Figure 10-2; Autogenous deformation, calculated and measured

The autogenous deformation contribution to the restrained strain based on the investigated guidelines are given in Table 10.4. For autogenous deformation, only CIRIA and CEOS.fr include reduction factors reflecting the relaxation of restrained stresses over time. It should be noticed that NS3473 does not provide a method to model autogenous deformation, and hence the autogenous deformation measured under isothermal temperature conditions was applied. The autogenous deformation measured at 90 days is considerably higher than the corresponding modelled deformation found from the model-codes. However, this was not found to have similarly high impact on the succeeding calculated crack width. The lack of a reduction factor in the NS3473, on the other hand, was found to have a considerable impact.

Table 10.4: Restrained strain due to autogenous deformation

	Reduction factor	Net autogenous deformation	
		28 days	90 days
CIRIA	0.65	42	55
Eurocode 2	-	65	85
Annex D	-	84	103
MC2010 and CEOS.fr	0.50	42	52
NS3473	-	64	160
JCI	-	89	111

Drying shrinkage

Drying shrinkage is generally ignored in a short-term perspective when it comes to massive concrete structures. However, for the given example, drying shrinkage was decided included in the calculations for comparison reasons. Drying shrinkage development modelled by CIRIA (i.e. Eurocode 2), Annex D (i.e. revised Eurocode 2), CEOS.fr (i.e. Model Code 2010) and NS3473 are presented in Figure 10-3. The obtained shrinkage curves are quite similar.

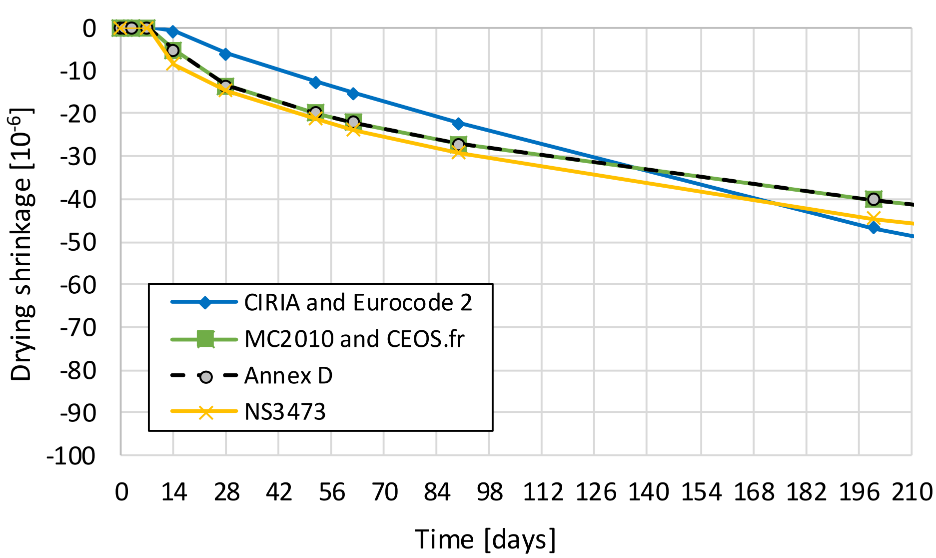

Figure 10-3; Drying shrinkage

10.4 **Restraint**

Some of the guidelines provide calculation methods to assess which restraint factor to be used. The revised Eurocode 2 suggests a restraint equal to 0.75 at the base of a wall, it was however decided to use R = 0.65 as to include an adjustment to account for a reduction in restraint at a distance 800 mm from the joint. In the existing Eurocode 2, the suggested restraint factor of 0.5 includes a reduction factor to account for the relaxation of stresses which should be kept in mind when compared with the other approaches. NS3473 did not provide any input on the restraint, and it was decided to assume a restraint factor of 0.6. The restraint factors used in the current calculations are presented in Figure 10-4.

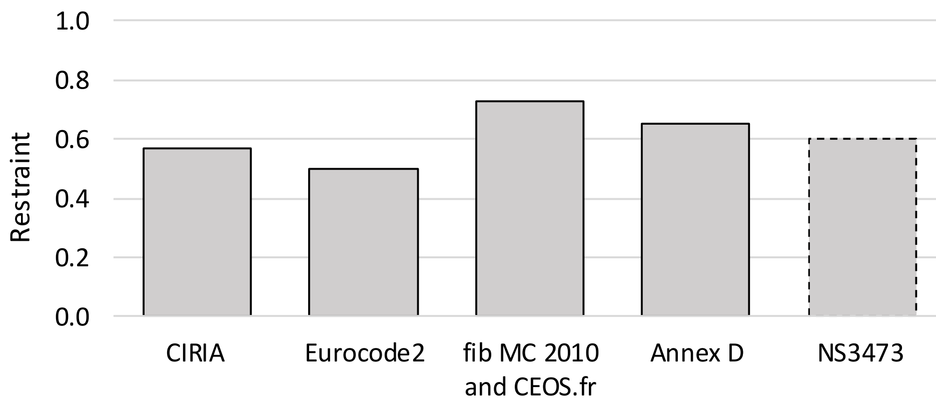

Figure 10-4; Restraint factors

10.5 **Restrained strain and cracking risk**

Both the suggested time for crack estimations and the corresponding restrained strain varied between the evaluated calculation approaches. Even so, all calculations and guidelines predicted that the structure would crack, i.e. that the restrained strain would exceed the strain capacity of 111 $\mu\varepsilon$ at 28 days. Restrained strain calculated by the various investigated approaches are presented in Table 10.5.

Table 10.5: Restrained strain and crack-inducing strain [$\mu\varepsilon$]

	Restrained strain		Crack-inducing strain	
	28 days	90 days	28 days	90 days
CIRIA	245	251	177	183
Eurocode2	331	349	331	349
Annex D	390	411	352	373
MC2010 / CEOS.fr	353	364	353	364
NS3473	401	467	401	467
JCI	-	-		

Both CIRIA and the revised Eurocode 2 include the effect of tension stiffening by differing between the restrained strain and the crack-inducing strain. The crack-inducing strain, i.e. the net strain used in the crack width calculations, was for both approaches found by subtracting the assumed residual tensile strain in the concrete after cracking from the estimated restrained strain, see Table 10.5 and Figure 10-5. Figure 10-5 also shows the strain capacity for the given concrete, as well as a corresponding measured strain development for the given concrete subjected to a realistic temperature history and applied a restraint factor of 0.5. The CIRIA approach provided a considerable lower crack-inducing strain than the other calculation approaches. The main reason for this was that both the short-term and long-term strains were multiplied with a reduction factor K_1 (= 0.65) in CIRIA, compensating for the relaxation of stresses. NS3473, on the other hand, does not specify any reduction factors, and therefore provided a crack-inducing strain which was considerably higher than the other approaches for the given case.

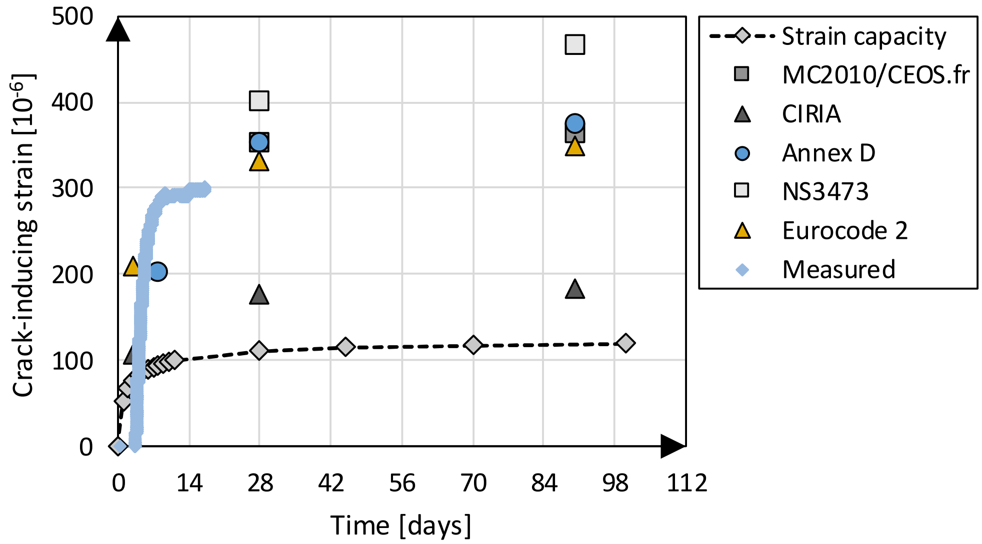

Figure 10-5; Crack inducing strain

10.6 **Force transfer length**

Most of the crack width calculation approaches are based on the principle of multiplying the restrained strain with the force transfer length between concrete and reinforcement, and the force transfer length will therefore directly affect the calculated crack width. However, it should be noticed that this approach is derived from the theory of a tie. In a thick member, the stress distribution over the width is not uniform and the above described tie-theory will thus not accurately represent the real behaviour of a thick member. The BAW guideline defines a crack spacing between the primary cracks, and is thus not comparable with the force transfer lengths found from the other guidelines and regulations. The force transfer lengths calculated from the various approaches are presented in Table 10.6.

Table 10.6: Force transfer length [mm]

	Force transfer length [mm]
CIRIA and Eurocode 2	618
Annex D, revised EC2	359
MC2010 / CEOS.fr	356
NS3473	478
JCI	-

10.7 **Crack width**

Crack widths calculated by the various approaches at 28 and 90 days are presented in Table 10.7, and illustrated in Figure 10-6. Calculated crack widths versus time are presented in Figure 10-7.

Table 10.7: Crack width calculated at 28 and 90 days [mm]

	$w_{28\ days}$	$w_{90\ days}$
CIRIA	0.109	0.113
Eurocode2	0.204	0.215
Annex D	0.126	0.134
MC2010 / CEOS.fr	0.126	0.130
NS3473	0.254	0.295
JCI	0.130	0.130
BAW	0.229	0.229

Despite the differences between the investigated calculation approaches, the resulting crack widths found from several of the calculation approaches were quite similar. However, NS3473, the existing Eurocode 2 and the BAW guideline provided crack widths that were somewhat higher. The considerably higher crack widths found from NS3473 were caused by the lack of a dedicated reduction factor taking into account stress relaxation, combined with a somewhat higher transfer length. Also the BAW guideline provided quite high crack widths, however, it should be noticed that the BAW approach was established in order to satisfy a crack width criteria. The approach includes several safe side assumptions, and hence the currently performed reversed calculation was expected to give a crack with in the upper range. The existing Eurocode 2 gave a crack-inducing strain very similar to several of the other approaches, however, the transfer length was considerable higher. For the given example, the calculations based on the revised Eurocode 2 decreased the calculated 28-day crack width by approximately 38% when compared with the existing Eurocode 2. The crack widths found from the revised Eurocode 2 also gave very good agreement with the crack widths found based on CIRIA, CEOS.fr (MC2010) as well as the JCI guideline.

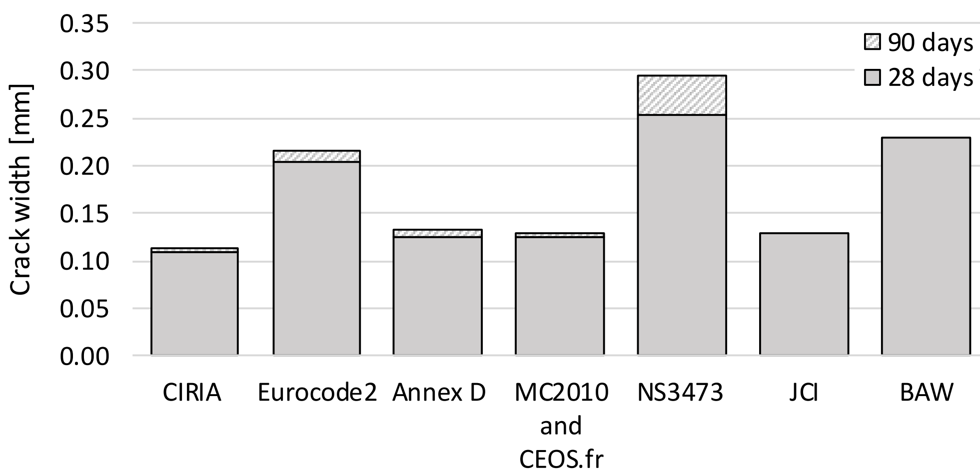

Figure 10-6; Crack widths at 28 and 90 days

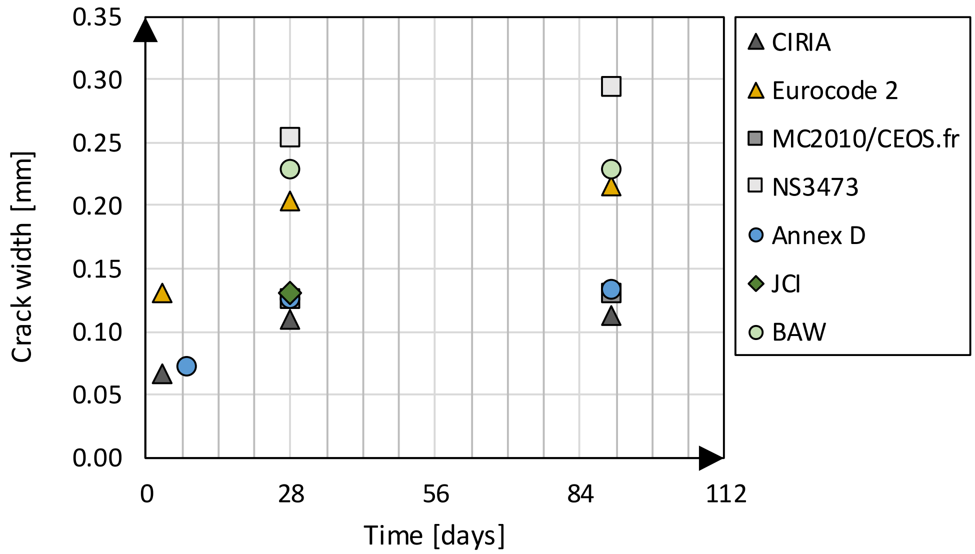

Figure 10-7; Crack widths versus time

CIRIA provided crack-inducing strains which were considerably lower than all the other investigated guidelines. Despite this, the appurtenant crack widths calculated by CIRIA were very similar to crack widths obtained from CEOS.fr (Model Code 2010), Annex D (revised Eurocode 2) and the JCI guideline. This was caused by the considerable higher transfer length found in CIRIA (and Eurocode 2), equalizing the effect of the emphasized lower crack-inducing strain.

The JCI guideline constituted a very different calculation approach than the other guidelines and standards. Despite this, the resulting crack width gave very good agreement with the other investigated approaches.

11 Conclusions

The investigated standards and guidelines all stated that imposed deformations, temperature, creep and shrinkage should be included when it comes to crack risk assessment and crack width calculations. It could thus be strongly argued that also early age effects, i.e. thermal dilation and shrinkage, should be taken into consideration during design. However, in general, limited information was provided on how to include such early age effects, and hence several assumptions and adjustments had to be made during the currently performed calculations.

All the investigated calculation approaches predicted that cracking would occur for the given wall example. It was seen that several of the determined parameters varied considerably between the different approaches, e.g. reduction parameters, restraint, crack-inducing strain and transfer length. However, for CIRIA, Model Code 2010, Annex D and JCI, these different parameters seemed to neutralize each other, resulting in quite similar calculated crack widths. The calculation approaches found in Eurocode 2, BAW, and NS3473, on the other hand, provided crack widths that were somewhat larger. For the given example, the calculated crack width at 28 days determined by seven different calculation approaches varied between 0.109 mm and 0.254 mm. For future development, it is suggested that the factors introduced in order to take into account different physical effects are clearly separated so that the user can easily understand the background and make refined estimates when needed.

The upcoming revision of Eurocode 2, including Annex D, seems to be somewhat different from the existing Eurocode 2 when it comes to crack width calculations and early age volume effects, e.g. the differentiation between restrained strains and crack-inducing strains. In addition, the revised Eurocode 2 has adopted several of the models found in Model Code 2010, e.g. autogenous deformation development and transfer length estimations. For the given example, the calculations based on the revised Eurocode 2 decreased the calculated 28-day crack width by approximately 38% when compared with the existing Eurocode 2. The crack widths found from the revised Eurocode 2 also gave very good agreement with the crack widths found based on CIRIA, CEOS.fr (MC2010) as well as the JCI guideline. However, it should be noticed that the current calculation approaches should be interpreted as simplified and rough estimation methods, meaning that the user should be encouraged to take the step towards more advanced methods if necessary.

The current investigation only considered early age effects, without addressing the effects of other imposed live loads or gravity loads. Further work should focus on how all these effects could be treated in a unified manner, ensuring the existence of simplified estimation methods and also clear guidance on when a step towards more refined calculation methods should be advised, in order to avoid un-necessary conservatism. Finally, it should be noted that several interpretations and assumptions were made during the calculations presented in the previous sections. These interpretations and assumptions must be considered subjective, and the calculations and appurtenant results are thus to some degree affected by the background and experience of the author.

References

1. NS-EN 1992 (1992) Eurocode 2: Design of concrete structures. Norsk Standard, Norway
2. NS 3473:2003 (2003) Prosjektering av betongkonstruksjoner, Beregnings- og konstruksjonsregler. Norsk standard, Norway
3. NS-EN 1992-1-1:2004 (2004) NS-EN 1992-1-1:2004+NA:2008, Eurocode 2: Design of concrete structures. Part 1-1: General rules and rules for buildings. Norsk Standard,
4. NS-EN 1992-3:2006 (2006) NS-EN 1992-3:2006+NA:2009, Eurocode 2: Design of concrete structures. Part 3: Liquid retaining and containment structures. Norsk Standard,
5. ***fib*** (2010) Model Code for Concrete Structures 2010. Berlin, Germany
6. CIRIA C660 (2007) Early age thermal crack control in concrete. London, England
7. CEOS.fr RP (2016) Control of Cracking in Reinforced Concrete Structures. France
8. JCI JCI (2008) JCI Guidelines for Control of Cracking of Mass Concrete. Tokyo, Japan
9. BAW BfW (2011) Rissbreitenbegrenzung für frühen Zwang in massiven Wasserbauwerken (MFZ). Karlsruhe, Germany
10. Kanstad T, Bosnjak D, Øverli JA (2001) 3D Restraint Analyses of Typical Structures with Early age Cracking Problems. IPACS report. Luleå University of Technology, Sweden
11. Klausen AE (2016) Early age crack assessment of concrete structures, experimental determination of decisive parameters. PhD Thesis, Norwegian University of Science and Technology (NTNU), Trondheim, Norway
12. Klausen AE, Kanstad T, Bjøntegaard Ø, Sellevold EJ (2017) Comparison of tensile and compressive creep of fly ash concretes in the hardening phase. Cement and Concrete Research 95 (2017):pp. 188-194
13. Klausen AE, Kanstad T, Bjøntegaard Ø, Sellevold EJ (2018) The effect of curing temperature on autogenous deformation of fly ash concretes. Article under review