Dual Dynamic Programming for Long-Term Receding-Horizon Hydro Scheduling with Head Effects

Benjamin Flamm, Annika Eichler, Joseph Warrington, John Lygeros
Automatic Control Laboratory, ETH Zurich

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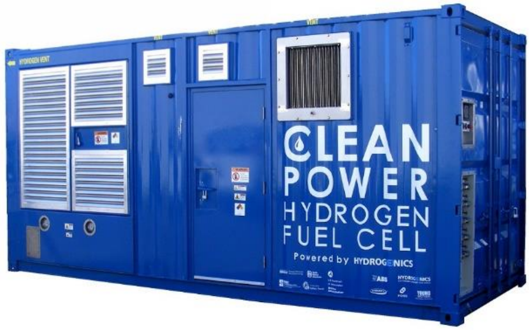
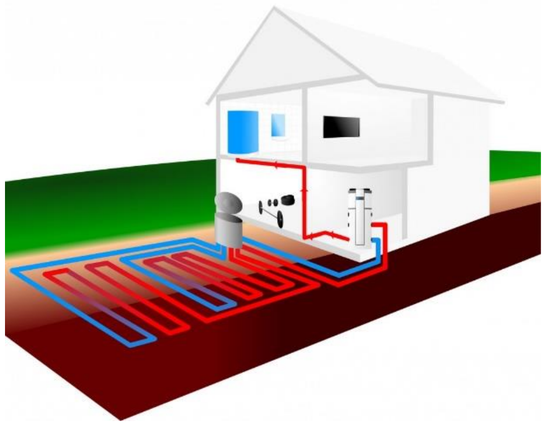
Motivation: Long-term control of energy storage

- Nonlinear, complex models
- Receding horizon setting
- Applications:
  - Seasonal demand
  - Peak shaving
  - Price arbitrage





Seasonal storage problem

- Want to solve long-term nonlinear problem of the form **(NLP)**

$$
\begin{aligned}
&\min_{x,u} \sum_{t=0}^{T-1} g_t(x_t, u_t) + g_T(x_T) \\
&\text{s.t.} \quad x_{t+1} = f_t(x_t, u_t), && t = 0, \ldots, T-1 \\
&\qquad (u_t, x_t) \in \mathcal{Z}_t, && t = 0, \ldots, T-1 \\
&\qquad x_0 \text{ given.}
\end{aligned}
$$

… but this can be computationally intractable.

Split-horizon approximation

- To make problem tractable, approximate second part of horizon: **(NLP+LP)**

$$
\begin{aligned}
\min_{x,u} \quad & \sum_{t=0}^{T_1-1} g_t(x_t, u_t) + \sum_{t=T_1}^{T-1} \left(c_t^\top x_t + d_t^\top u_t\right) + c_T^\top x_T \\
\text{s.t.} \quad & x_{t+1} = \begin{cases} f_t(x_t, u_t), & t = 0, \dots, T_1 - 1 \\ A_t x_t + B_t u_t, & t = T_1, \dots, T - 1 \end{cases} \\
& (u_t, x_t) \in \mathcal{Z}_t, \qquad t = 0, \dots, T_1 - 1 \\
& E_t x_t + F_t u_t \le h_t, \qquad t = T_1, \dots, T - 1 \\
& x_0 \text{ given.}
\end{aligned}
$$

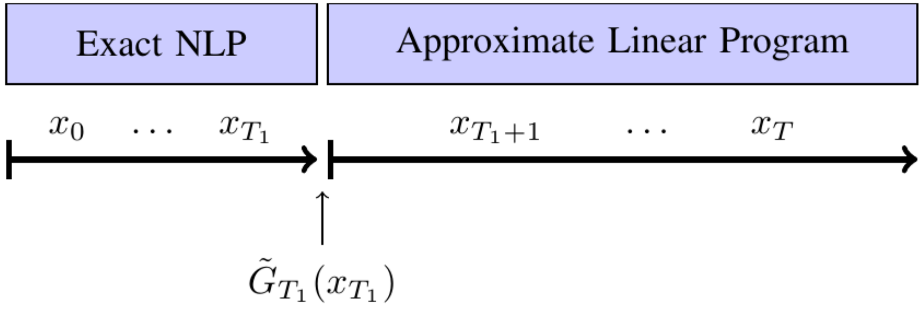
Split-horizon approximation, continued

- Split problem into nonlinear part **(NLP1)**:

$$\begin{aligned} \min_{\substack{u_0,\ldots,u_{T_1-1}\\ x_1,\ldots,x_{T_1}}} \quad & \sum_{t=0}^{T_1-1} g_t(x_t,u_t) + \tilde{G}_{T_1}(x_{T_1}) \\ \text{s.t.} \quad & x_{t+1} = f_t(x_t,u_t), \quad t = 0,\ldots,T_1-1 \\ & (u_t,x_t) \in \mathcal{Z}_t, \qquad t = 0,\ldots,T_1-1 \\ & x_0 \text{ given.} \end{aligned}$$

- And linear part **(LP2)**:

$$\begin{aligned} \tilde{G}_{T_1}(x_{T_1}) = \min_{\substack{u_{T_1},\ldots,u_{T-1}\\ x_{T_1+1},\ldots,x_T}} \quad & \sum_{t=T_1}^{T-1} (c_t^\top x_t + d_t^\top u_t) + c_T^\top x_T \\ \text{s.t.} \quad & x_{t+1} = A_t x_t + B_t u_t, \\ & E_t x_t + F_t u_t \leq h_t, \\ & t = T_1,\ldots,T-1. \end{aligned}$$

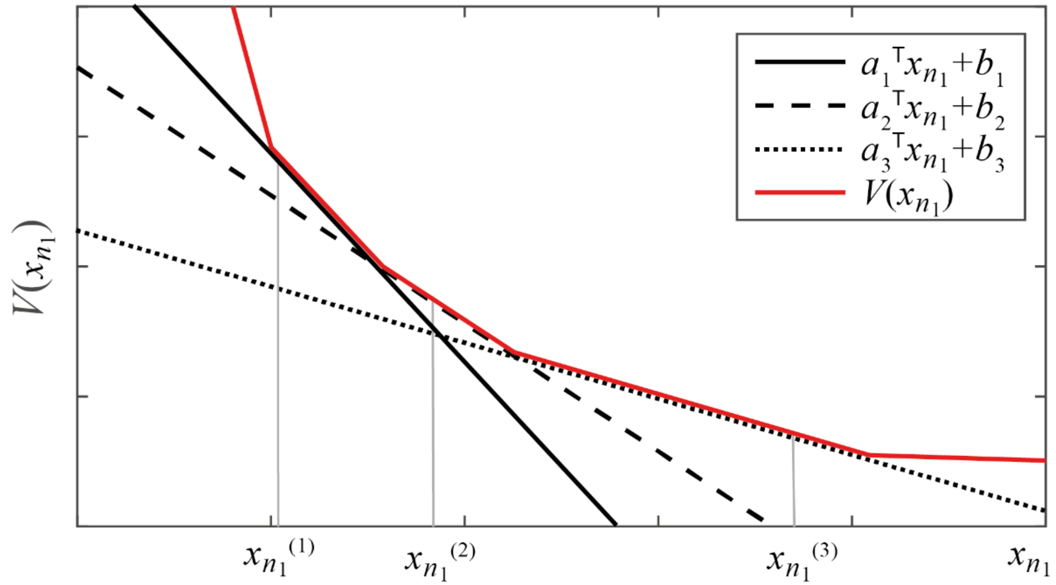


Complete recourse assumption

Assumption 1: The linear second-stage problem **(LP2)** is feasible for all x_{T_1} in the domain of the first-stage problem **(NLP1)**.

Benders decomposition review

- Want to find value function for **(LP2)**, as function of terminal state x_{T_1}



- Approximate value function: $\tilde{G}_{T_1}^{(j)}(x_{T_1}) = \max_{i=1,\ldots,j} \{a_i^\top x_{T_1} + b_i\}$

Pereira and Pinto, 1991

Benders decomposition bounds

Given feasible solutions for each stage, can form

$$LB = \sum_{t=0}^{T_1-1} g_t(x_t, u_t) + \tilde{G}_{T_1}^{(j)}(x_{T_1})$$

$$UB = \sum_{t=0}^{T_1-1} g_t(x_t, u_t) + \sum_{t=T_1}^{T-1} \left(c_t^\top x_t + d_t^\top u_t\right) + c_T^\top x_T$$

Dual dynamic programming algorithm with approximate solutions of nonlinear first stage

- Nonlinear first stage **(NLP1)**:

$$\begin{aligned}\min_{\substack{u_0,\dots,u_{T_1-1}\\x_1,\dots,x_{T_1}}} \quad & \sum_{t=0}^{T_1-1} g_t(x_t,u_t) + G_{T_1}(x_{T_1})\\ \text{s.t.} \quad & x_{t+1} = f_t(x_t,u_t), \quad t=0,\dots,T_1-1\\ & (u_t,x_t)\in\mathcal{Z}_t, \qquad t=0,\dots,T_1-1\\ & x_0 \text{ given.}\end{aligned}$$

- Linear second stage **(LP2)**:

$$\begin{aligned}G_{T_1}(x_{T_1}) = \min_{\substack{u_{T_1},\dots,u_{T-1}\\x_{T_1+1},\dots,x_T}} \quad & \sum_{t=T_1}^{T-1} (c_t^\top x_t + d_t^\top u_t) + c_T^\top x_T\\ \text{s.t.} \quad & x_{t+1} = A_t x_t + B_t u_t,\\ & E_t x_t + F_t u_t \le h_t,\\ & t = T_1,\dots,T-1.\end{aligned}$$

Algorithm 1 Split-horizon DDP with nonlinear first horizon

while $(UB - LB > \epsilon)$ **do**

1. Solve (NLP1) using $\tilde{G}_{T_1}^{(j)}(x_{T_1}) = \max_{i=1,\dots,j}\ \{a_i^\top x_{T_1} + b_i\}$;
2. Generate LB using x, u from step 1 and current $\tilde{G}_{T_1}^{(j)}(x_{T_1})$;
3. Solve (LP2) using x_{T_1} from step 1;
4. Generate UB using x, u from steps 1 and 3;
5. Improve estimate of $G_{T_1}(x_{T_1})$ using dual variables from step 3;
6. $j = j + 1$;

end

Dual dynamic programming algorithm convergence

Theorem 1: When using Algorithm 1 to solve a split-horizon problem of the form (NLP+LP), the algorithm will terminate in a **finite number of iterations**, returning a **feasible** solution of the split-horizon problem. In addition, two special cases apply:

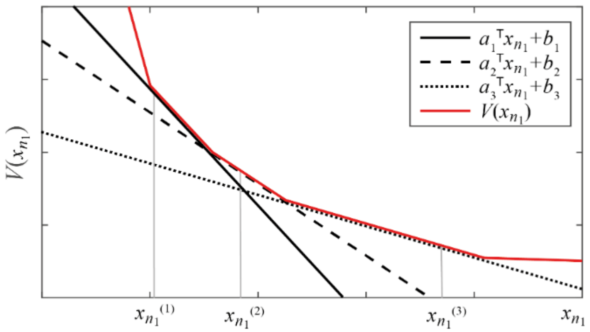
(a) If the nonconvex first stage is solved in step 1 to **local optimality**, then Algorithm 1 will return a **locally-optimal** solution to the two-stage problem.

(b) If the nonconvex first stage is solved in step 1 to **global optimality**, then Algorithm 1 solves the two-stage problem to **global optimality**.

Proof sketch: Since second stage is linear, finite number of lower-bounding hyperplanes. Eventually find all relevant hyperplanes to set LB = UB at given point.

$$LB = \sum_{t=0}^{T_1-1} g_t(x_t, u_t) + \tilde{G}_{T_1}^{(j)}(x_{T_1})$$

$$UB = \sum_{t=0}^{T_1-1} g_t(x_t, u_t) + G(x_{T_1})$$



Error bound on two-stage approximation

We restrict ourselves to the following tractable problem class:

Assumption 2: There exists a map M_t which takes a solution for (LP2) and produces a unique solution for the second stage of (NLP) such that

(a) At each timestep, the objective of the LP is an underestimate of (NLP), with a maximum error of δ_t.

(b) Through M_t , each feasible solution of (NLP) is mapped to by a feasible solution of (LP2).

In other words, any feasible second-stage solution of (NLP) is underapproximated by a feasible solution of (LP2), with a maximum underestimate of δ_t .

Theorems on suboptimality

- **Theorem 2**: When using Algorithm 1 to solve (NLP+LP), if Assumption 2 holds, and step 1 is solved to ϵ -suboptimality each iteration, then the resulting solution $\underline{G}_0^{(J)}(x_0)$ is such that:

$$\underline{G}_0^{(J)}(x_0) \leq G_0(x_0) \leq \underline{G}_0^{(J)}(x_0) + \epsilon + \sum_{t=T_1}^{T-1} \delta_t .$$

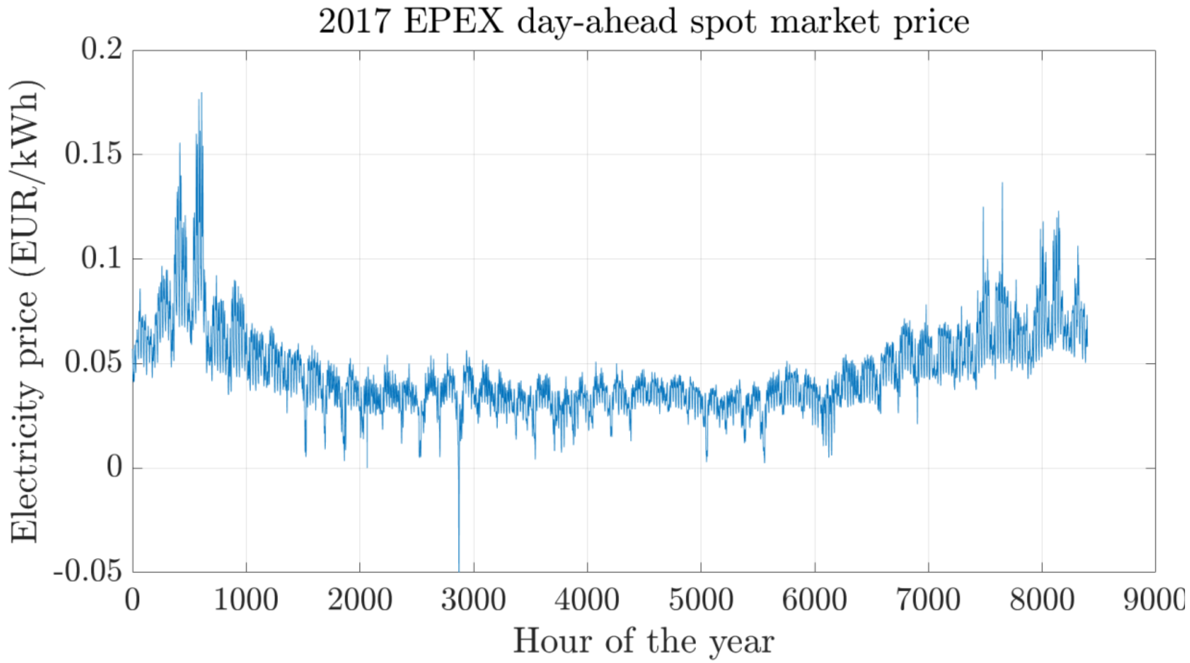
- **Theorem 3**: When using Algorithm 1 to solve (NLP+LP), if Assumption 2 holds, and step 1 is solved to ϵ -suboptimality each iteration, then when the resulting arguments $(\tilde{x}^{(J)}, \tilde{u}^{(J)})$ are evaluated in the objective of (NLP):

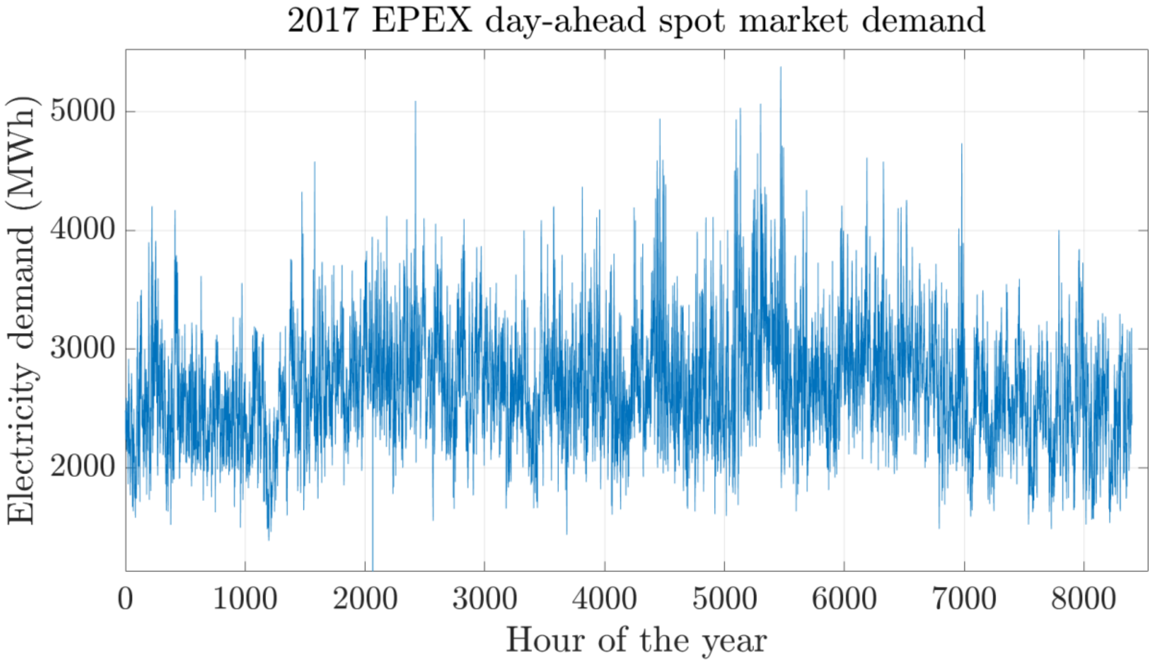
$$G_0(x_0) \leq \sum_{t=0}^{T_1-1} g_t(\tilde{x}_t^{(J)}, \tilde{u}_t^{(J)}) + G_{T_1}(\tilde{x}_{T_1}^{(J)}) \leq G_0(x_0) + \epsilon + \sum_{t=T_1}^{T-1} \delta_t .$$

Inspired by Guigues, 2018

Application: hydro reservoir scheduling with head effects

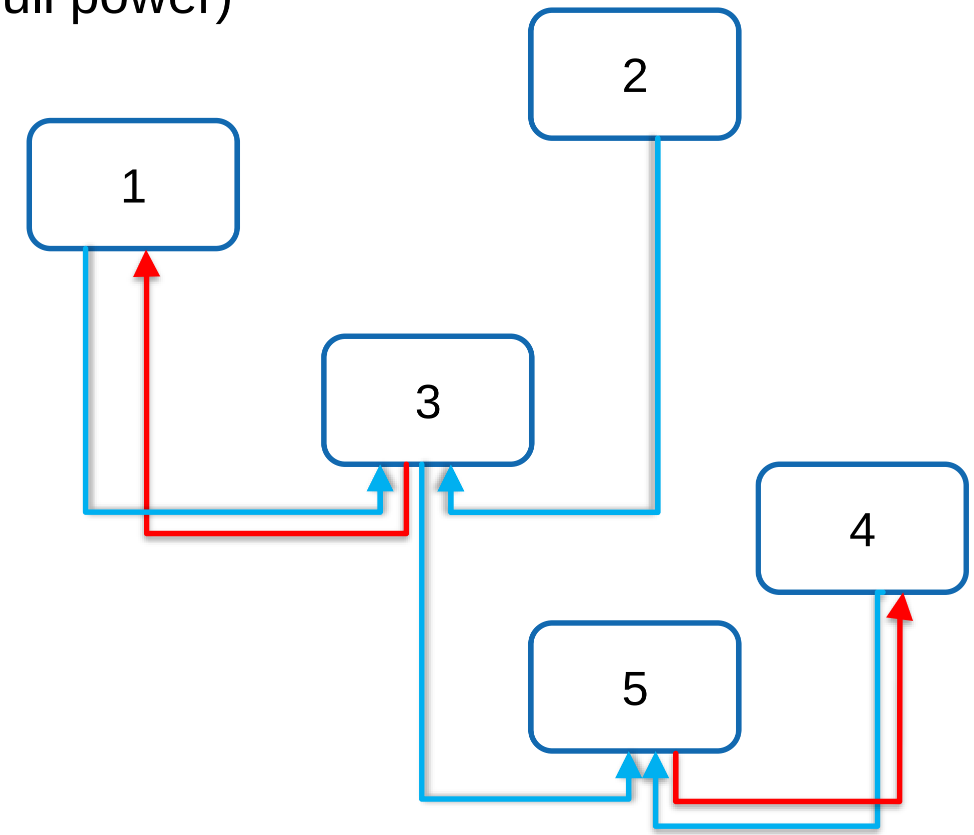
- System of multiple, interconnected reservoirs, with head-dependent power
- Objective: meeting electricity demand, with deterministic electricity spot prices





Application: sample hydro reservoir system parameters

- Five equally sized reservoirs
- 35 MW turbines and pumps with system topology as below
- Drainage time roughly 10 days (from full to empty at full power)
- No spillage – constant amount of water in system
- Reservoirs are rectangular in profile
  - Head can vary by a factor of 5 (40 m to 200 m)



Application: hydro reservoir scheduling with head effects

- Energy per unit volume depends on net head between reservoirs:

$$E^{i\to j}[t] = \alpha^{i\to j} + \beta^{i\to j}\left(\ell_i[t] - \ell_j[t]\right), \quad P^{i\to j}[t] = V^{i\to j}[t]E^{i\to j}[t]$$

- First stage exact NLP:

$$\begin{aligned}
\min_{V} \quad & \sum_{t=0}^{T-1} p[t]\Big(D[t] + \sum_{i=1}^{N}\sum_{j\in\mathcal{N}^{i\to}} P^{i\to j}[t]\Big) + G(\ell[T]) \\
\text{s.t.} \quad & P^{i\to j}[t] = V^{i\to j}[t]\ \left(\alpha^{i\to j} + \beta^{i\to j}\left(\ell_i[t] - \ell_j[t]\right)\right) \quad \forall\, j\in\mathcal{N}^{i\to} \\
& \ell_i[t+1] = \ell_i[t] + \frac{1}{\gamma_i}\Big(\sum_{k\in\mathcal{N}^{\to i}} V^{k\to i}[t] - \sum_{j\in\mathcal{N}^{i\to}} V^{i\to j}[t]\Big) \\
& \underline{\ell}_i[t] \le \ell_i[t] \le \bar{\ell}_i[t], \quad \ell_i[0] \text{ given} \\
& \underline{V}^{i\to j}[t] \le V^{i\to j}[t] \le \bar{V}^{i\to j} \quad \forall\, j\in\mathcal{N}^{i\to}[t] \\
& \quad i = 1,\ldots,N; \quad t = 0,\ldots,T-1.
\end{aligned}$$

Bilinear approximation: McCormick envelope

We replace the problematic constraint

$$P^{i\to j}[t] = V^{i\to j}[t]\ \left(\alpha^{i\to j} + \beta^{i\to j}\left(\ell_i[t] - \ell_j[t]\right)\right)$$

with

$$P^{i\to j}[t] = V^{i\to j}[t]\ \alpha^{i\to j} + \beta^{i\to j}(\chi_i^{i\to j}[t] - \chi_j^{i\to j}[t])$$

where:

$$\chi_i^{i\to j}[t] \geq V^{i\to j}[t]\bar{\ell}_i + \bar{V}^{i\to j}\ell_i[t] - \bar{V}^{i\to j}\bar{\ell}_i,$$

$$\chi_i^{i\to j}[t] \geq V^{i\to j}[t]\underline{\ell}_i + \underline{V}^{i\to j}\ell_i[t] - \underline{V}^{i\to j}\underline{\ell}_i,$$

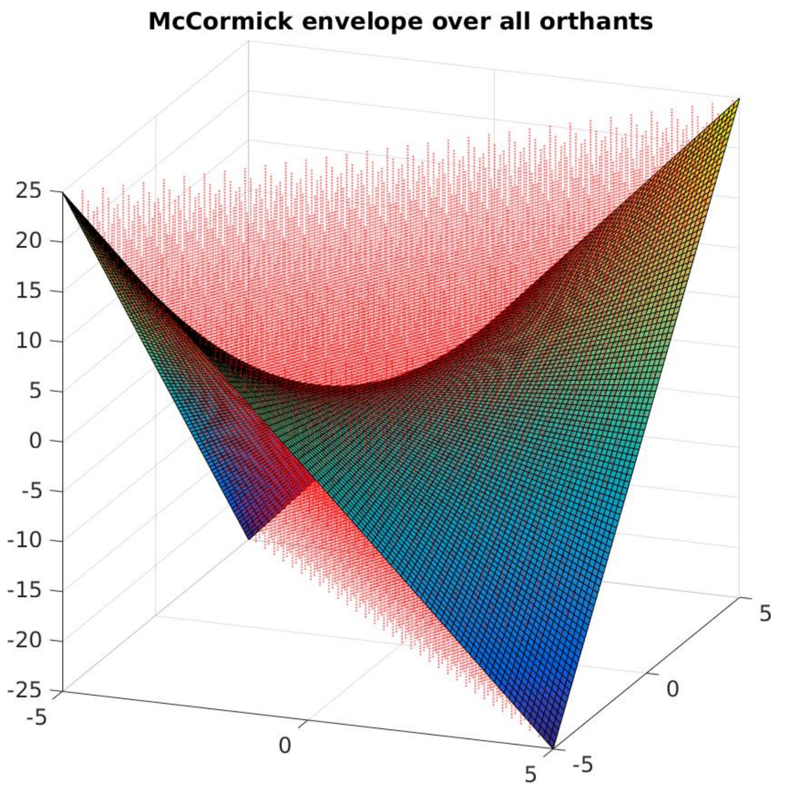
$$\chi_i^{i\to j}[t] \leq V^{i\to j}[t]\bar{\ell}_i + \underline{V}^{i\to j}\ell_i[t] - \underline{V}^{i\to j}\bar{\ell}_i,$$

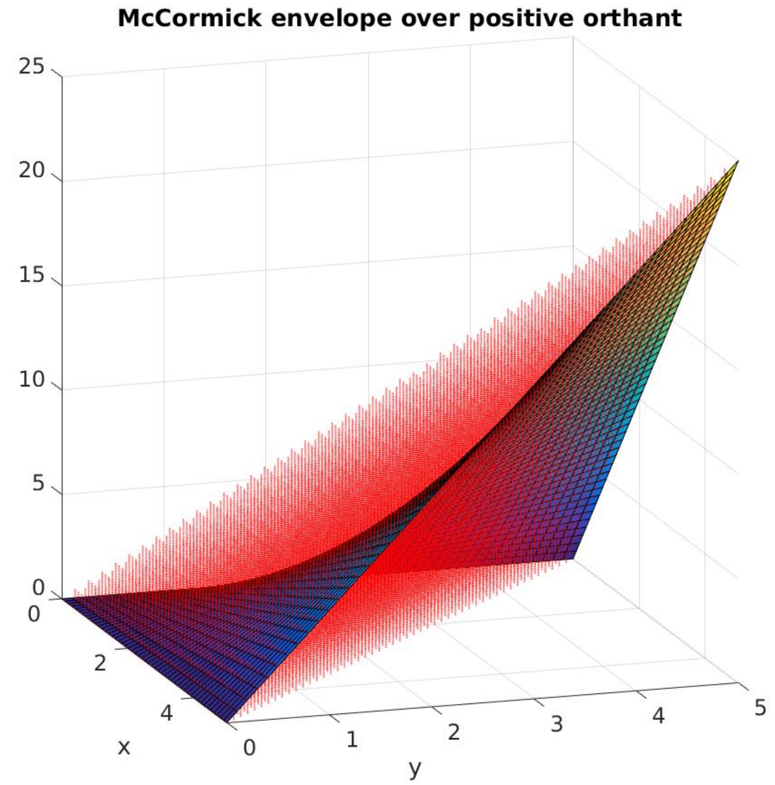
$$\chi_i^{i\to j}[t] \leq V^{i\to j}[t]\underline{\ell}_i + \bar{V}^{i\to j}\ell_i[t] - \bar{V}^{i\to j}\underline{\ell}_i.$$

Inspired by Cerisola, 2012

ETH zürich

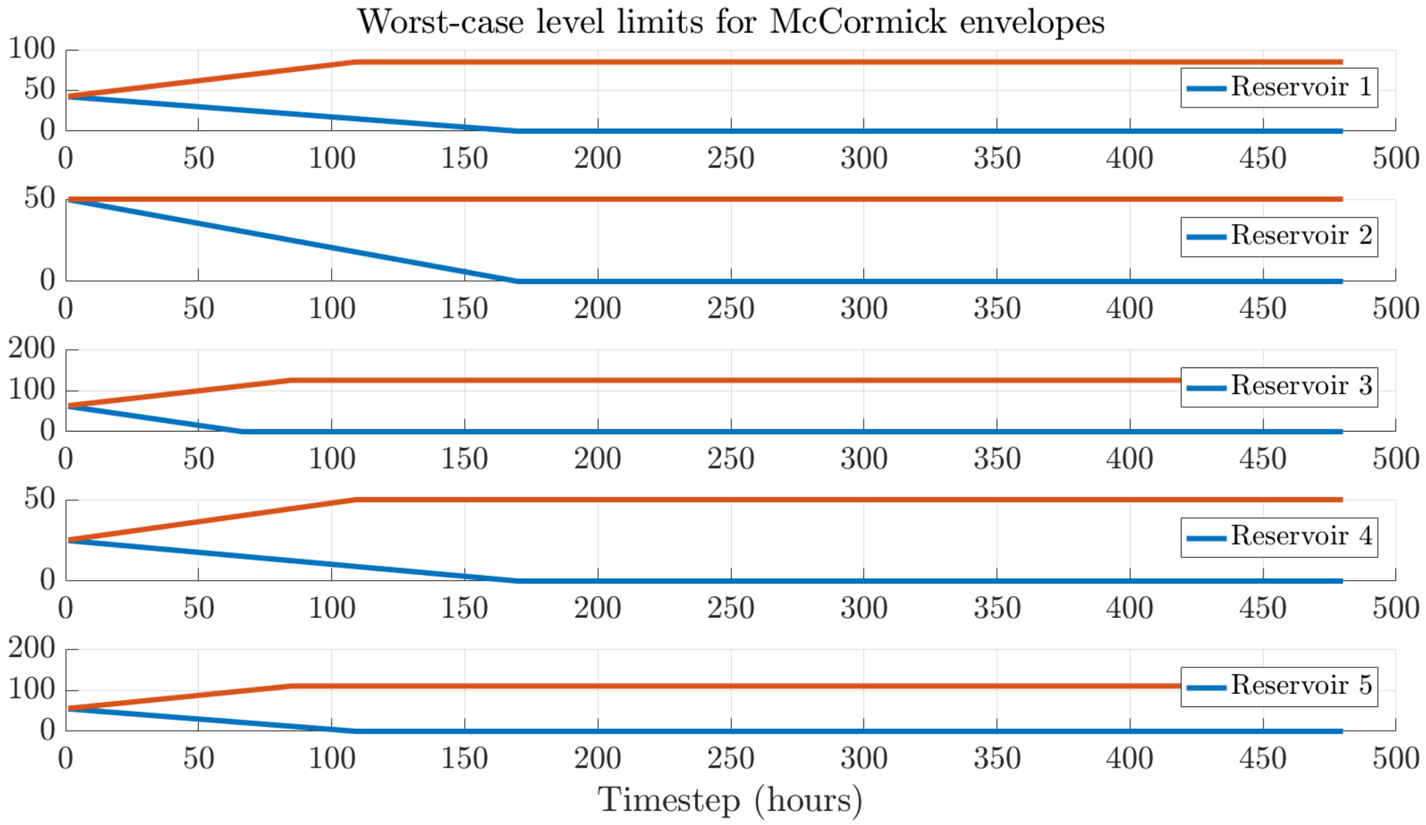
Bilinear approximation: McCormick envelope





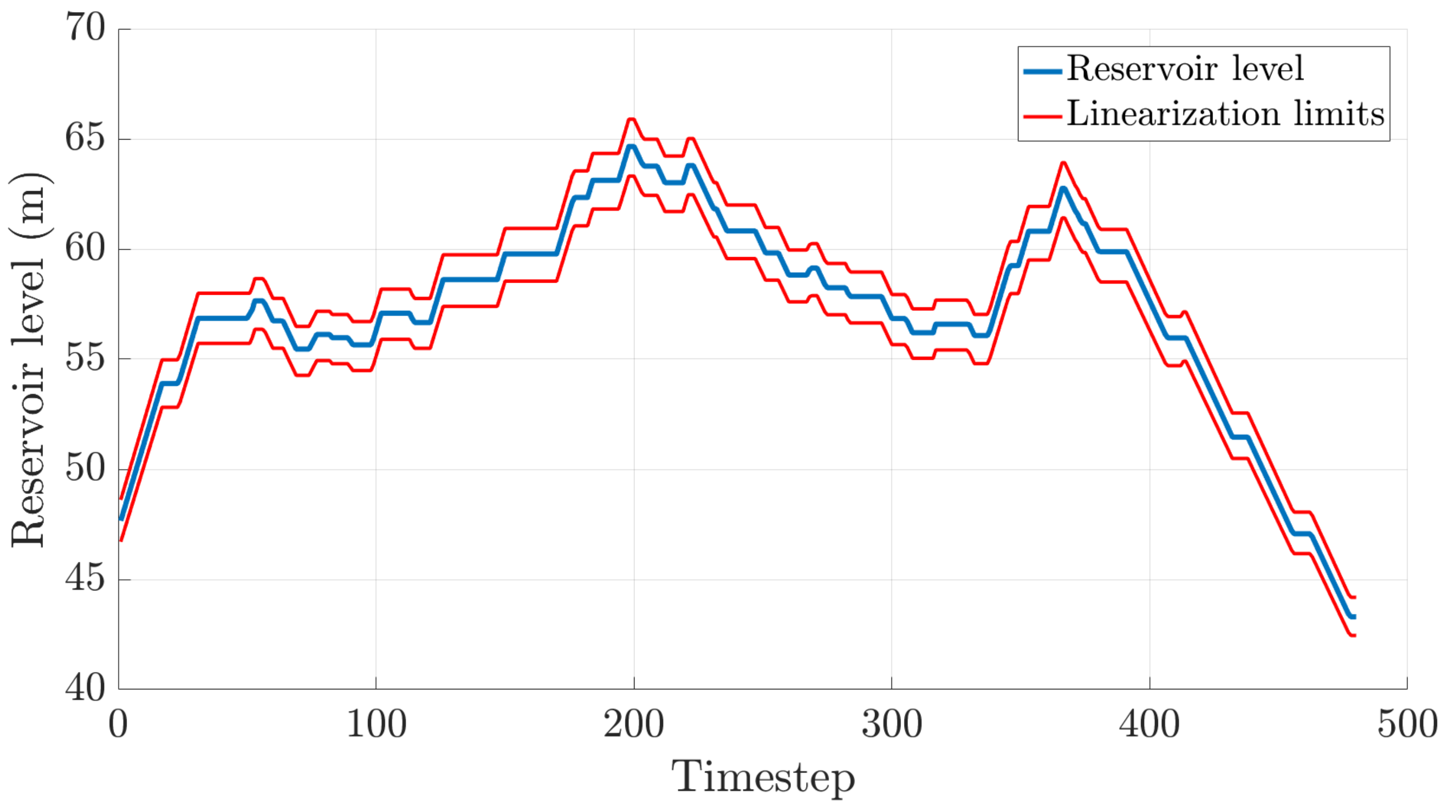
Improved bounds on McCormick envelopes

- Set linearization bounds to worst-case combination of inflows and outflows



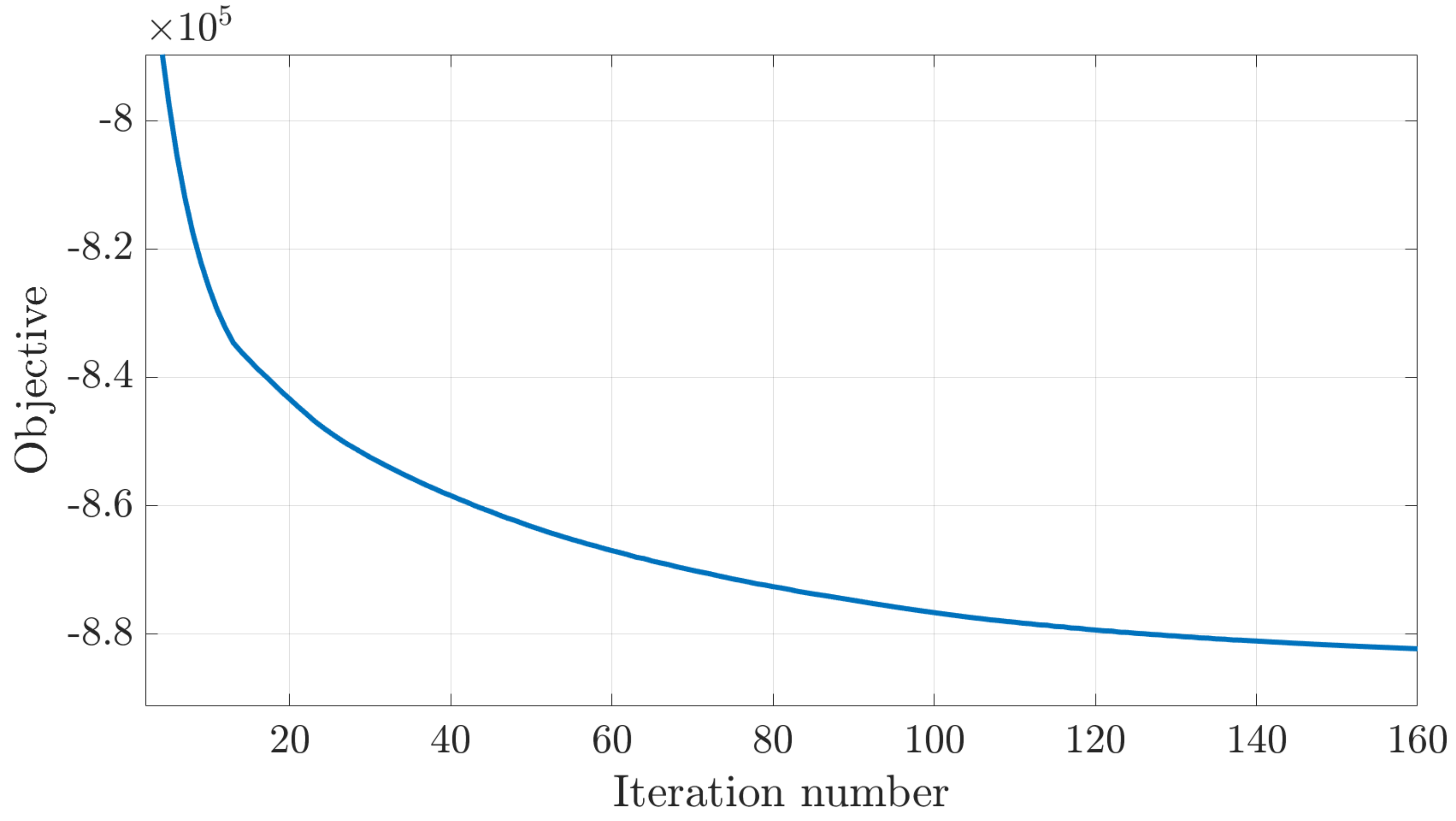
Successive linearization with McCormick envelopes

- Set linearization bounds to envelope about previously-found trajectory



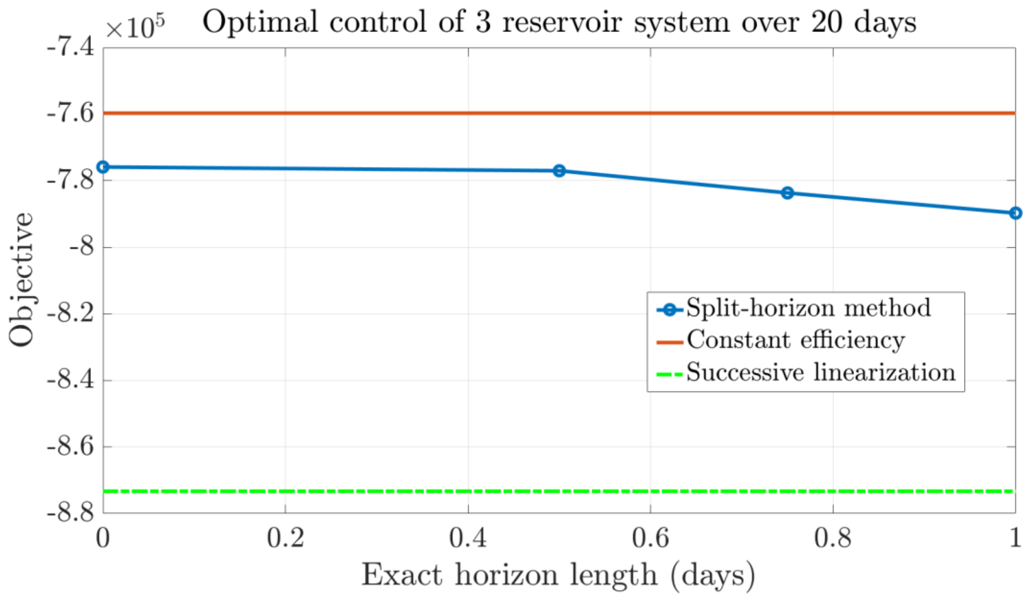
Successive linearization with McCormick envelopes

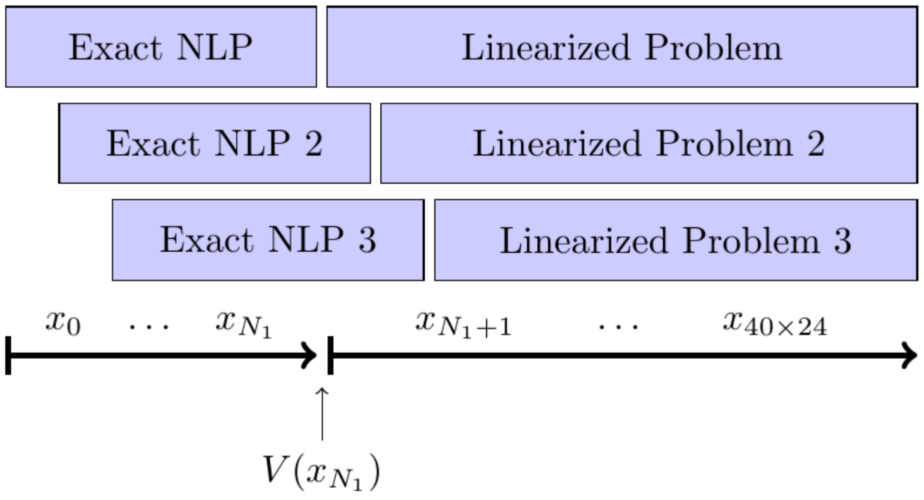
- Objective improves rapidly with heuristic algorithm



Experimental results: objective

- Compute objective over receding horizon for 20 days
  - 1 day split-horizon method is 4% better than constant efficiency approximation, but only 90% of objective found by successive linearization





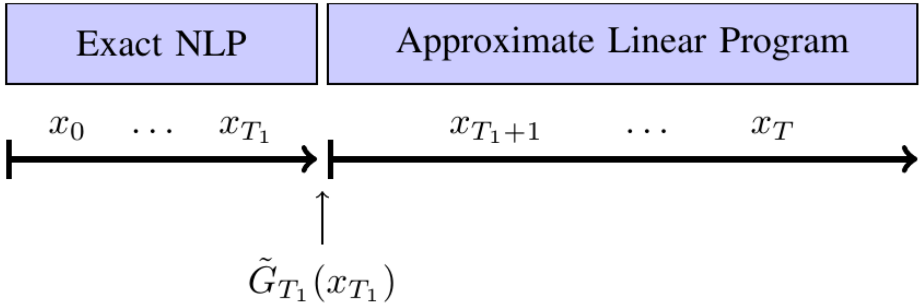
Discussion

- For larger systems and longer time horizons, worst-case bounds are quite poor
  - Modeling exact near term is beneficial
- Solving exact problem scales exponentially in problem size
  - But longer linear stage is relatively cheap

Conclusion

- Solved long-term nonlinear problem via multistage approximation
- Extended DDP to case where first stage is nonlinear
- Provided bounds on suboptimality

- Different nonlinear problem classes?



Thanks!

Reference:

B. Flamm, A. Eichler, J. Warrington, J. Lygeros, “Dual Dynamic Programming for Nonlinear Control Problems over Long Horizons,” European Control Conference, 2018. https://polybox.ethz.ch/index.php/s/woxUYLtSEKRlfKB