Valuation and Operation of Three Types of Power Plants using Continuous Time Stochastic Control Models

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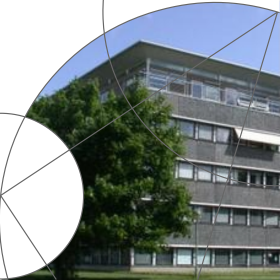
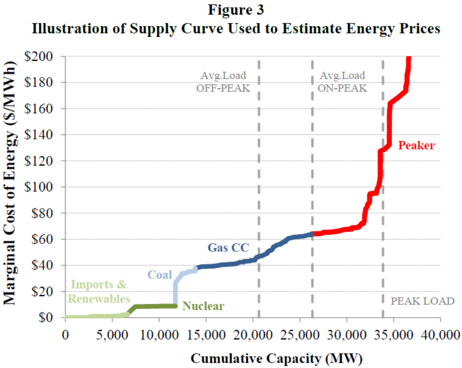


Overview

1. Motivation
2. Power plant models
   - Wind power plant
   - Gas power plant
   - Hydro power plant
3. Stochastic Optimal Control
   - Heuristics of HJB equation
   - Applying the equation
4. Price model
   - Calibration with EM algorithm

Motivation

Energy prices based on supply and demand.



Increasing the amount of renewables will changes price dynamics in energy market.

Motivation

Investigate impact of changes in dynamics on:

1. Wind power plant
   - Non-controllable
   - Stochastic generation
   - No variable cost
2. Gas power plant
   - Controllable generation
   - Variable cost
3. Hydro plant
   - Controllable output/input levels
   - Stochastic input

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Wind power plant

$$V_{wind}(t,P,W) = E\left[\int_t^T e^{-\rho(s-t)} P_s h(W_s)\,\mathrm{d}s | P_t = P, W_t = W\right]$$

$$st.\ \mathrm{d}P_t = \mu_P(P_t,t)\,\mathrm{d}t + \sigma_P(P_t,t)\,\mathrm{d}Z_t^P + \gamma(P_t,t)\,\mathrm{d}J_t$$

$$\mathrm{d}W_t = \mu_W(W_t,t)\,\mathrm{d}t + \sigma_W(W_t,t)\,\mathrm{d}Z_t^W$$

Z_t is a Brownian Motion and J_t is a pure jump process.

V_{wind}: Value of wind power plant given wind and price.
P_t: Spot price
W_t: Wind speed
ρ: Discount factor
h: Power curve

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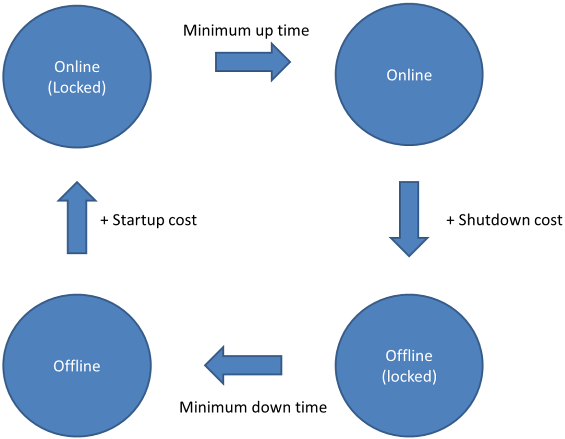
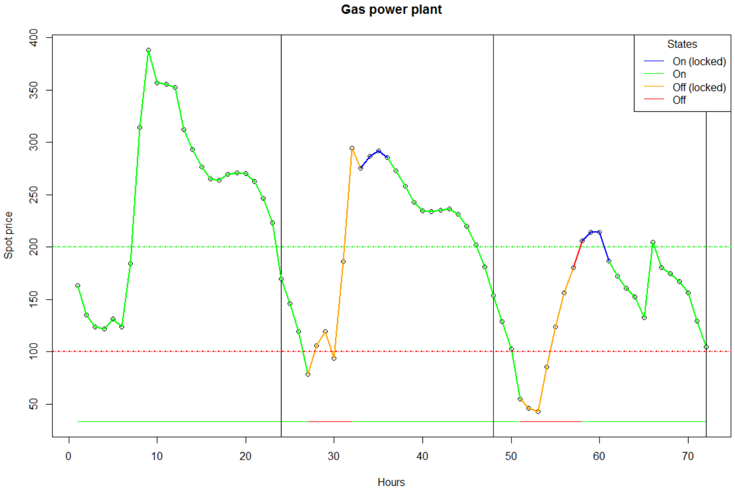
Gas power plant



Gas power plant



Gas power plant

$$V_{gas}(t,P,t_0,1) = \max_{\tau,Q_s} E\Big[\int_t^{\tau} e^{-\rho(s-t)}\Pi(Q_s,P_s,1)\,\mathrm{d}s + e^{-\rho(\tau-t)}V_{gas}(t+\tau,P_{t+\tau},0,2)\Big|P_t=P\Big] \quad \text{[On locked]}$$

$$V_{gas}(t,P,t_0,2) = \max_{\tau,Q_s} E\Big[\int_t^{\tau} e^{-\rho(s-t)}\Pi(Q_s,P_s,2)\,\mathrm{d}s + e^{-\rho(\tau-t)}(C_{off}+V_{gas}(t+\tau,P_{t+\tau},0,3))\Big|P_t=P\Big] \quad \text{[On]}$$

$$V_{gas}(t,P,t_0,3) = \max_{\tau,Q_s} E\Big[\int_t^{\tau} e^{-\rho(s-t)}\Pi(Q_s,P_s,3)\,\mathrm{d}s + e^{-\rho(\tau-t)}V_{gas}(t+\tau,P_{t+\tau},0,4)\Big|P_t=P\Big] \quad \text{[Off locked]}$$

$$V_{gas}(t,P,t_0,4) = \max_{\tau,Q_s} E\Big[\int_t^{\tau} e^{-\rho(s-t)}\Pi(Q_s,P_s,4)\,\mathrm{d}s + e^{-\rho(\tau-t)}(C_{on}+V_{gas}(t+\tau,P_{t+\tau},0,1))\Big|P_t=P\Big] \quad \text{[Off]}$$

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Hydro Power Plant

$$V_{hydro}(t, P, I, L) = \max_{(v_s)_{s\in[t,T]}} E\left[\int_t^T e^{-\rho(s-t)} P_s H(v_s, L_s)\,\mathrm{d}s | P_t = P, I_t = I\right]$$

$$st.\ \mathrm{d}P_t = \mu_P(P_t, t)\,\mathrm{d}t + \sigma_P(P_t, t)\,\mathrm{d}Z_t^P + \gamma(P_t)\,\mathrm{d}J_t$$

$$\mathrm{d}I_t = \mu_I(I_t, t)\,\mathrm{d} + \sigma_I(I_t, t)\,\mathrm{d}Z_t^I$$

$$\mathrm{d}L_t = (I_t - v_t)\,\mathrm{d}t$$

$$(I_t - v_t)1_{(L_t > L_{min})} \leq I_t - v_t$$ Positive inflow at lower bound

$$(I_t - v_t)1_{(L_t < L_{max})} \geq I_t - v_t$$ Negative inflow at upper bound

$$v_{min} \leq v_t \leq v_{max}$$

P_t: Spot price
I_t: Inflow rate to storage facility (melt water and rainfall etc.)
L_t: Water level in hydro power plant
v_t: outflow rate
H: Power produced
$-v_{min}$: Max inflow rate
v_{max} : Max outflow rate

Overview

Stochastic Optimal Control

Let $u(t)$ be the control at time t.

Minimize

$$J_t = E_{tx}\left[\int_t^T L(s, X_s, u(s))\,\mathrm{d}s\right]$$

Define value function

$$V(t,x) = \inf_{u \in C} J_t$$

C is set of admissable controls.
Bellman's principle of dynamic programming:

$$V(t,x) = \inf_{u \in C} E_{tx}\left[\int_t^{t+h} L(s, X_s, u(s))\,\mathrm{d}s + V(t+h, X_{t+h})\right]$$

Stochastic Optimal Control

Minimizing

$$J_t = E_{tx}\left\{\int_t^T L(s, X_s, u(s))\,\mathrm{d}s\right\}$$

with

$$V(t,x) = \inf_{u\in C} J_t$$

becomes equivalent to, under some regularity conditions, to determining $V(t,x)$ through the HJB equation

$$\inf_{u\in C}\{A^u V(t,x) + L(t,x,v)\} = 0$$

with $V(T,x) = 0$.
A^u is the "derivative of the expected value" defined as

$$A^v V(t,x) = \lim_{h\to 0^+} \frac{1}{h}\left[E_{tx}V(t+h, X_{t+h}) - V(t,x)\right]$$

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Wind power plant

$$V(t,P,W) = E\left[\int_t^T e^{-\rho(s-t)} P_s h(W_s)\,\mathrm{d}s | P_t = P, W_t = W\right]$$
$$st.\ \mathrm{d}P_t = \mu_P(P_t,t)\,\mathrm{d}t + \sigma_P(P_t,t)\,\mathrm{d}Z_t^P + \gamma(P_t)\,\mathrm{d}J_t$$
$$\mathrm{d}W_t = \mu_W(W_t,t)\,\mathrm{d}t + \sigma_W(W_t,t)\,\mathrm{d}Z_t^W$$

V solves the Hamilton-Jacobi-Bellman equation

$$V_t + \mu_P V_P + \mu_W V_W + \frac{1}{2}\sigma_P^2 V_{PP} + \frac{1}{2}\sigma_W^2 V_{WW} + \rho_{PW}\sigma_P\sigma_W V_{PW}$$
$$+ \lambda E\left[V^+ - V | P_t = P, W_t = W\right] + Ph(W) - \rho V = 0$$
$$V(T,P,W) = 0$$

Gas power plant

The Hamilton-Jacobi-Bellman PIDEs are

$$A(V_{gas}(t,P,t_0,1)) + \max_Q\{\Pi(P,Q,1)\} - \rho V_{gas}(t,P,t_0,1) = 0,$$

$$A(V_{gas}(t,P,t_0,2)) + \max_Q\{\Pi(P,Q,2)\} - \rho V_{gas}(t,P,t_0,2) = 0,$$

$$A(V_{gas}(t,P,t_0,3)) + \max_Q\{\Pi(P,Q,3)\} - \rho V_{gas}(t,P,t_0,3) = 0,$$

$$A(V_{gas}(t,P,t_0,4)) + \max_Q\{\Pi(P,Q,4)\} - \rho V_{gas}(t,P,t_0,4) = 0,$$

with boundary conditions $V_{gas}(T,P,t_0,i) = 0,\ i = 1,2,3,4$, and

$$\begin{aligned} V_{gas}(t,P,T_{up},1) &= V_{gas}(t,P,0,2), \\ V_{gas}(t,\mathcal{P}^*_{off}(t),t_0,2) &= V_{gas}(t,\mathcal{P}^*_{off}(t),0,3) - C_{shutdown}, \\ V_{gas}(t,P,T_{down},3) &= V_{gas}(t,P,0,4), \\ V_{gas}(t,\mathcal{P}^*_{on}(t),t_0,4) &= V_{gas}(t,\mathcal{P}^*_{on}(t),0,1) - C_{startup}. \end{aligned}$$

Storage facility (Hydro Power Plant)

$$V(t,P,I,L) = \max_{(v_s)_{s\in[t,T]}} E\left[\int_t^T e^{-\rho(s-t)} P_s H(v_s, L_s)\,\mathrm{d}s | P_t = P, I_t = I\right]$$

$$
\begin{aligned}
st.\ \mathrm{d}P_t &= \mu_P(P_t,t)\,\mathrm{d}t + \sigma_P(P_t,t)\,\mathrm{d}Z_t^P + \gamma(P_t)\,\mathrm{d}J_t \\
\mathrm{d}I_t &= \mu_I(I_t,t)\,\mathrm{d} + \sigma_I(I_t,t)\,\mathrm{d}Z_t^I \\
\mathrm{d}L_t &= (I_t - v_t)\,\mathrm{d}t \\
&(I_t - v_t)1_{(L_t > L_{min})} \leq I_t - v_t \\
&(I_t - v_t)1_{(L_t < L_{max})} \geq I_t - v_t \\
&v_{min} \leq v_t \leq v_{max}
\end{aligned}
$$

HJB equation:

$$
\begin{aligned}
&V_t + \mu_P V_P + \frac{1}{2}\sigma_P^2 V_{PP} + \mu_I V_I + \frac{1}{2}\sigma_I^2 V_{II} + \rho_{PI}\sigma_I\sigma_P V_{PI} \\
&\quad + \max_v \{PH(v,L) + (I-v)V_L\} \\
&\quad + \lambda E\left[V^+ - V | P_t = P, I_t = I, L_t = L\right] - \rho V = 0
\end{aligned}
$$

$$V(T,P,W) = 0$$

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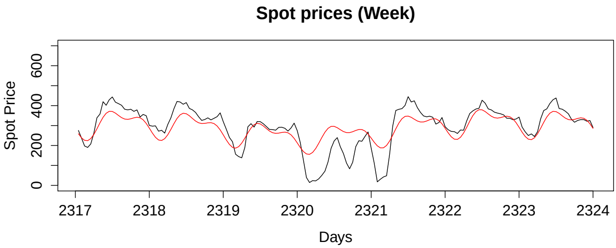
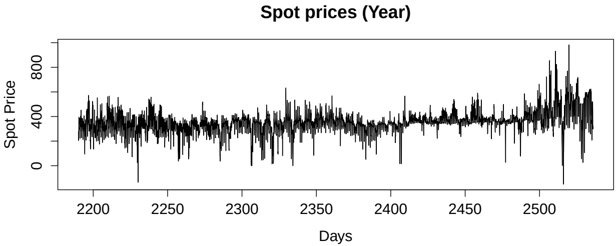
Price model



- Seasonality
- Mean reversion



- Jumps
- Negative prices

Price model

Spot priced modelled as

$$P_t = e^{\theta_t} e^{X_t} - M$$

where

$$\mathrm{d}X_t = \kappa(\alpha - X_t)\,\mathrm{d}t + \sigma\,\mathrm{d}Z_t^P + \,\mathrm{d}J_t$$

θ_t: Seasonal component
M: Constant to allow for negative prices

Explicit solution

$$P_t = e^{\theta_t} \exp\left\{X_s e^{-\kappa(t-s)} + \alpha(1 - e^{-\kappa(t-s)}) + \sigma \int_s^t e^{-\kappa(t-u)}\,\mathrm{d}Z_u^P + \sum_{n=N_s+1}^{N_t} e^{-\kappa(t-T_n)} Y_n\right\} - M$$

Price model

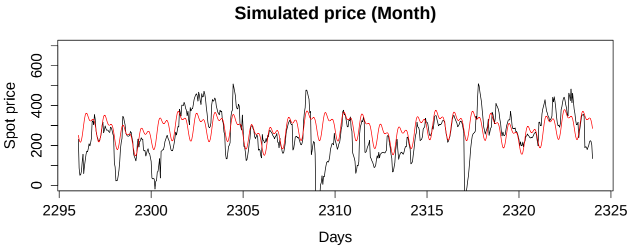
Calibration gives

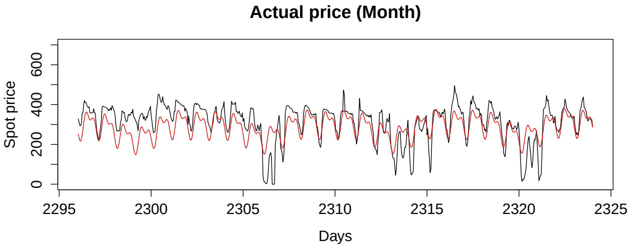
- Expected price 297.10
- Hourly volatility $\sim$ 17 (95% in (-33.3,33.3))
- Mean reversion level 279.57
- Mean reversion ($\sim$ 3.5 hours to get halfway to mean)

Jumps modelled as medium and small jumps

- Small jumps ($\sim$ 3.5 hours between jumps, 95% in $(-100, 100)$)
- Medium jumps ($\sim$ 38 hours between jumps, 95% in $(-360, 360)$)

Price model


Simulated price (Month)
Spot price
0
200
400
600
2295
2300
2305
2310
2315
2320
2325
Days


Actual price (Month)
Spot price
0
200
400
600
2295
2300
2305
2310
2315
2320
2325
Days

Conclusion

- Possible to determine value and control of power plants over long time horizons
- Problem is solved locally
- Running time increases linearly with T
- Incorporates non-linear payoff functions
- Capture specific marginal distributions
- Approximate discrete problem with continuous time problem

References

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Questions?