Modeling high-dimensional natural inflows for stochastic-dynamic optimization

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Stochastic Dynamic Optimization

- Dynamic decision problem with T time stages

  $\xi = (\xi_1, \dots, \xi_T)$ with $\xi_t(\omega) \in \mathbb{R}^m$ and $\xi^t = (\xi_1, \dots, \xi_t)$

  $x = (x_1, \dots, x_T)$ decisions with $x^t = (x_1, \dots, x_t)$ and $x_t \in \mathcal{X}_t$

  x_t measurable w.r.t. $\sigma(\xi^t)$

- Model formulation using value / cost-to-go functions

$$V_T(x^{T-1}, \xi^T) = \max_{x_T \in \mathcal{X}_T} R_T(x^T, \xi^T)$$

$$V_t(x^{t-1}, \xi^t) = \max_{x_t \in \mathcal{X}_t} R_t(x^t, \xi^t) + \mathbb{E}\left[V_{t+1}(x^t, \xi^{t+1}) | \xi^t\right], \ \forall t : 1 \leq t < T$$

- Numerical solution

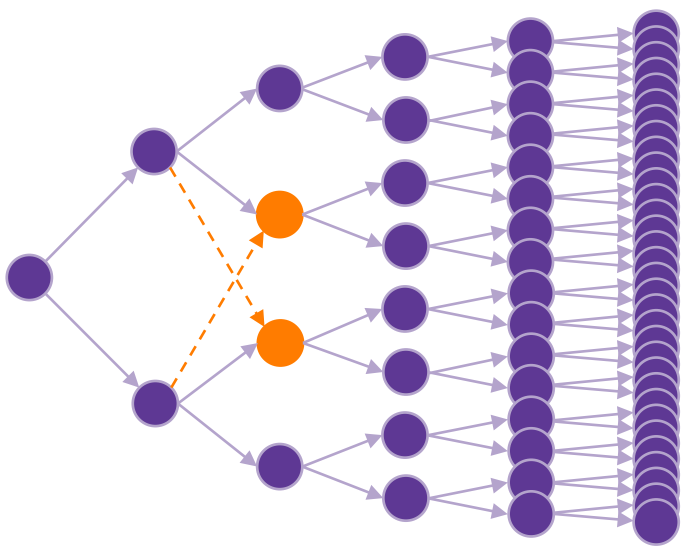
  Integrals $\mathbb{E}\left[V_{t+1}(x^t, \xi^{t+1}) | \xi^t\right]$ have to be calculated

  Discretization of ξ required for numerical solutions

From Trees to Lattices

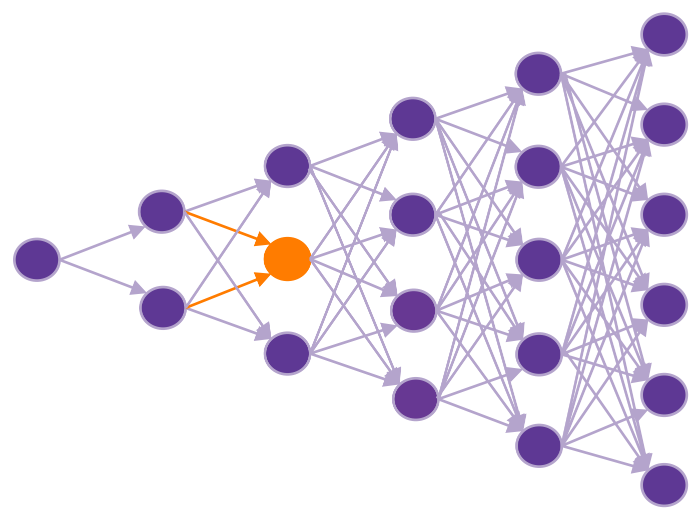
Scenario Generation for Stochastic Optimization

Scenario Tree



State-time-*history* graph with
63 nodes and 32 scenarios

Scenario *Lattice*



State-time graph with
21 nodes and 720 scenarios

Scenario Lattices
A Compressed Scenario Representation

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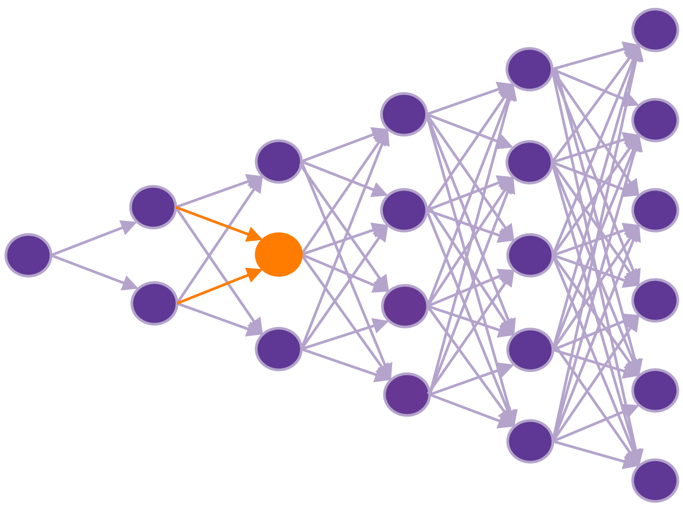
- Lattices carry no information about the history of the process
- Suitable for Markov processes where

  $$P(\xi_{t+1} \in A|\xi^t) = P(\xi_{t+1} \in A|\xi_t)$$

- Covers all state space type stochastic models:
  - Autoregressive models
  - Dynamic factor models
  - Exponential smoothing
  - Hidden Markov models
  - Stochastic volatility models

Scenario *Lattice*



Stochastic Optimization on Lattices
From optimal decisions to value functions

- Decisions dependent on path leading to node *n*

$$x_n = x_n(x^{n-1}, \xi^t)$$

- Past decisions aggregate into a *resource* state (inventory, reservoir, …)

$$S_t(x_1, \xi_1, \ldots, x_{t-1}, \xi_{t-1}) = S_t(S_{t-1}, x_{t-1}, \xi_{t-1})$$

- Decisions are based on current state of the 'world'

$$x_t(x_1, \xi_1, \ldots, x_{t-1}, \xi_{t-1}, \xi_t) = x_t(S_t, \xi_t)$$

- *Markov Decision Process (MDP)* if the random process is memoryless and the value depends only on the current state

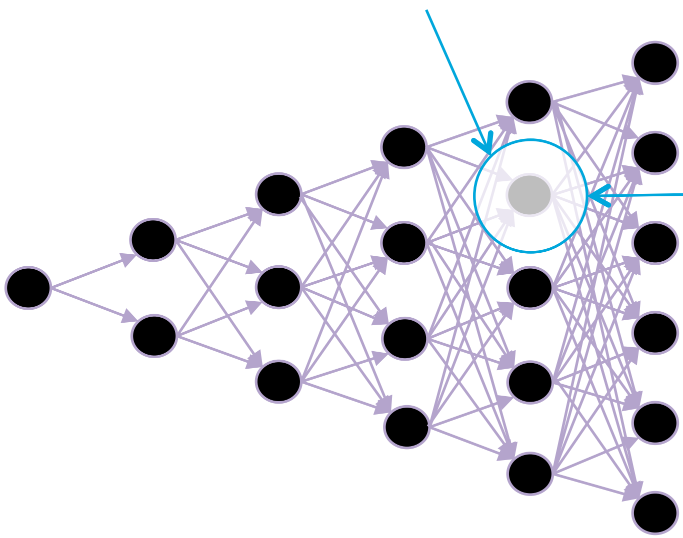
$$\mathbb{E}\left[V_t(x^{t-1}, \xi^t) | \xi^{t-1}\right] = \mathbb{E}\left[V_t(S_t, \xi_t) | \xi_{t-1}\right]$$

Approximate Dual Dynamic Programming

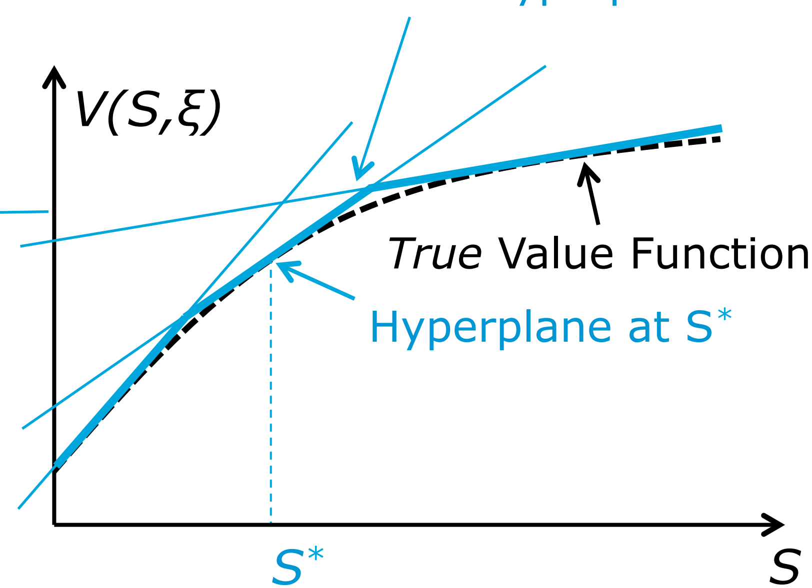
A solution strategy using scenario lattices

1. Construct a scenario lattice from a stochastic process
2. Find an approximate value function for each node

Each node of the lattice holds a value function



Approximate the value function as minimum of a set of hyperplanes



How Do We Fit a Lattice From Data?

- Parametric approach
  1. Estimate a Markovian time series process
  2. Reduce the continuous Markovian process to a lattice
  3. Determine an optimal policy using ADDP
  4. Evaluate the policy on the original time series model
- Data-driven approach
  1. Estimate a lattice directly from data
  2. Determine an optimal policy using ADDP

Skip scenario reduction

+

No policy validation needed

Multivariate Time Series Models

- Time series models with many variables
  - Vector autoregressive (VAR) models
  - Vector error correction (VEC) models
  - Multifactor models of state space type
- Dynamic factor model
  - Markovian time series model
  - Observations as linear combination of a few hidden factors
  - Less parameters through dimensionality reduction

Dynamic Factor Model

t: time index
F_t: vector of factors
A: transition matrix
X_t: vector of observations
V: dynamic factor loadings
B: parameter matrix of exogenous predictors
Z_t: exogenous predictors
u_t, v_t: error term

$$F_t = AF_{t-1} + u_t$$

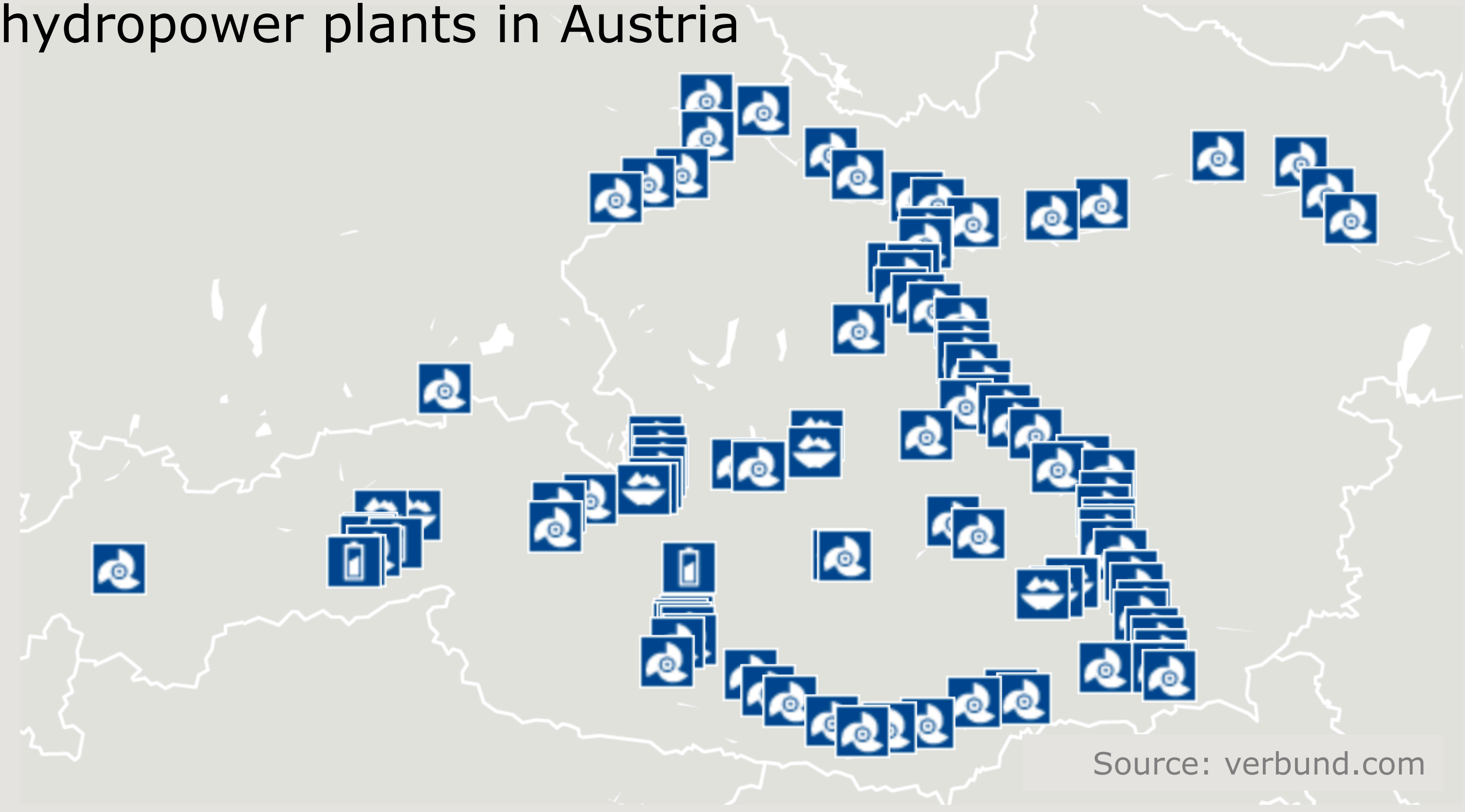
$$X_t = VF_t + BZ_t + v_t$$

Dynamic Factor Lattice

- Building blocks
  - Represent factor space by set of discrete factors $\bar{F}$
  - Replace linear state transition equation with transition matrices $P_t(\bar{F}_t|\bar{F}_{t-1})$
- Learning a lattice from data
  1. Choose optimal quantizers of the data at stage *t*
  2. Count transitions between observations in the neighborhood of quantizers at successive stages
  3. We use a moving time window of 30 days of observed transitions to obtain a large enough sample

Verbund Case Study

- Inflow data
  - Historical data from 1990 to 2012
  - Daily incremental inflows of 50 rivers
- Verbund hydropower plants in Austria

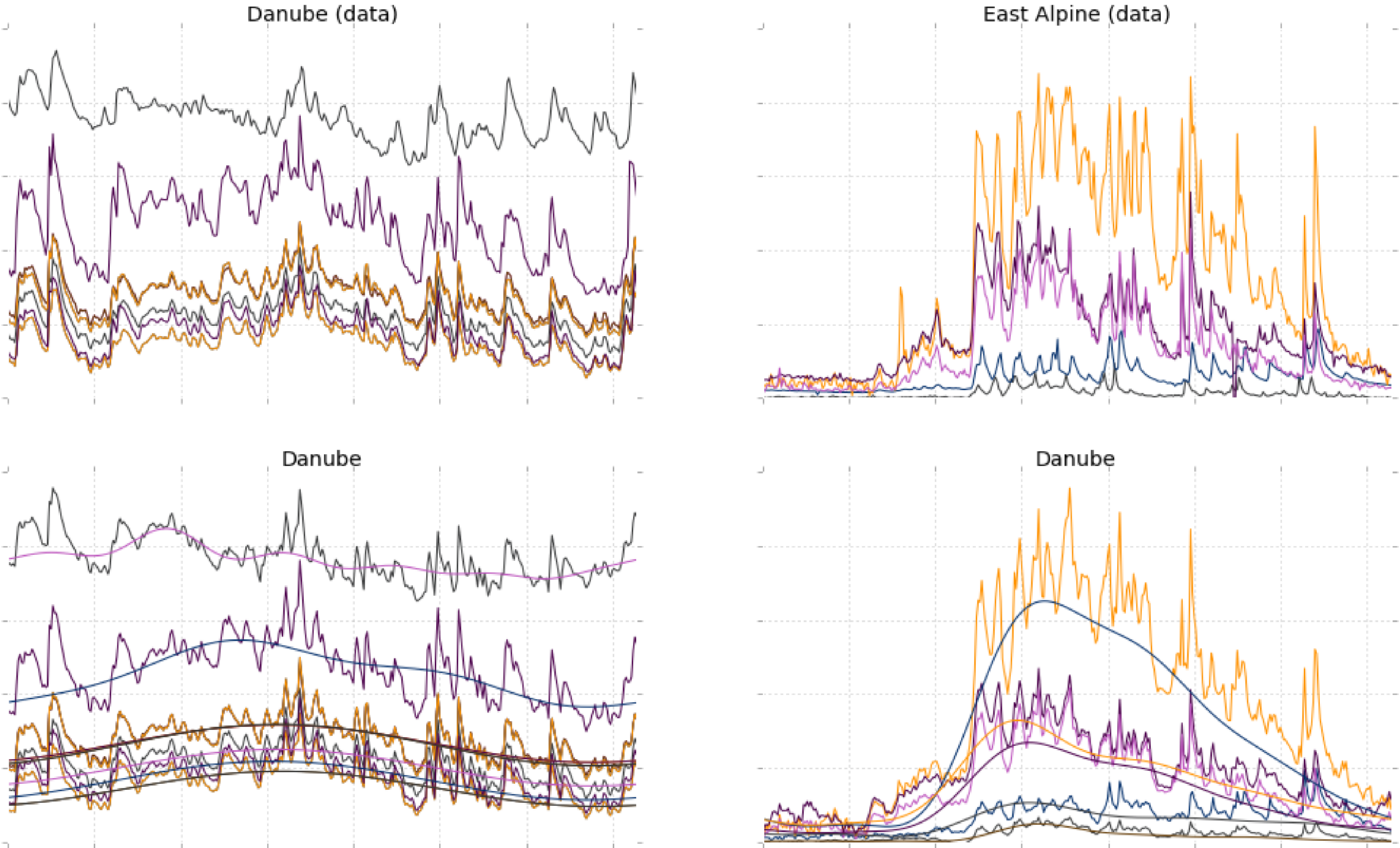


Source: verbund.com

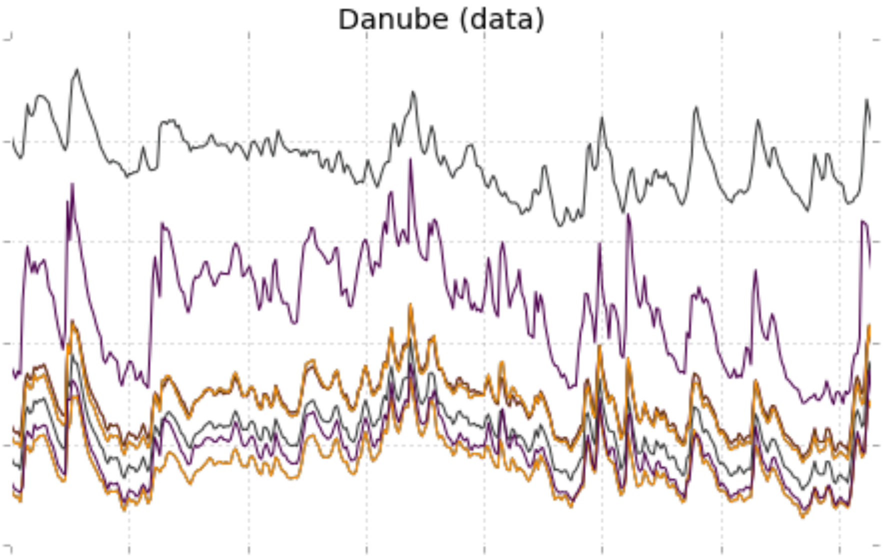
Methodology

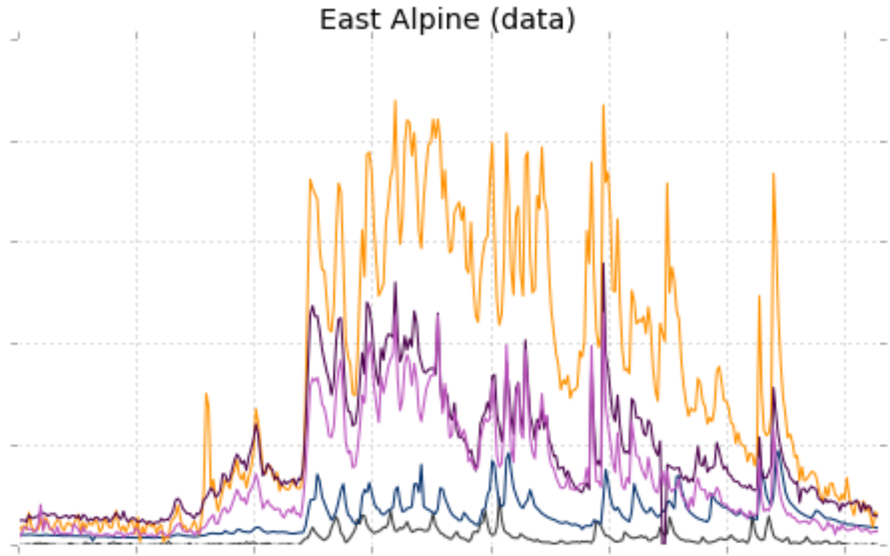
1. Transformation
   - Negative inflows prohibit Box-Cox transformation
   - Inverse hyperbolic sine transformation
2. Time series regression
   - Seasonal component as Fourier series
   - Model selection using BIC
3. Missing values
   - Singular value decomposition
   - Threshold value selected via cross-validation
4. Dynamic Factor Model
   - Static approach using PCA
   - 3 factors capture 75% of the variance
5. Lattice
   - 50 states + 364 transition matrices

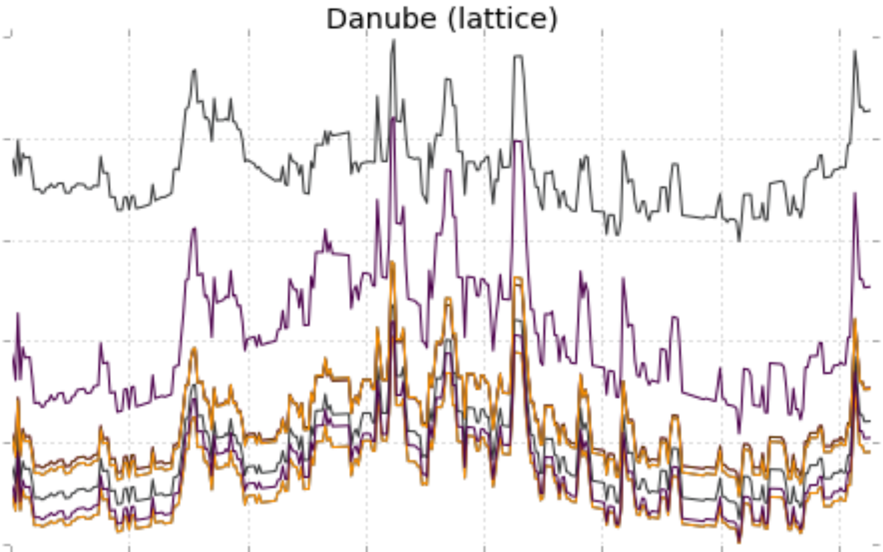
Original vs Reduced Time Series

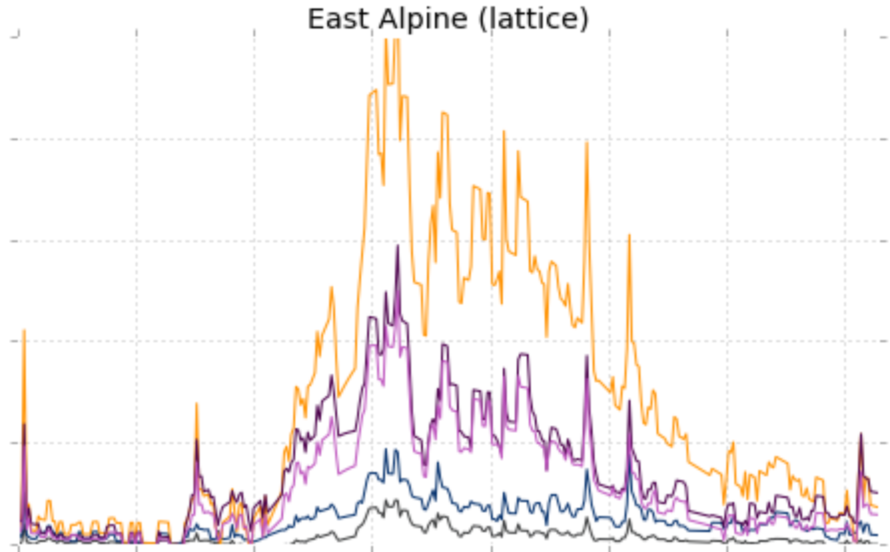


Original vs Simulated Time Series

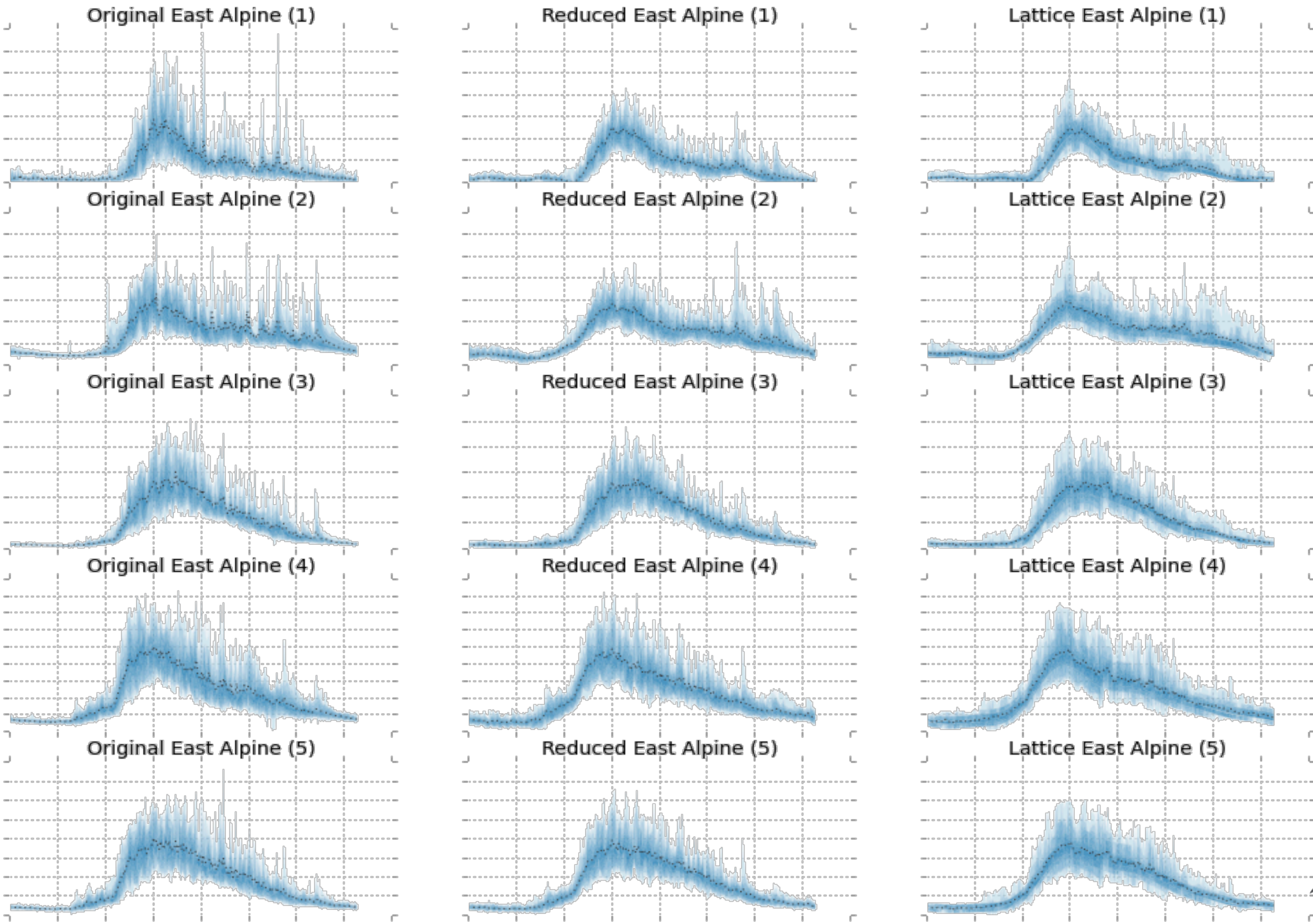






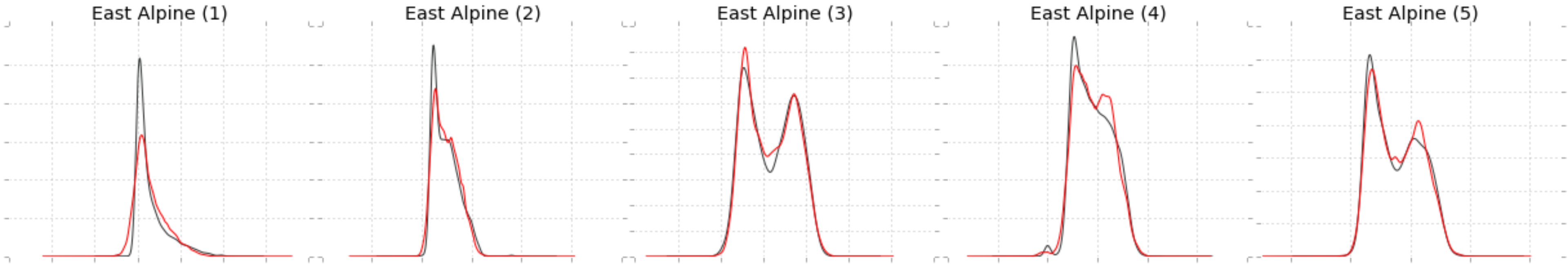


Data → Factors → Lattice

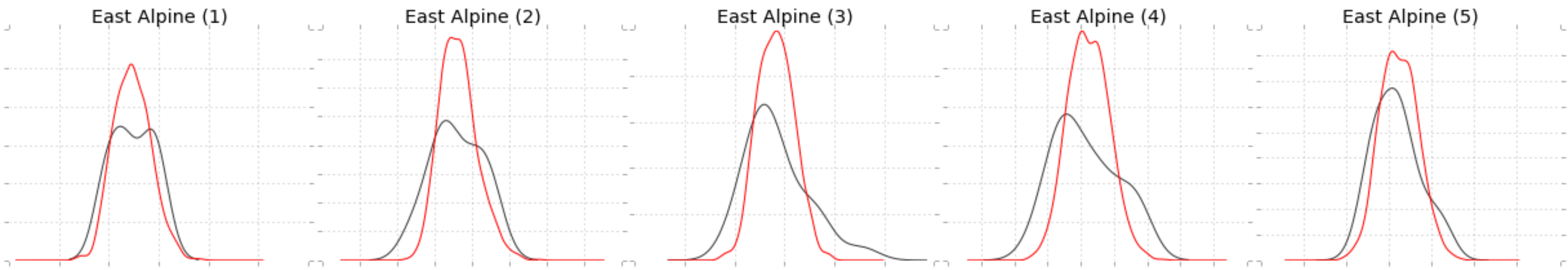


Kernel Density Estimation

Mean daily inflows



Mean annual inflows



– original data – lattice simulation

Conclusion

- Results
  - A small number of discrete states is sufficient to explain a high-dimensional inflow process.
  - Factors achieve longitudinal smoothing which helps to extract long-range information
- Outlook
  - Semi-parametric extensions to overcome sparse state transition samples
- Future Work
  - How do we measure the goodness of fit of a lattice?

Quantego QUASAR
A general-purpose stochastic optimizer