Turbulent and financial time series analysis

Amjad Mohammed (amjad.a.mohammed@uni-oldenburg.de)

Abstract: Some of the characteristics of turbulence are its randomness, nonlinearity, diffusivity, and dissipation, just to name a few. But couldn’t we characterize financial data in the same way? The answer is no, not exactly. Some of the extra descriptions for financial data, which makes it different than the steady experimental turbulence, are its Markovity and non-stationarity.

Turbulent signals:

Fig. 1 shows a non-trended non-corrependable stationary turbulent velocity signal, regime, made at the experiment in Oldenburg university. The spectrum of turbulent signals was chosen by Kolmogorov in the form:

\[ E(k) = C k^{-5/3} \]

where \( E(k) \) is the spectrum as a function of the wave number, \( C \) is Kolmogorov’s constant, \( k \) is the wave number, and \( C k^{-5/3} \) is the slope of the spectrum represented by the blue color in Fig. 1. It is not equal to \( 5/3 \) while the green color represents the dissipation spectrum and its slope equals \( 1/3 \), which verifies the theory in addition to that, the energy spectrum shows three distinct regions: the large scales range, the dissipation range and the random region. Taking a look at Figs. 2 and 4 we see that the same three regions are more or less represented. First comes the Taylor microscale \( l_{t} \) which is the curvature of the autocorrelation which is approximately equal for both data sets. (Fig. 3) data set is in the same as in Fig. 1 above and in Fig. 4 we show another data set with a higher Reynolds number \( R e = 400 \). This means that in both cases the autocorrelation is noise and the zero crossing in the beginning of the random region. Fig. 2 shows the same noise, but in the structure function of the high frequency number data and we notice the Kolmogorov inertial upon reaching the zero crossing point.

A useful tool in studying turbulence is the structure function of the increment and is equal to \( y_{i+1} - y_{i} \) for \( i = 1, 2, \ldots, n \), where \( y \) is the velocity at position \( x \), and \( \tau \) is the lag. The higher order moment \( \langle y_{i+1} - y_{i} \rangle^{n} \) describes the cascading of the energy from large scales to small scales. This cascading follows a power law as is shown in the following figures:

\[ \langle y_{i+1} - y_{i} \rangle^{n} \sim C_{n} \tau^{n} \]

In Fig. 5 the structure functions for the exponent \( n = 2 \) were calculated and in Fig. 6 we see the scaling of these structure functions according to \( \langle y_{i+1} - y_{i} \rangle^{2} \sim \tau \). In Fig. 7 we have used the standard Kolmogorov to show the power law scaling of the structure functions. We see also that the best fit is Kolmogorov’s logarithmic scaling model which is:

\[ \langle y_{i+1} - y_{i} \rangle^{2} \sim \tau^{11/3} \]

In Fig. 8 the dissipation time \( t_{d} \) is the logarithmic scale by tuning the parameter \( \tau = 2 \) and the constant is a constant that fits the data shown in Fig. 7. The question now is whether the above tools are suitable for non-stationary time series like the global warming temperature. In Figs. 9, 10, and 11 we show the autocorrelation functions, the dissipation time, and the dissipation time constant.

Financial signals:

Financial time series are non-stationary, i.e., the moments are a function of time. This is evident from Fig. 11a, which shows the DAX index for the period from 6/2/2001 to 3/2/2001. In Fig. 11b the autocorrelation function which shows a long memory. In Fig. 11c we see the spectrum which scales as \( -1/2 \) and not \( 5/3 \) as in the turbulent, while the probability density function, however, shows another scaling first and by increasing the lag they reach probably one could say, a von-Bruun distribution form upon reaching the zero crossing point.

Non-stationary time series are marked by a weakening of the Markov chain. This is evident from the figure above that the tools that were used to analyze stationary turbulence are not helpful in analyzing non-stationary time series. Again, this is clear from Fig. 11c.

Fig. 12 only shows that the data of the DAX index are not stationary. In the same way, this is also evident from the changes in the periods of the graph in Fig. 13 and the presence of the periodicity in the data. In Fig. 14 we show a random process and in Fig. 15 we show a random process and its autocorrelation. In Fig. 16 we show a random process and its autocorrelation. In Fig. 17 we show a random process and its autocorrelation.

At last we show in Figs. 17 and 18 a random analysis of the data that appeared in Figs. 9 and 12a respectively.

References:

- Jürgen Franke, Wolfgang Härdle, Christian Hafner, Einführung in die Statistik der Finanzmärkte (Springer 2004)