A comparison of methods for the nonlinear optimization problem of hydropower generation in a reservoir cascade

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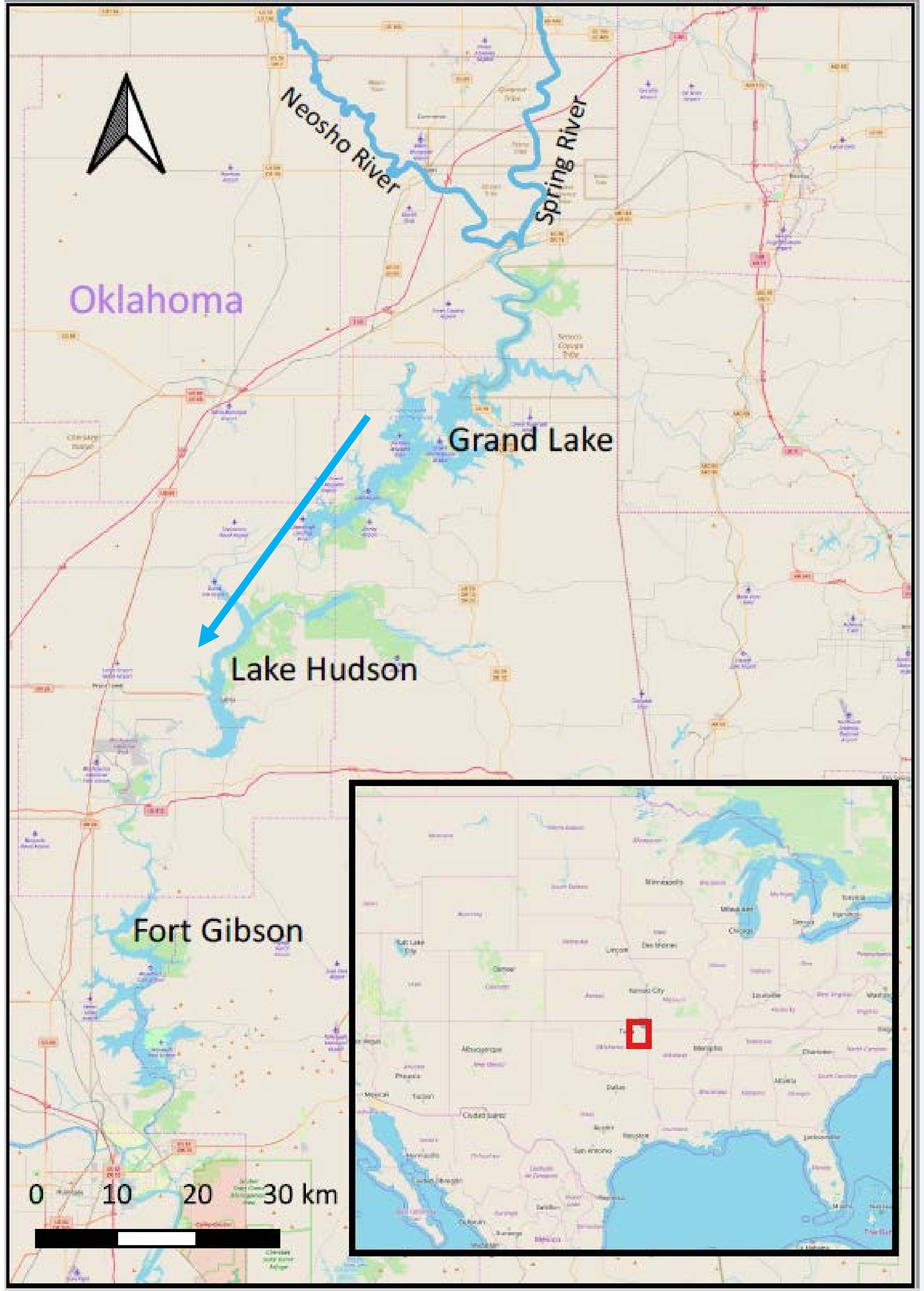
Reservoir cascade

- Three reservoirs, Inspired by the Grand River system (USA)
- Question: When to release how much?
  - o Reservoir inflow
  - o Load balance
  - o Flood control
  - o Generator limit
  - o Dam safety
  - o …



Equations

- Reservoir equation (linear)
- Turbine flow (power generation) and spill flow (no power generation) (linear)
- Relation between water level and volume (nonlinear)
- Power generation (nonlinear)
- Head difference (nonlinear)
- Tailwater level (linear)

$$\frac{\partial V}{\partial t} = Q_{\text{in}} - Q_{\text{out}}$$

$$Q_{\text{out}} = Q_{\text{turbine}} + Q_{\text{spill}}$$

$$h = f(V)$$

$$P = \Delta h \cdot Q_{\text{turbine}} \cdot \eta \cdot \rho \cdot \mathrm{g}$$

$$\Delta h = h - h_{\text{tailwater}}$$

$$h_{\text{tailwater}} = \gamma(Q_{\text{out}})$$

Optimization problem

- Equality constraints:
  - o physical equations (previous slide)
- Bounds: physical limits:
  - o Minimum and maximum volume / pool elevation
  - o Maximum turbine admission
  - o Generator limits
  - o Spillway capacity
- Operational goals
  - o Generation targets (load)
  - o Operational range for water levels
  - o Priorities (goal programming) and weights (Pareto optimization)
  - o Discharge targets


Edertalsperre dam, Germany (1908–1914)

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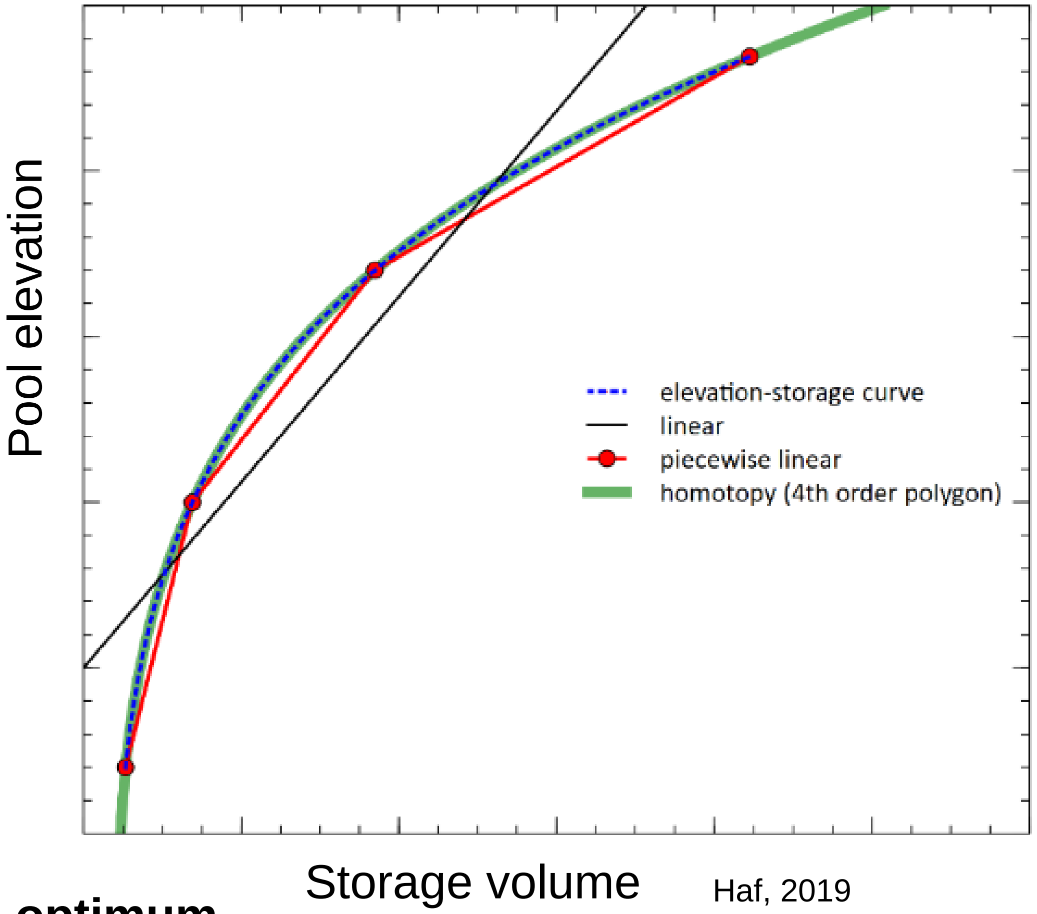
(Nonlinear) reservoir optimization - linearization

1. Volumes and flows only
   - Very fast
   - Limited in equations to volumes and flows:
     - no hydropower, no water level
   - Compute hydropower in post-processing:
     - no optimization to maximize hydropower
     - no incentive to keep the water level high
2. Linear approximation
   - Compute linear although not linear
3. Piecewise-linear
   - Split nonlinear function into linear pieces
   - Linear behavior for sections limits inaccuracy
   - Quite fast
   - Adds Boolean logic to the optimization



Haf, 2019

Linear optimization problems can be solved to a global optimum

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Nonlinear reservoir optimization

4. Homotopy
   - Novel approach (Deltares, software: RTC-Tools)
   - Functional principle: “bend” the equations from linear towards nonlinear equations with the help of a θ between 0 and 1 during the optimization (iteration steps)
   - Path-stable solution, connected to a global optimum of the linear part of the problem

$$P_{\mathrm{h}} = (1-\theta)\cdot \mathrm{g}\cdot\rho\cdot\eta\cdot Q_{\mathrm{turbine}}\cdot\Delta\widetilde{H} \qquad \text{linear part, } \Delta\text{H is constant}$$
$$+\,\theta\cdot(\mathrm{g}\cdot\rho\cdot\eta\cdot Q_{\mathrm{turbine}}\cdot\Delta H) \qquad \text{nonlinear part, } \Delta\text{H is variable}$$

$$\Delta\widetilde{H} = \mathrm{const}$$

5. Heuristic approaches
   - Functional principle:
     - e.g. genetic algorithm inspired by natural selection
   - Can optimize anything
   - No guarantee for a global optimum
     - Result can depend on seeding
     - a little change in inflow can lead to a completely different result → not suitable for operational use
   - Comparatively slow

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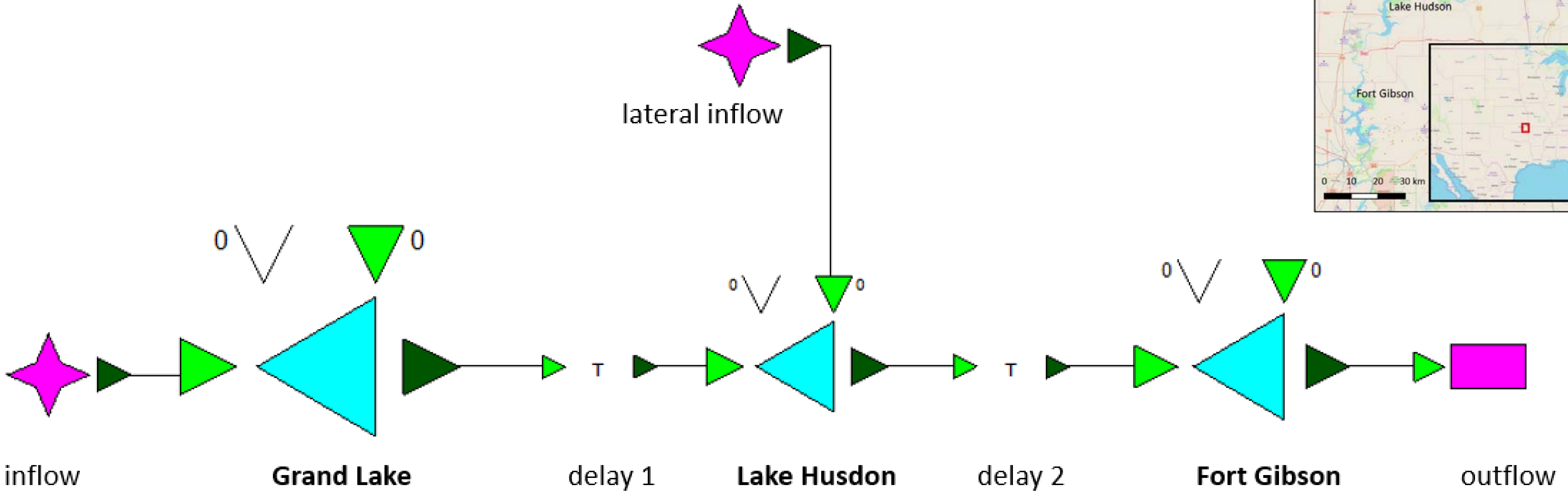
The RTC-Tools model for the Grand River cascade



Operational goals

Variable	Goal	Priority	Weight
H	Drawdown forebay < 0.0625 m/h → Bank stability	1	1.0
Q_{out}	$Q_{out,GL}$ < 1 000 m³/s $Q_{out,LH}$ < 1 000 m³/s $Q_{out,FG}$ < 1 500 m³/s → Flood protection goal	2	1.0
V	515 801 371 m³ < V_{GL} > 2 447 982 614 m³ 250 765 250 m³ < V_{LH} < 548 049 719 m³ 458 160 288 m³ < V_{FG} < 1 604 750 133 m³ → Water level range goals	3	1.0
V	1 639 356 717 m³ < V_{GL}< 1 949 341 935 m³ → Inner H goal for GL	4	1.0
P	P = power request timeseries → Load request goal	5	1.0
Q_{spill}	Minimize Q_{spill} → all water through the turbines	6	100.0
Q_{turb}	Minimize change in Q_{turb} → Smooth flow through turbines	6	1.0
Q_{spill}	Minimize change in Q_{spill} → Smooth flow through spillway	6	1.0

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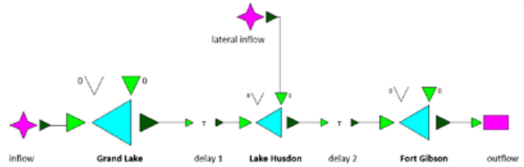
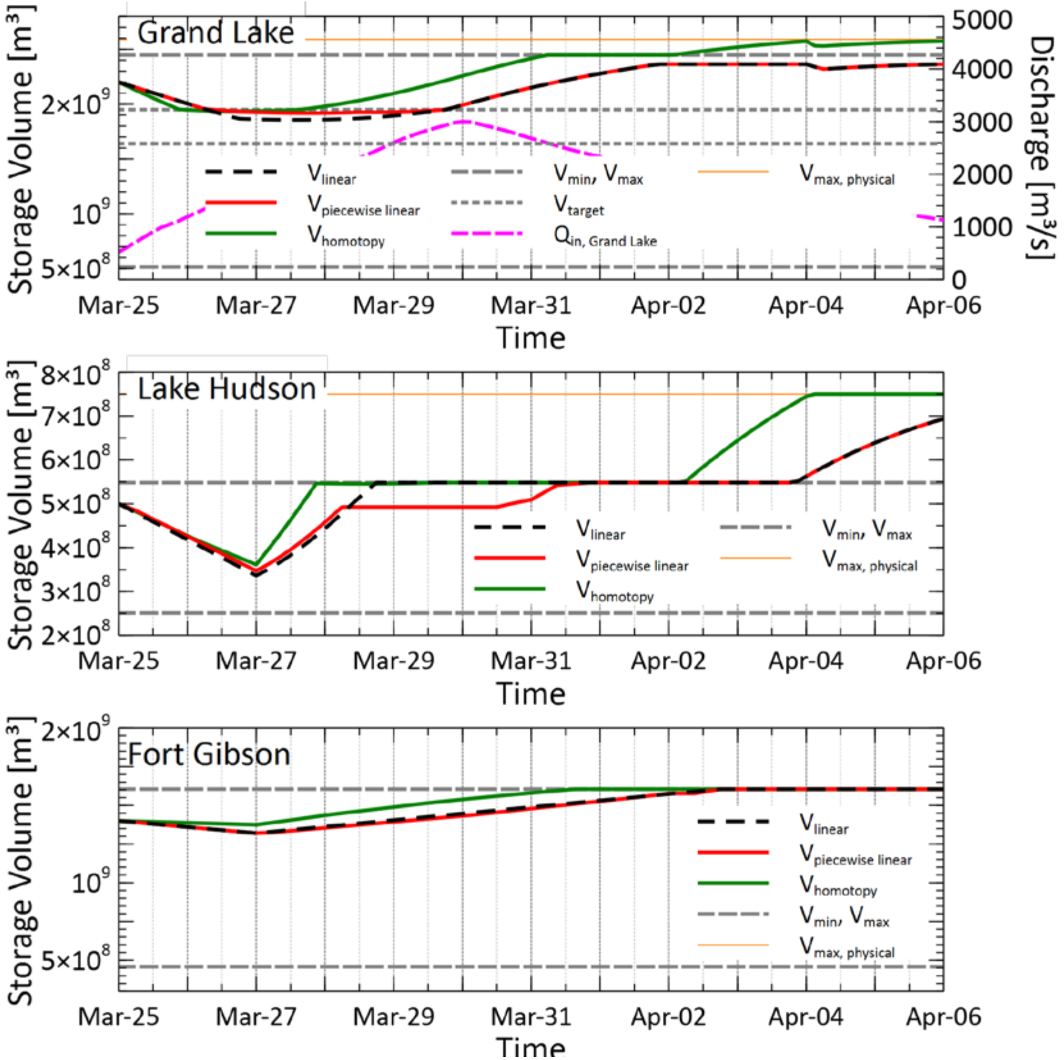
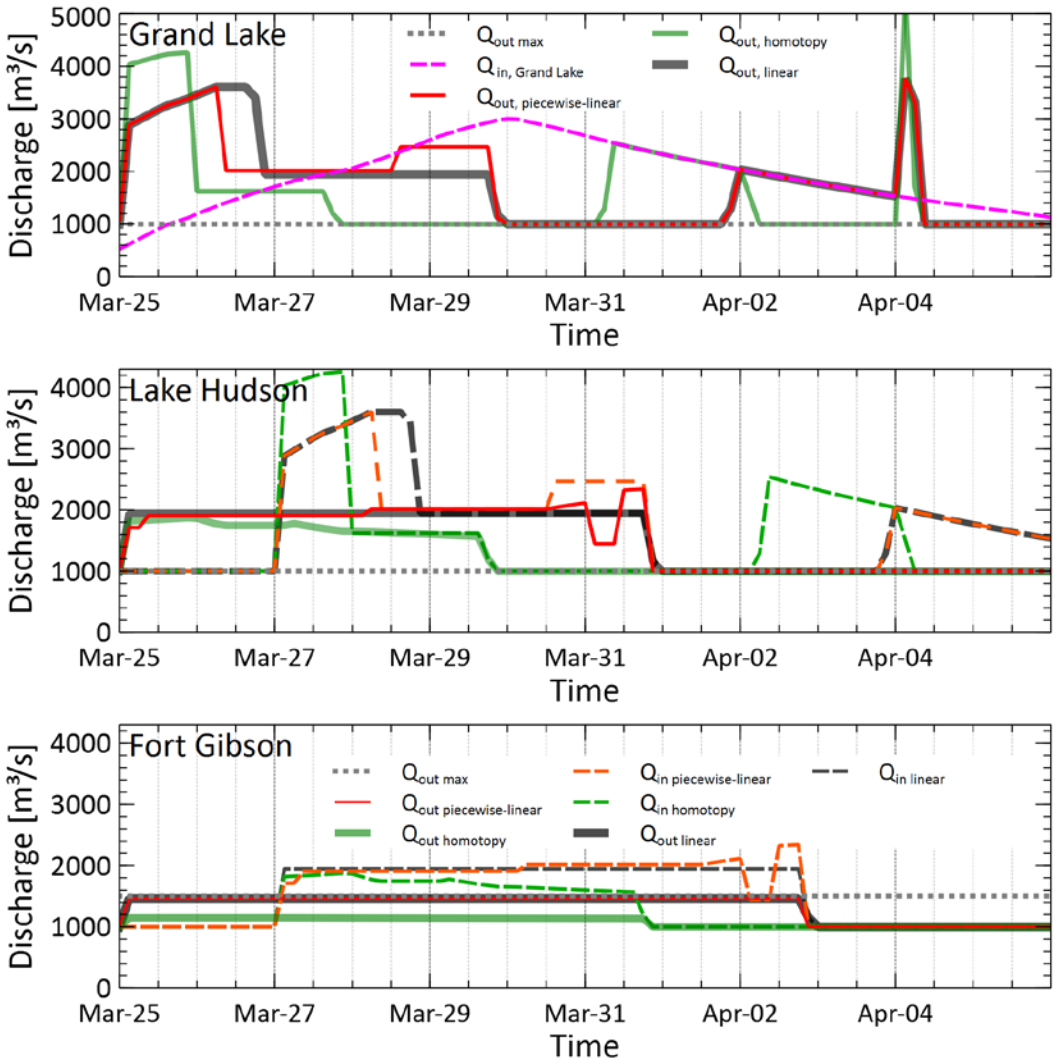
Flood scenario



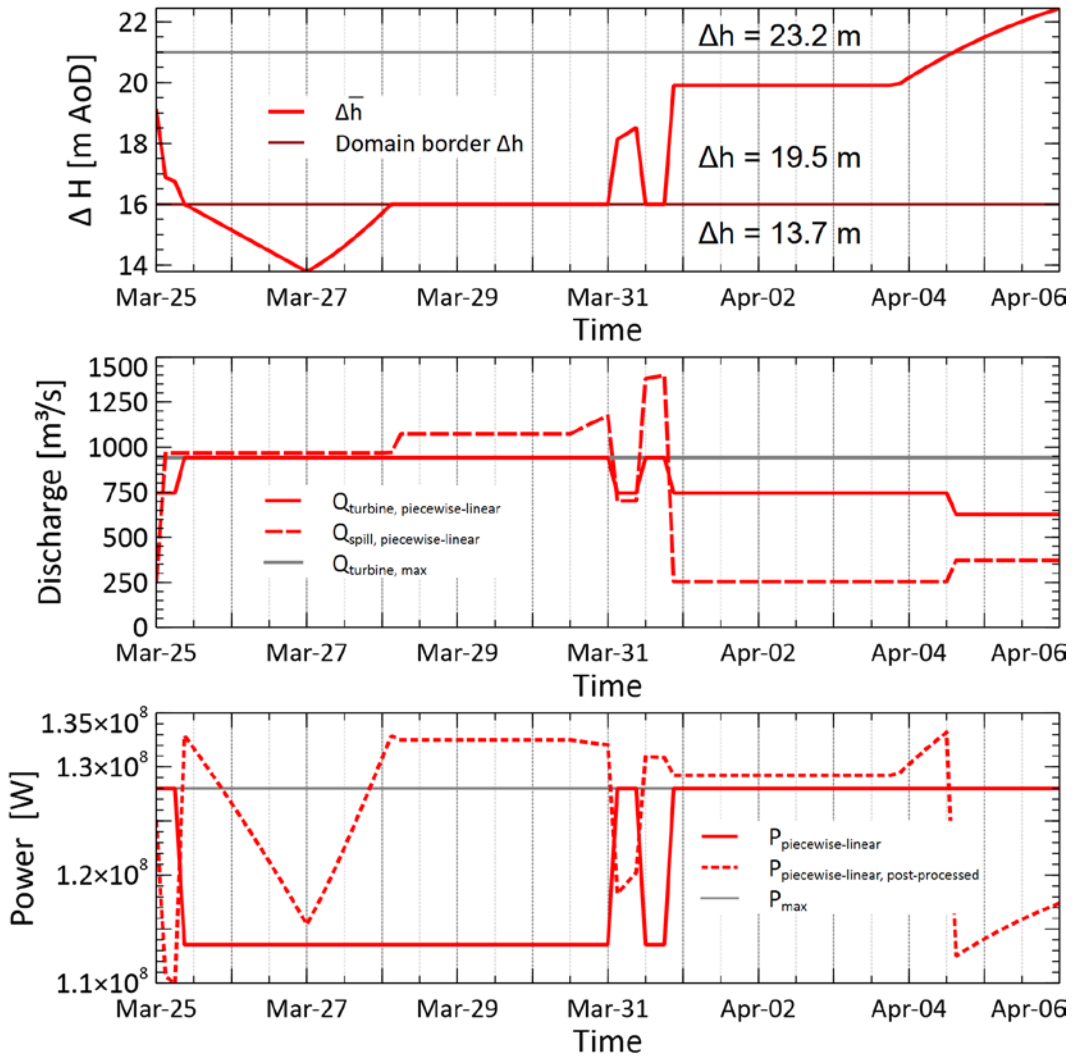


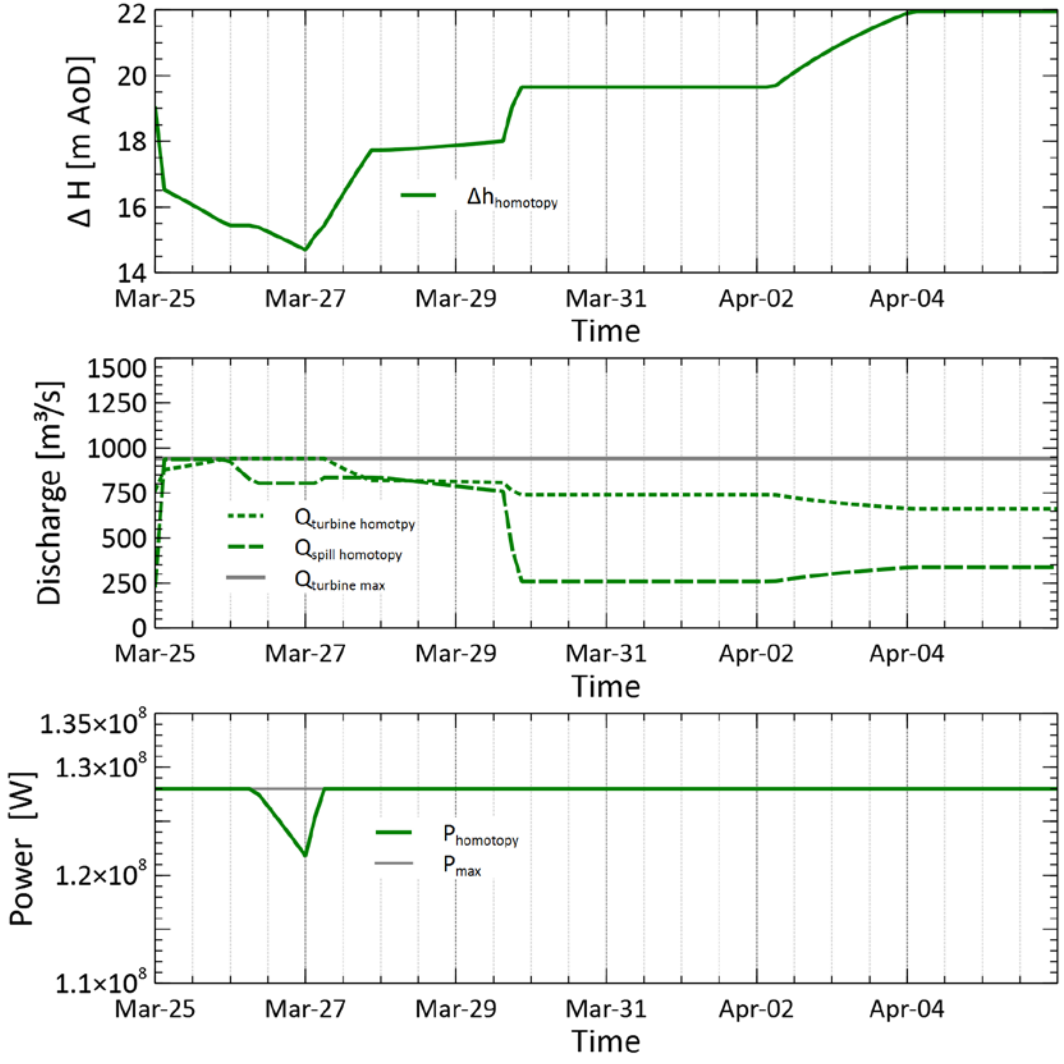
Reservoir volume



Reservoir outflow

Flood scenario: Lake Hudson



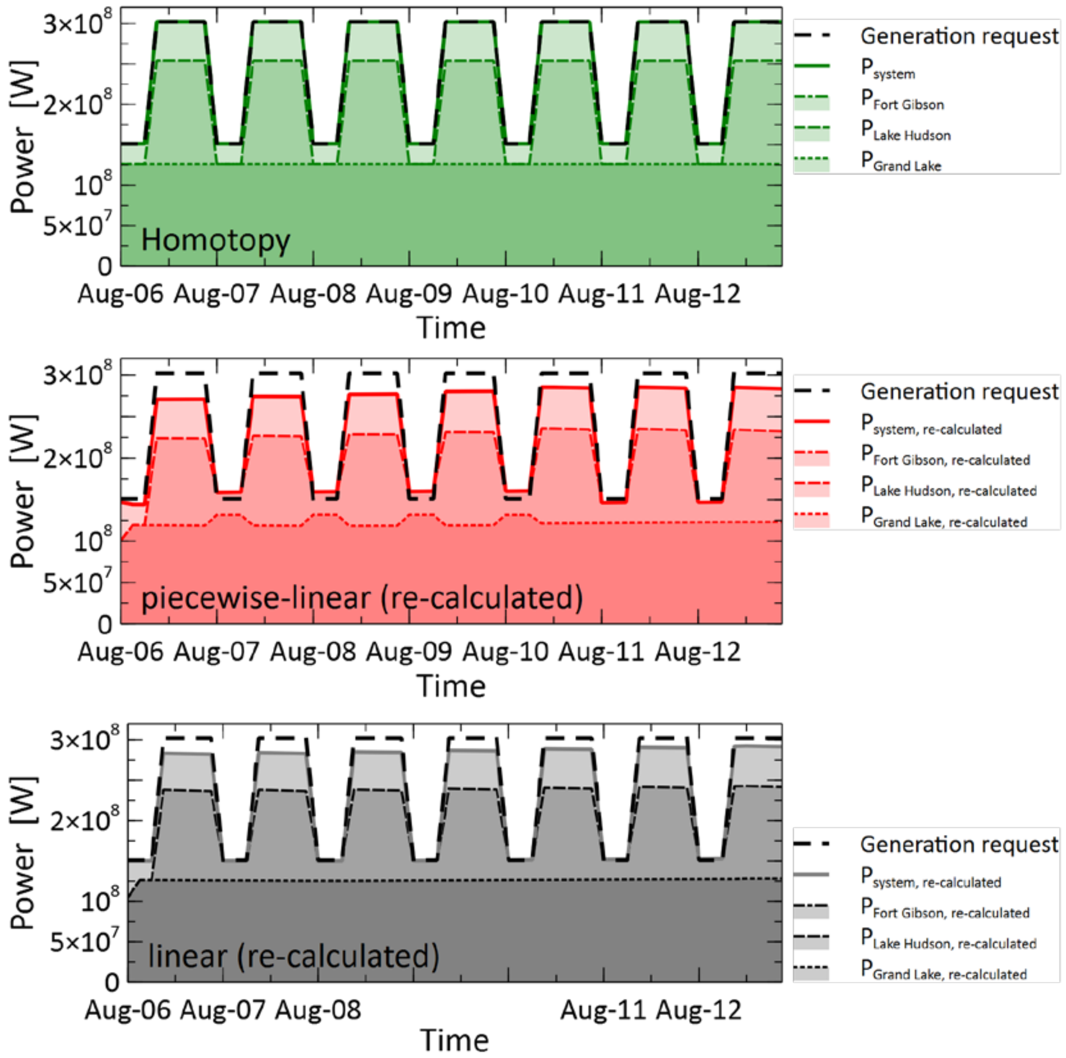


Piecewise linear:

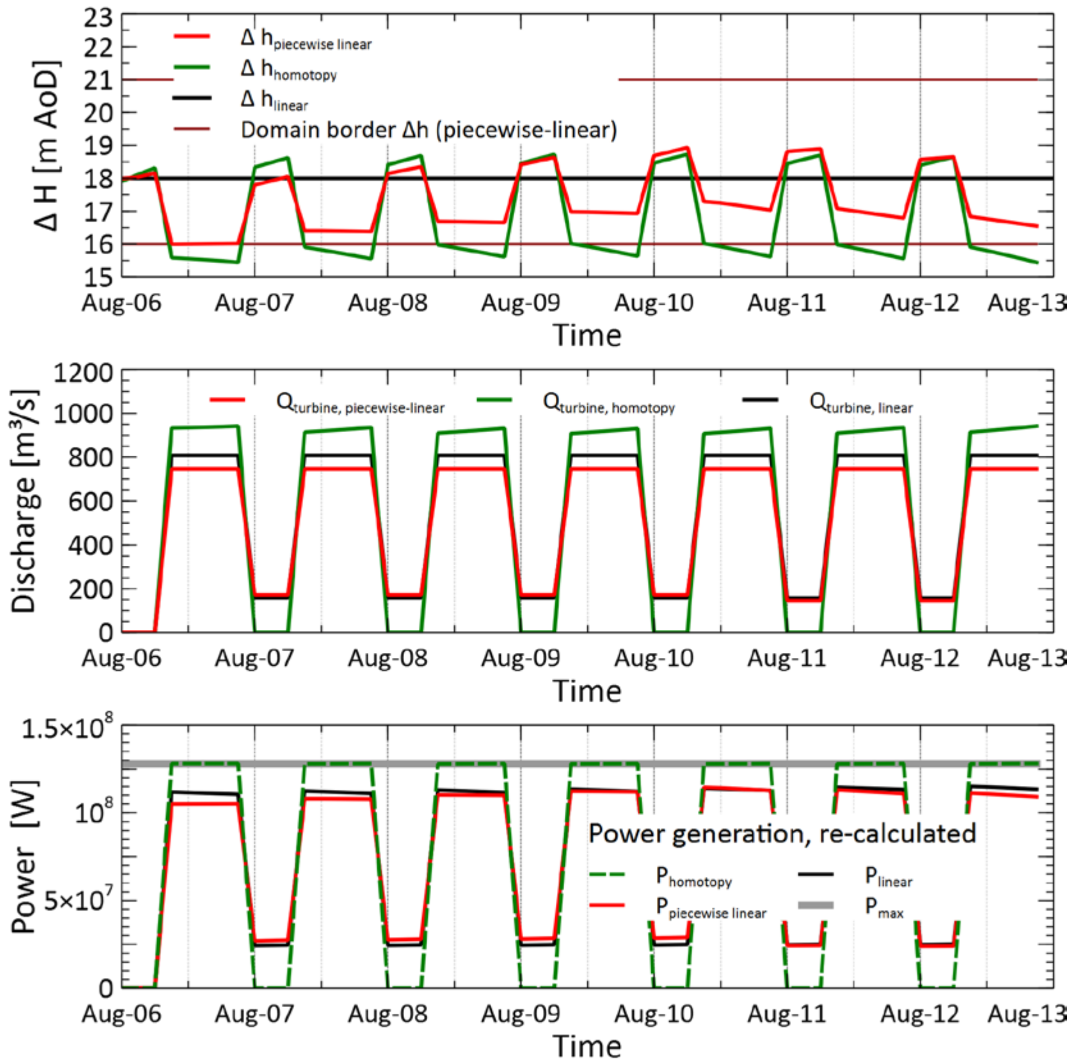
- optimizer seems to be reluctant to cross domain borders for Δh
- limit P_{max} not obeyed exactly (re-calculated)

Homotopy: P_{max} obeyed

Load balance scenario: system-wide generation



Generation request and system-wide power generation obtained with different methods



Optimization result for Lake Hudson

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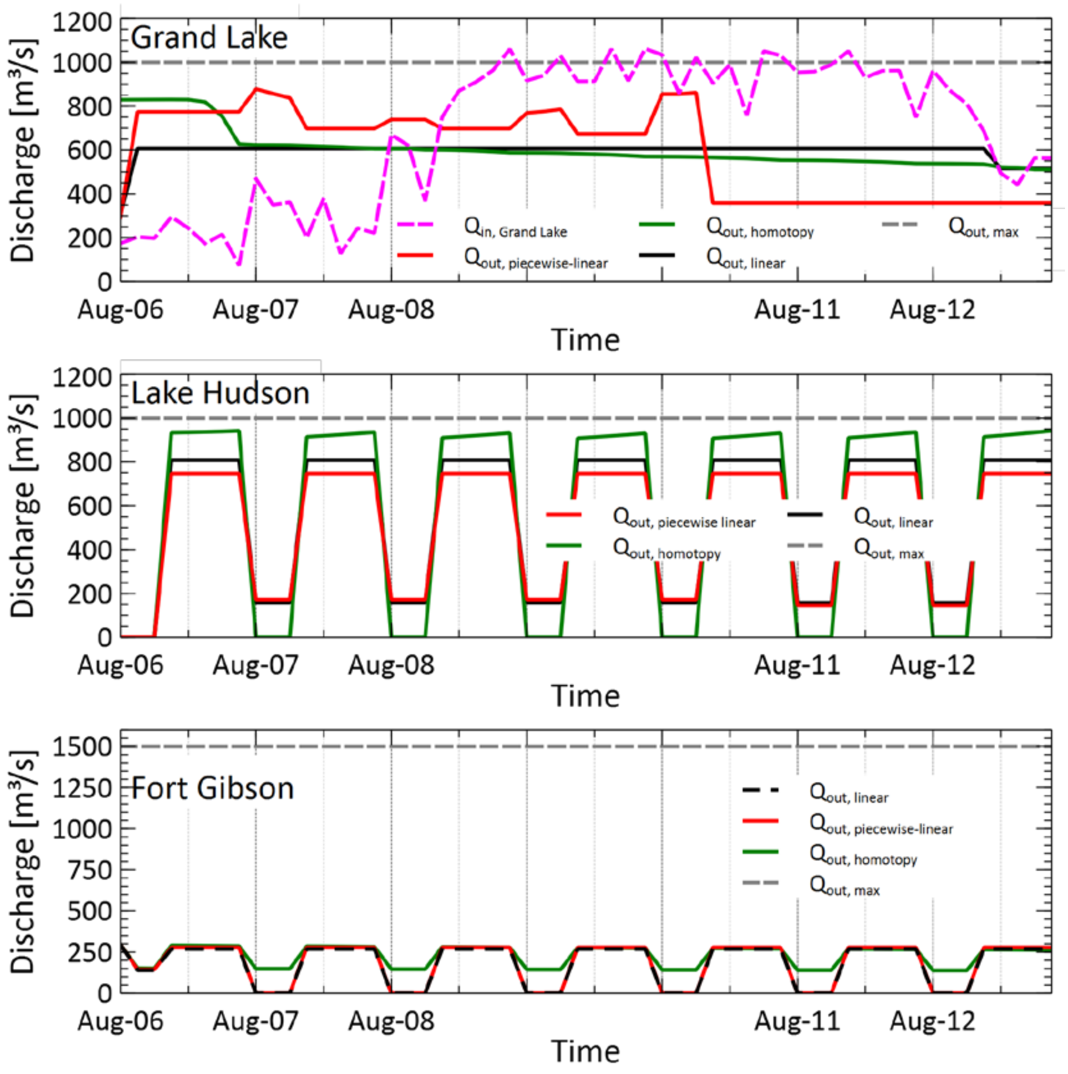
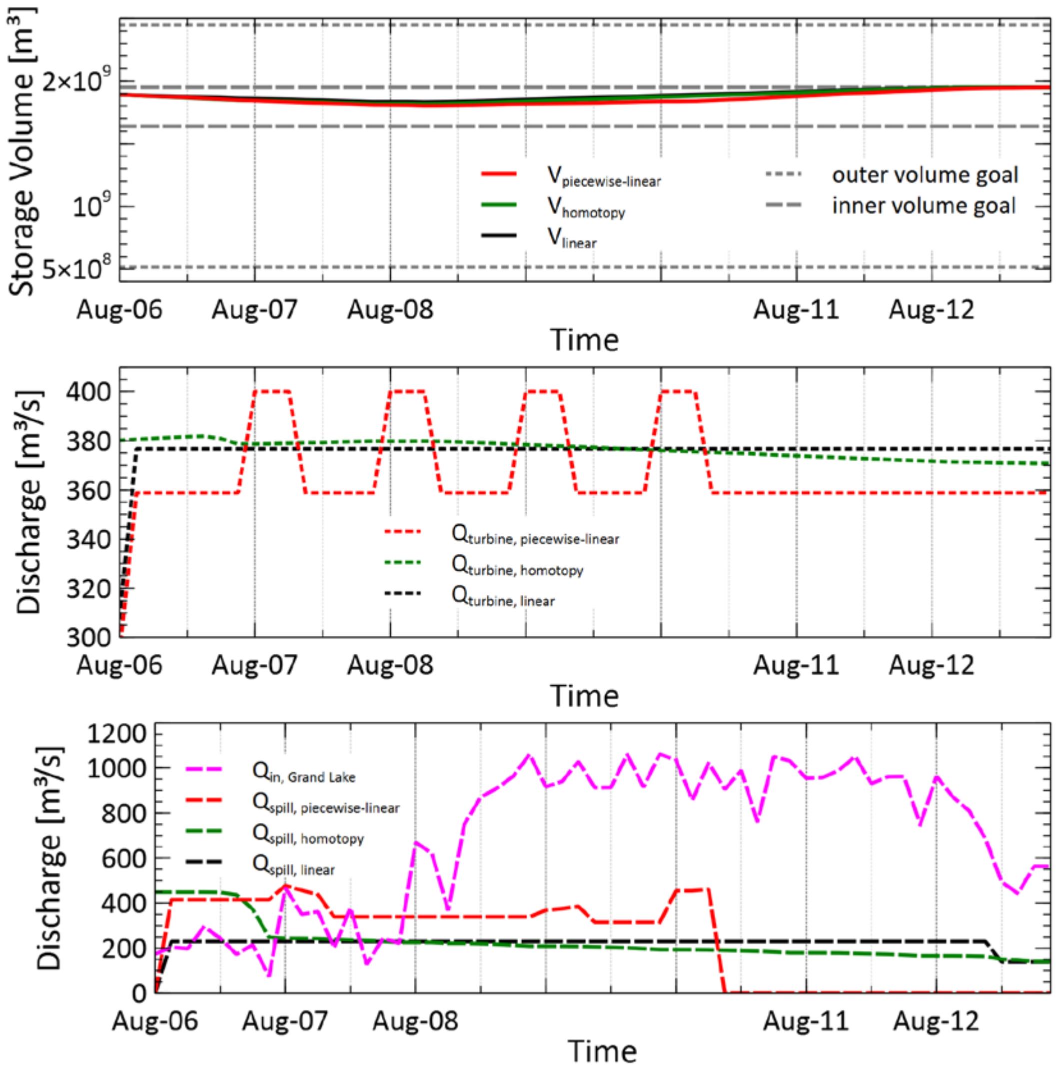
Load balance scenario: release and volume (GL)



Reservoir release



Volume, spill flow and turbine flow for Grand Lake

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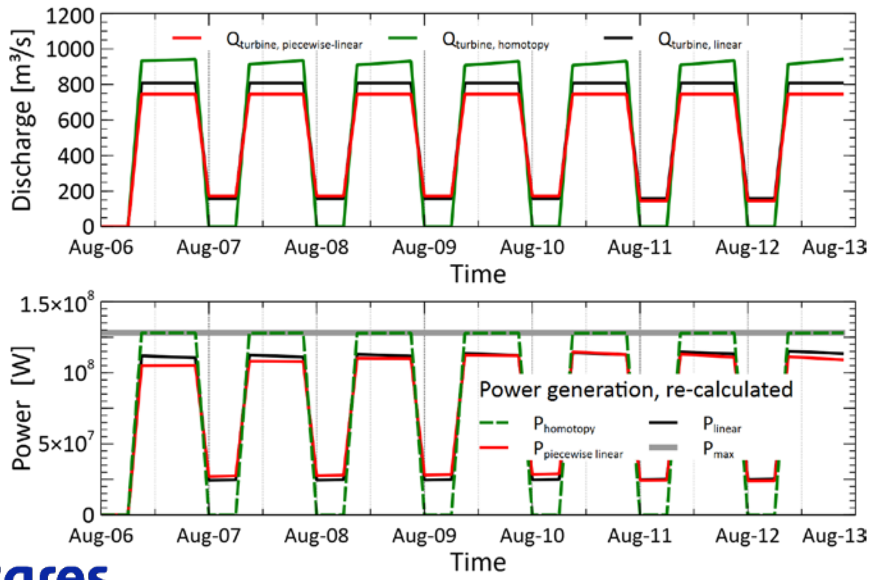
Technical remarks and conclusions

- Result quality
  - o All three methods produce good results
  - o Different accuracy for the physics:
    - Homotopy
      - shows most accurate results for Q - Δh in generation
    - Piecewise-linear
      - Represents interplay Q - Δh to some extent
      - Tends to not leave Δh domains
    - Linear
      - No incentive to keep water level high, only Q determines the generation

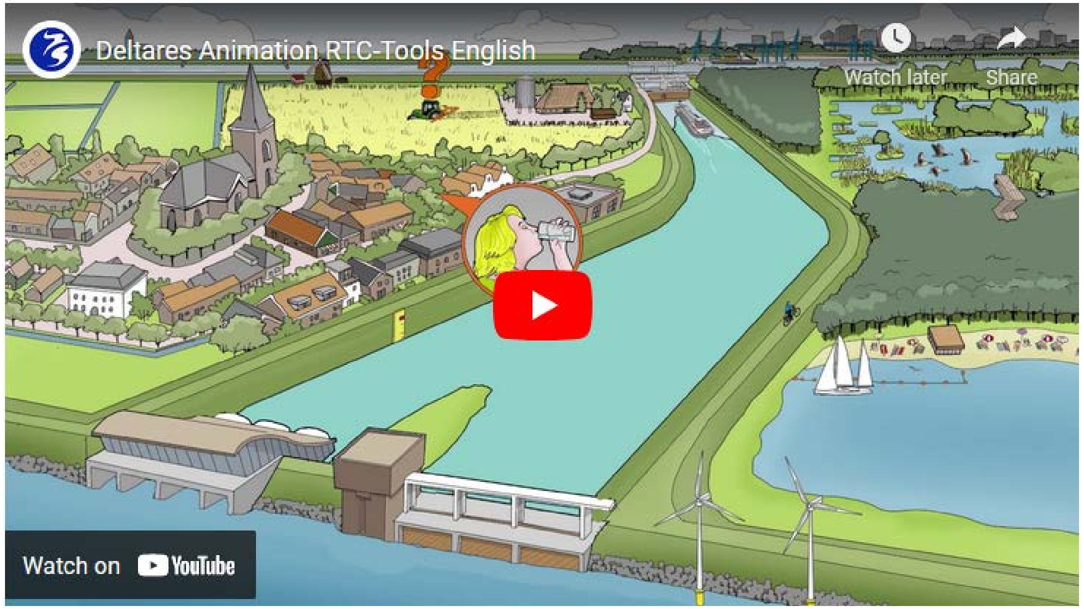


- Usability, performance and math
  - o Linear
    - Fast
    - Proven technology, straight-forward
    - Easy to understand
    - Global optimum for a simplified problem
  - o Piecewise-linear
    - Adds mixed integer logic
    - comes with user choices and tuning:
      - domain borders
      - scaling of the optimization problem
    - More domains (more accuracy) means exponentially more computing time
      (not shown here, see Haf 2019 for more details)
    - Global optimum for a simplified problem
  - o Homotopy
    - Easy to apply from a modeler's perspective
    - Must be supported by the software (here: RTC-Tools)
    - Novel method, not generally tested for other equations
    - Path-stable only
    - Not (yet) compatible with mixed integer logic
    - Not (yet) compatible with ensemble optimization

RTC-Tools

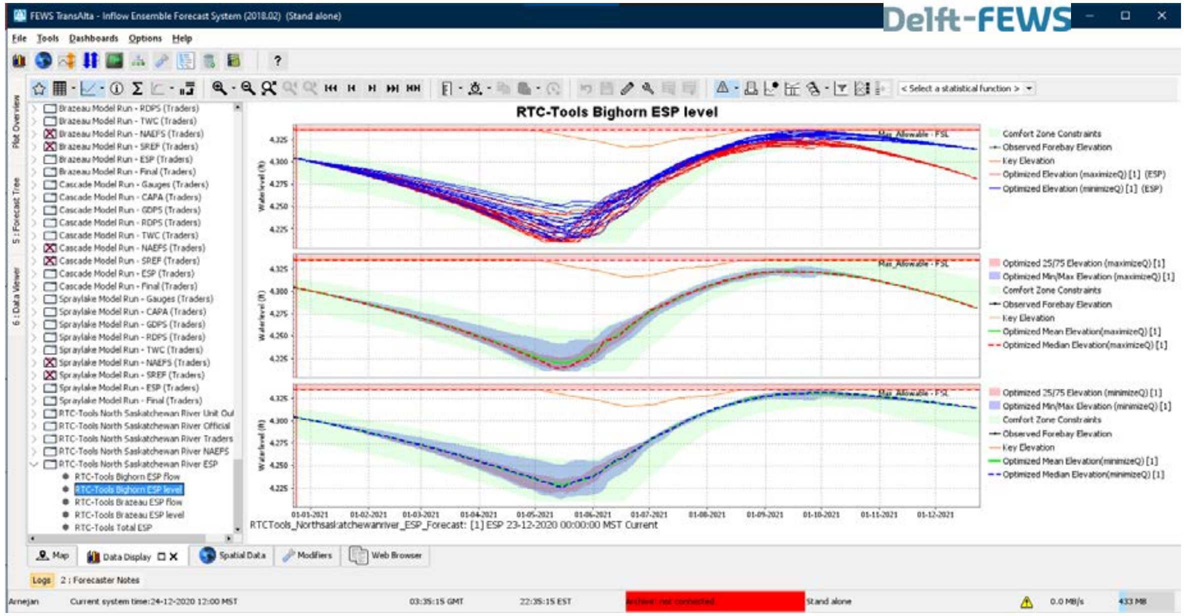
RTC-Tools

- A toolbox for (**r**eal-**t**ime) **c**ontrol and optimization of water systems
- Websites
  - o https://www.deltares.nl/en/software/rtc-tools/
  - o https://oss.deltares.nl/web/rtc-tools



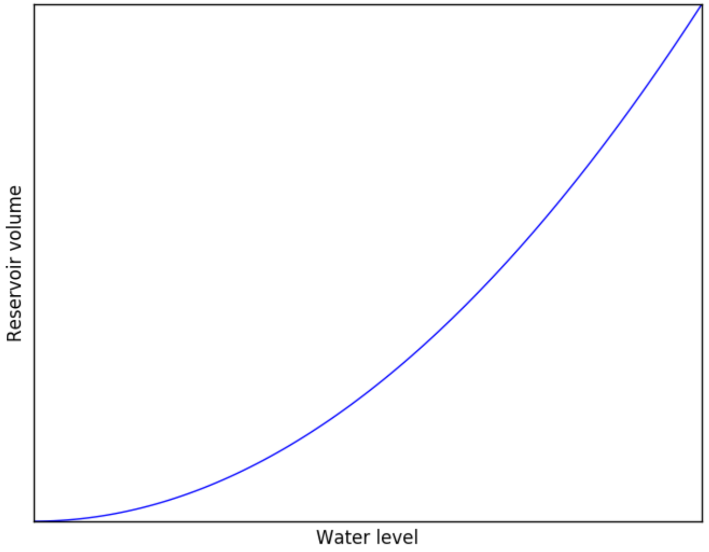
- Open source
- Modelica-library with various flow equations
- Comes as Python package
- Conflict resolution with goal programming and weighting factors
- Ensemble optimization
- Integrates in Delft-FEWS for operational use

Delft-FEWS



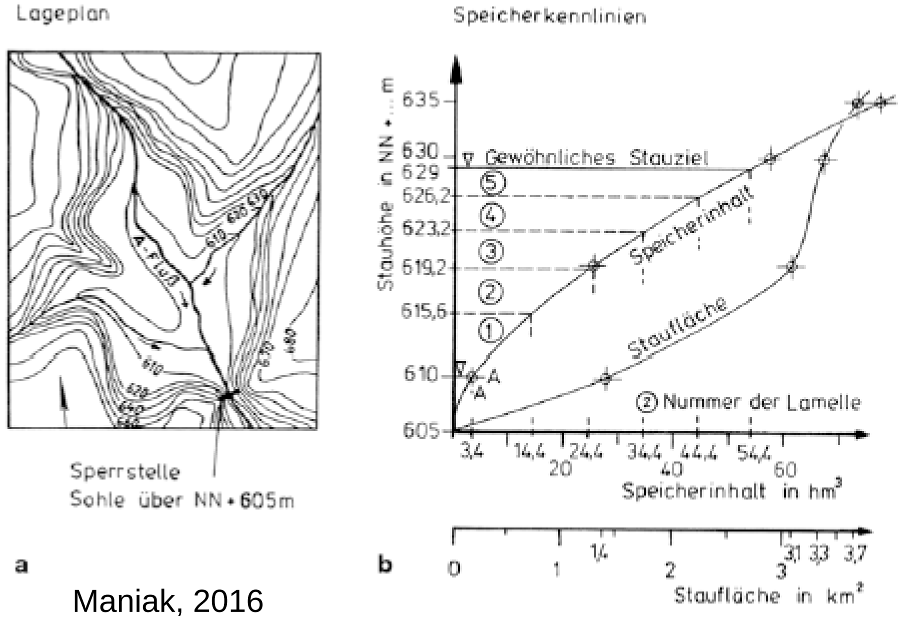
RTC-Tools optimization results in Delft-FEWS TransAlta (van Loenen & Fru, CEATI conference 2021)

Nonlinearity #1: Storage geometry



$$\frac{\partial V}{\partial t} = Q_{\text{in}} - Q_{\text{out}}$$

- The reservoir equation is linear
- The water volume in a reservoir is an <u>increasing</u>, but generally <u>nonlinear</u> function of water level.
- **Solution**: preprocess water level goals to volume goals



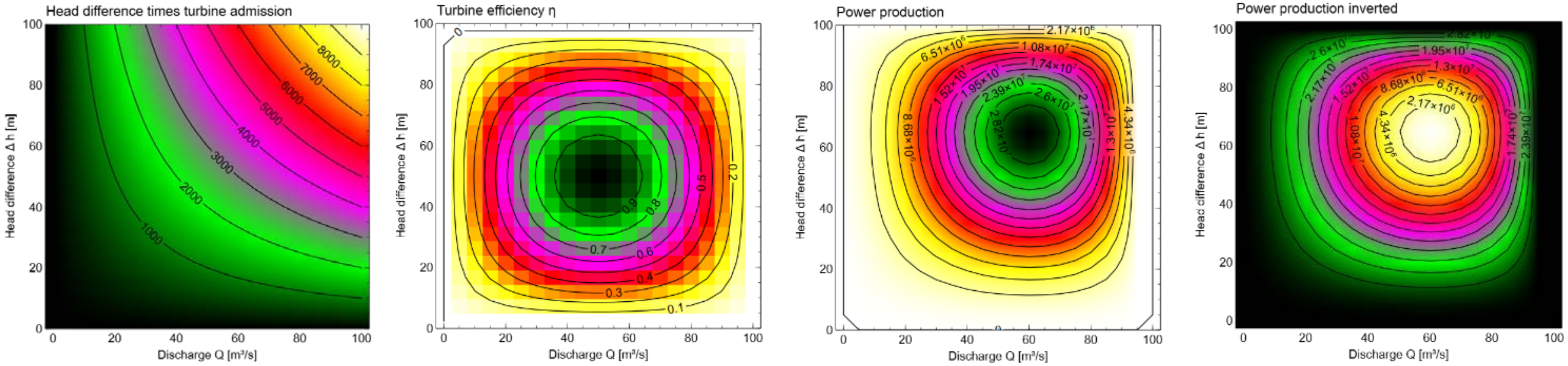
Maniak, 2016

Nonlinearity #2: Hydropower generation

- Instantaneous power from a hydroelectric turbine:

$$P = \eta \cdot \rho \cdot \mathrm{g} \cdot Q \cdot \Delta H.$$

- Q and ΔH are optimization variables.
- P is a nonlinear, even nonconvex, function of Q and ΔH.
- **Solution**: Choose of optimization variables and goals such that the problem is nonlinear but convex formulations
  - Assume $\Delta h = \mathrm{const}$ (no incentive to keep the water level high)
  - Maximize power as optimization goal



Hydropower production depends on Q • Δh (maximize is ok, but not load balance)

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