What is Isogeometric Analysis?

Carlo Lovadina, Alessandro Reali, Giancarlo Sangalli
University of Pavia and IMATI-CNR of Pavia

Milan, June 30, 2014

TERRIFIC
European Community's Seventh Framework Programme
Grant Agreement 284981
Call FP7-2011-NMP-ICT-FoF

TERRIFIC
Enhancing Interoperability

SEVENTH FRAMEWORK PROGRAMME

Isogeometric Analysis (IGA):

RECENT EMERGING technology for Scientifc Computing, stemming from OLD ideas

TERRIFIC
Enhancing Interoperability

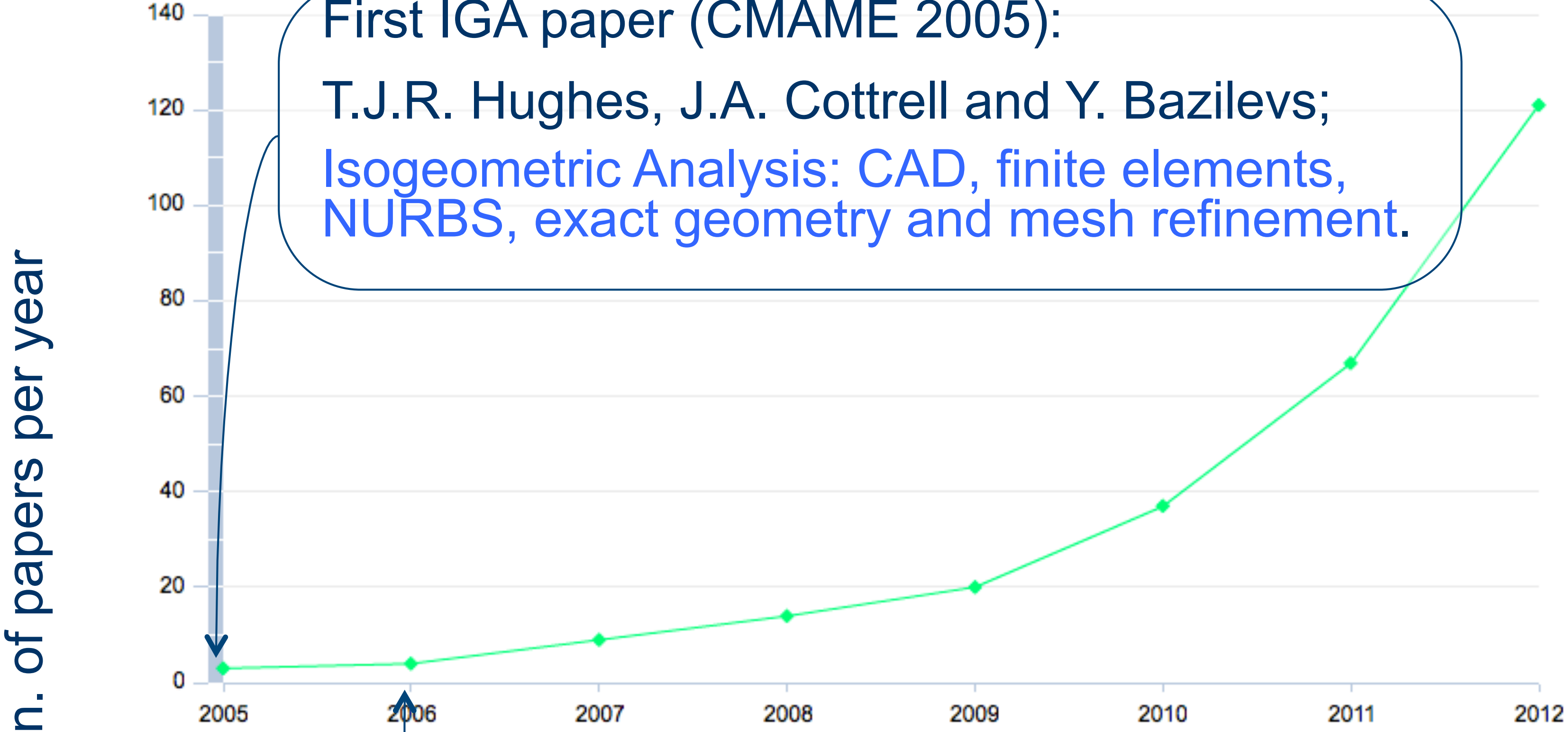
SEVENTH FRAMEWORK PROGRAMME

IGA timeline and diffusion (from SCOPUS)

First IGA paper (CMAME 2005):

T.J.R. Hughes, J.A. Cottrell and Y. Bazilevs;

Isogeometric Analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement.



First "math" IGA paper (M3AS 2006):

Y. Bazilevs, L. Beirão Da Veiga, J.A. Cottrell, T.J.R. Hughes, G. Sangalli

Isogeometric analysis: Approximation, stability and error estimates for h-refined meshes.

TERRIFIC
Enhancing Interoperability

SEVENTH FRAMEWORK PROGRAMME

Two fundamental steps in Virtual Manufacturing:

CAD (1970's - 1980's) – Engineering Design Process:

- ✓ engineering designs are encapsulated in CAD systems;
- ✓ CAD geometry is *exact*;
- ✓ hundreds of thousands analyses of CAD designs are performed in engineering offices throughout the world every day

FEM (1950's - 1960's) – Engineering Analysis Process:

- ✓ CAD geometry is replaced by FEM geometry ("mesh");
- ✓ mesh generation accounts for more than 80% of overall analysis time and is the major *bottleneck*;
- ✓ mesh refinement requires interaction with CAD geometry;
- ✓ the mesh is an *approximate* geometry

TERRIFIC
Enhancing Interoperability

SEVENTH FRAMEWORK PROGRAMME

Two fundamental steps in Virtual Manufacturing:

CAD (1970's - 1980's) – Engineering Design Process:

- ✓ engineering designs are encapsulated in CAD systems;
- ✓ CAD geometry is *exact*;
- ✓ hundreds of thousands analyses of CAD designs are performed in engineering offices throughout the world every day

FEM (1950's - 1960's) – Engineering Analysis Process:

- ✓ CAD geometry is replaced by FEM geometry ("mesh");
- ✓ mesh generation accounts for more than 80% of overall analysis time and is the major *bottleneck*;
- ✓ mesh refinement requires interaction with CAD geometry;
- ✓ the mesh is an *approximate* geometry

Critical issues

IDEA (Hughes et al., 2005): Isogeometric Analysis

In the Analysis framework, employ the same functions used to describe the geometry of the computational domain, i.e., typically, use B-Splines and Non-Uniform B-Splines (NURBS).

IDEA (Hughes et al., 2005): Isogeometric Analysis

In the Analysis framework, employ the same functions used to describe the geometry of the computational domain, i.e., typically, use B-Splines and Non-Uniform B-Splines (NURBS).

Alternative to standard FE analysis, *including isoparametric FEA as a special case*, but offering other features and possibilities:

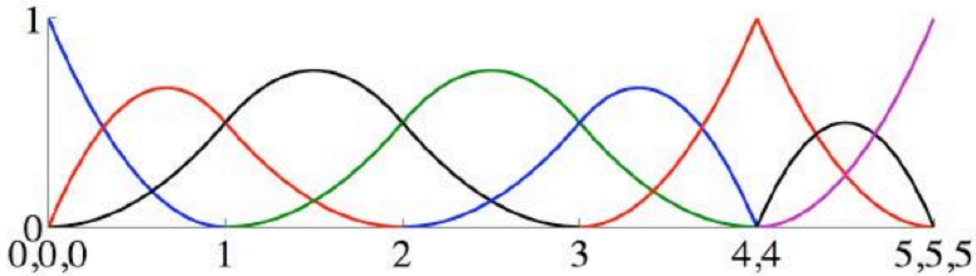
- ✓ CAD geometry is exactly and efficiently represented
- ✓ simplified mesh refinement
- ✓ *smooth* basis functions with *compact support*
- ✓ *superior* approximation properties
- ✓ *integration* of design and analysis

Starting point: Univariate B-Spline functions

Given a *non-uniform knot vector* $\mathcal{M} = \{\xi_1, ..., \xi_{n+p+1}\}$, in the parametric domain, B-spline functions are iteratively defined as:

$$B_{i,0}(\xi) = \begin{cases} 1 \text{ if } \xi_i \leq \xi < \xi_{i+1} \\ 0 \text{ otherwise.} \end{cases}$$

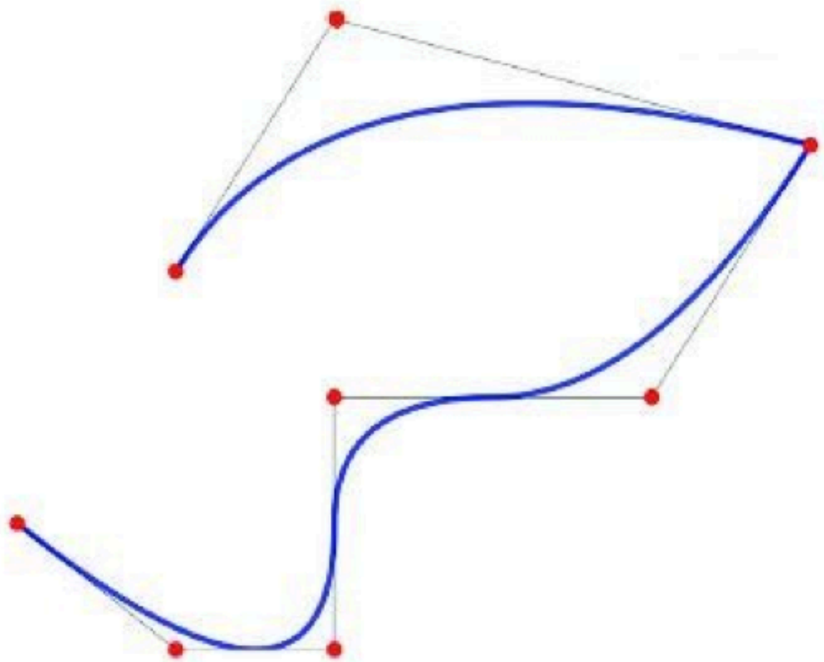
$$B_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} B_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} B_{i+1,p-1}(\xi).$$



The spline space is: $S_p(\mathcal{M}) = \text{span}\{B_{i,p}(\cdot)\}_{i=1,...,n}$

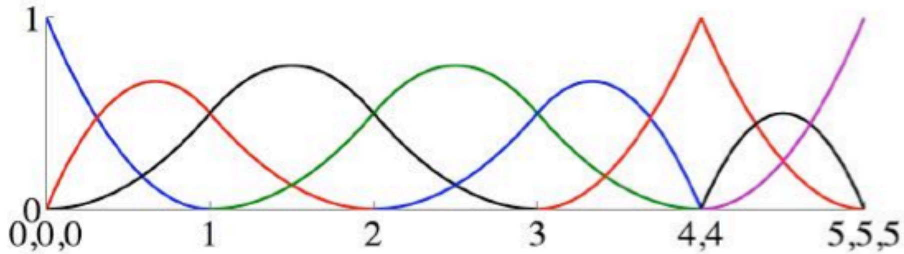
A spline curve

The image of $\mathbf{F}(\xi) = \sum_{\mathbf{i}} \mathbf{C_i} B_{\mathbf{i}}(\xi)$ is the B-spline curve:



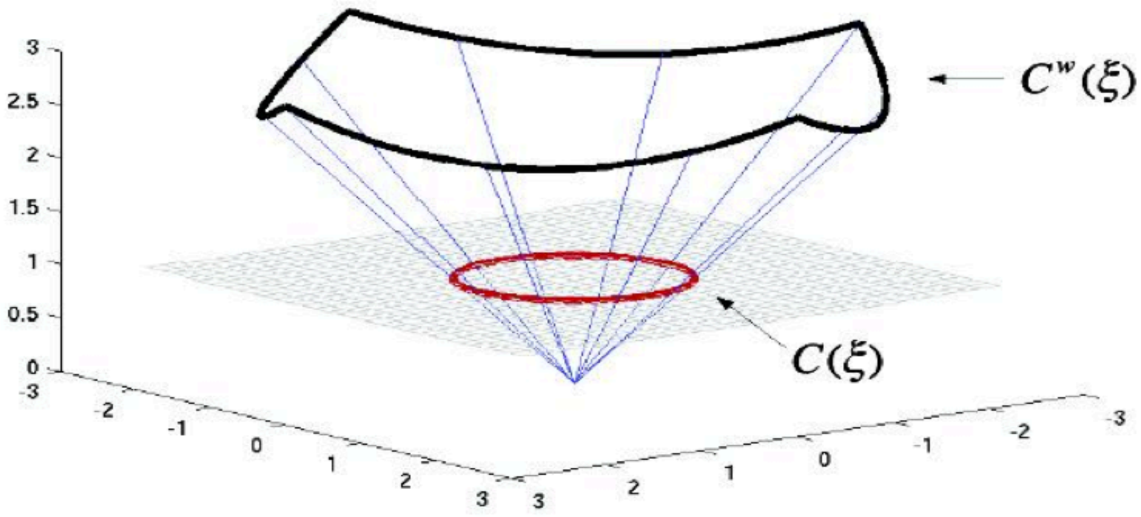
Control points

where $\mathbf{C_i} = \bullet$ and $B_{\mathbf{i}} =$



A NURBS curve

A NURBS curve in $\mathbb{R}^2$ is the projection of a B-spline in $\mathbb{R}^3$



$$C(\xi) = \frac{[C_x^w(\xi), C_y^w(\xi)]}{C_z^w(\xi)} = \sum_{i=1}^{n} \mathbf{C}_i \frac{w_i B_{i,p}(\xi)}{\sum_{\hat{i}=1}^{n} w_{\hat{i}} B_{\hat{i},p}(\xi)} = \sum_{i=1}^{n} \mathbf{C}_i R_{i,p}(\xi).$$

NURBS are able to exactly represent a vast set of geometrical objects, e.g. all the conic sections

TERRIFIC
Enhancing Interoperability

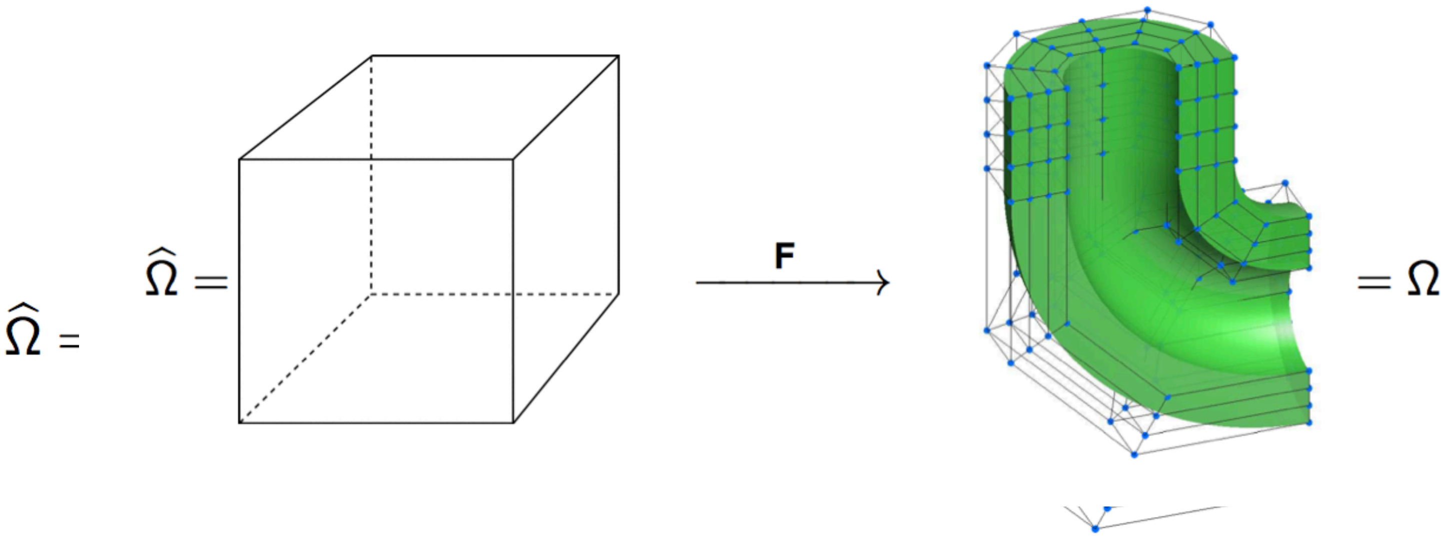
SEVENTH FRAMEWORK PROGRAMME

Spline/NURBS volumes

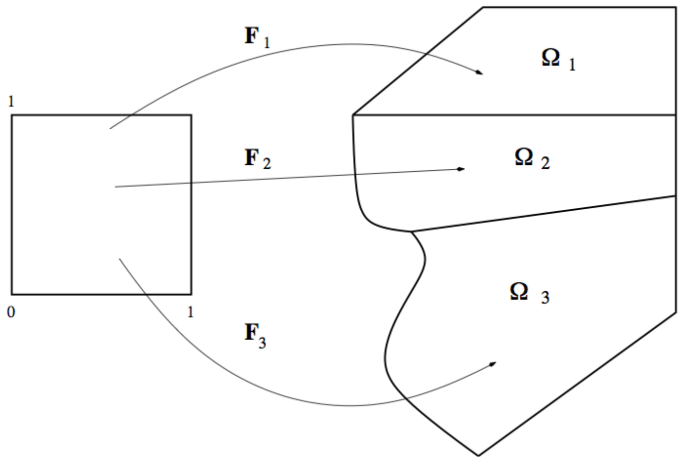
A spline space on a tensor product mesh $\mathcal{M} = \mathcal{M}_1 \otimes \mathcal{M}_2 \otimes \mathcal{M}_3$ is constructed by tensor product of univariate spaces:

$$S_{p_1,p_2,p_3}(\mathcal{M}) = S_{p_1}(\mathcal{M}_1) \otimes S_{p_2}(\mathcal{M}_2) \otimes S_{p_3}(\mathcal{M}_3)$$

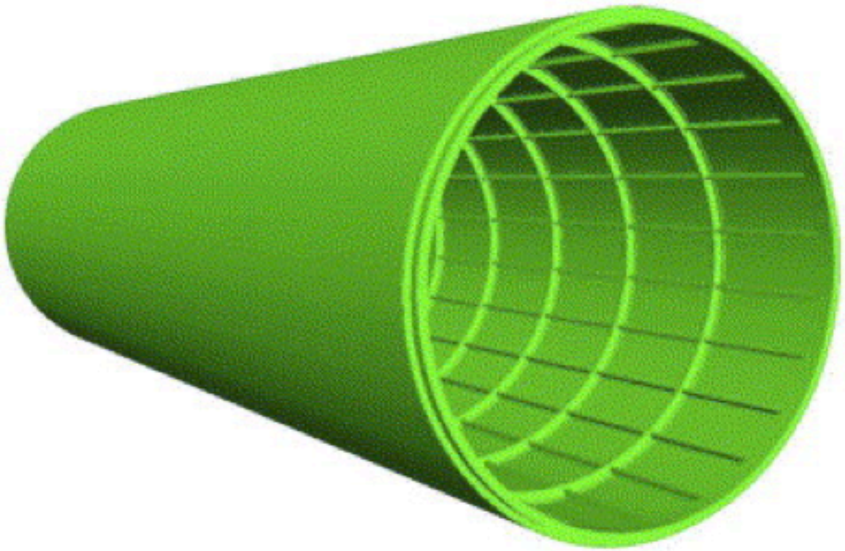
A *single-patch* spline geometry is parametrized by $\mathbf{F} \in (S_{p_1,p_2,p_3}(\mathcal{M}))^3$:

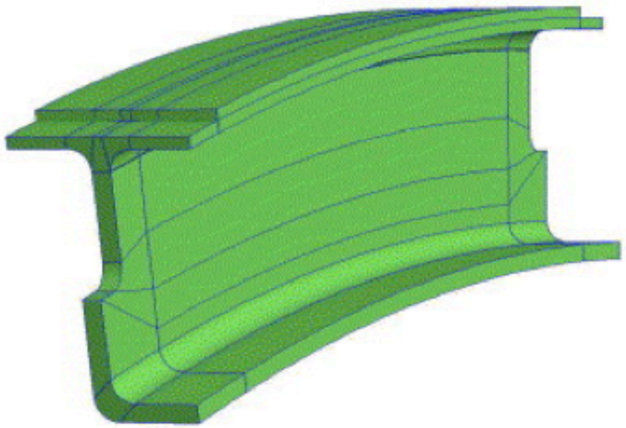


Spline/NURBS multi-patch volumes



Multi-patch geometries are typical in real-world applications



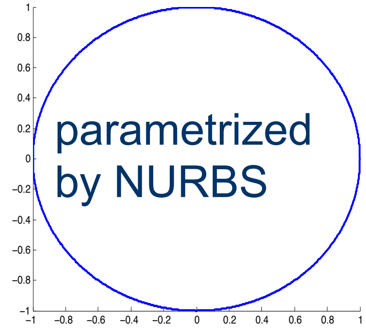


from T.J.R. Hughes group

FEM and IGA on a toy problem

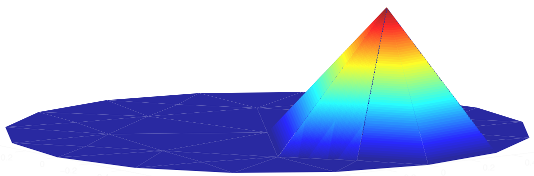
$$\begin{cases} -\Delta u = f \text{ in } \Omega \\ u = 0 \text{ on } \partial\Omega \end{cases}$$

CAD geometry

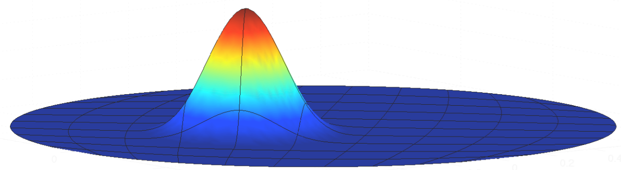


$$\text{Find } u_h \in V_h \text{ such that } \int_\Omega \nabla u_h \cdot \nabla v_h = \int_\Omega f\, v_h,\ \forall v_h \in V_h.$$

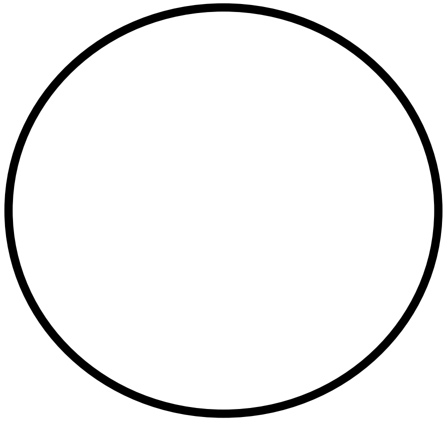
FEM



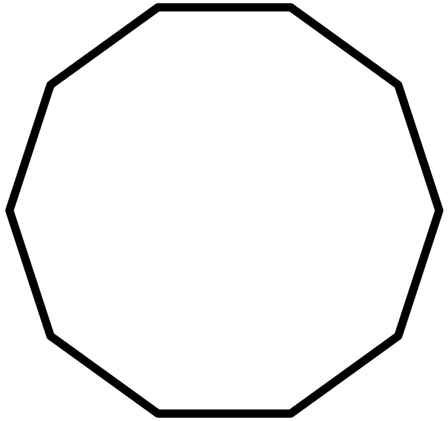
IGA



Exact geometry representation



Exact



FEM domain

✓Exact geometry provides more accurate results (computational domain is not altered)

TERRIFIC
Enhancing Interoperability

SEVENTH FRAMEWORK PROGRAMME

Exact geometry representation

Exact FEM domain

✓Exact geometry provides more accurate results (computational domain is not altered)

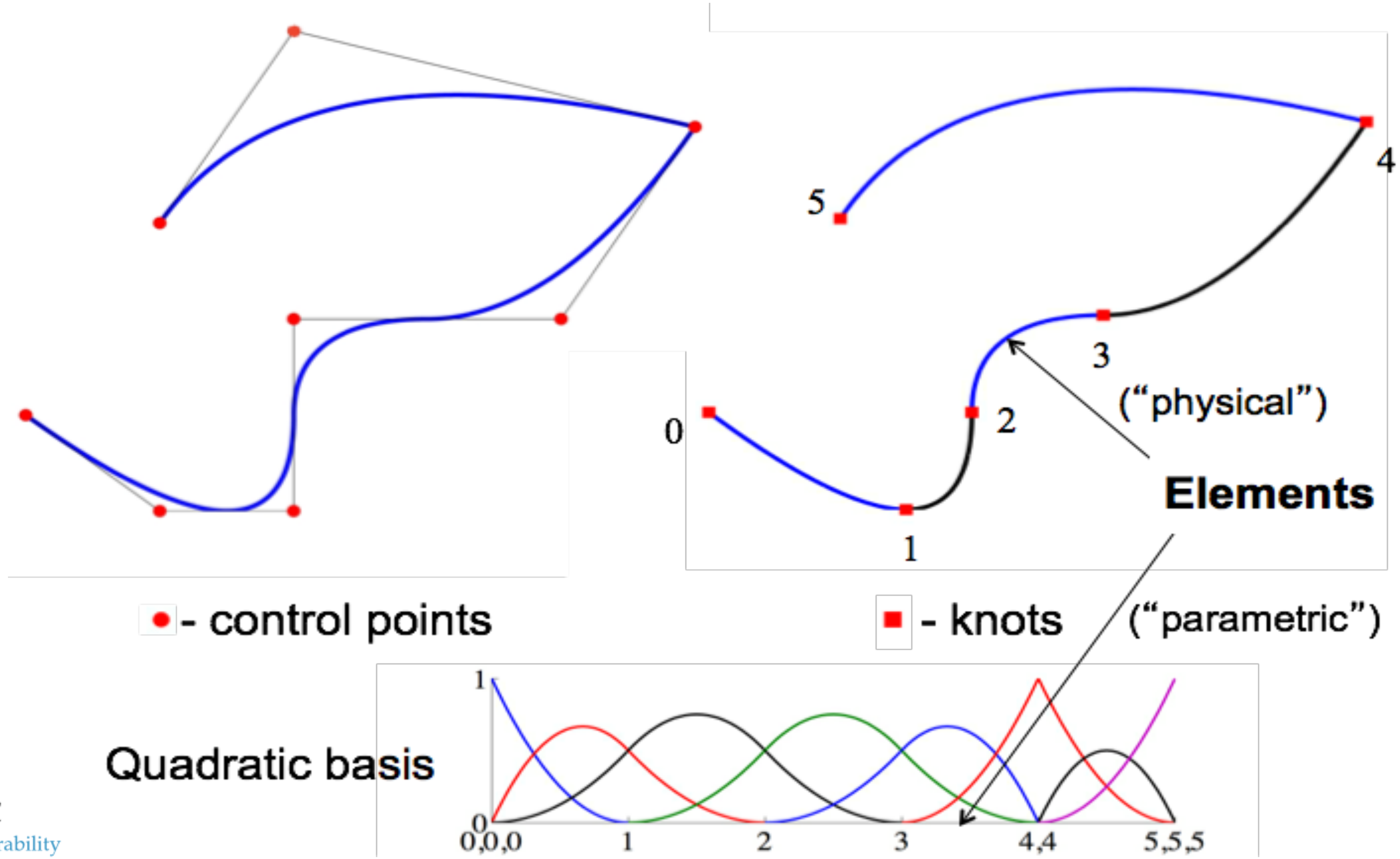
✓Sometimes the perturbed geometry causes wrong models (Babuska-Pitkaranta paradox 1990)

TERRIFIC
Enhancing Interoperability

SEVENTH FRAMEWORK PROGRAMME

Refinement strategies (1D case): strategies to improve accuracy, acting on the mesh and/or on the approximation space.

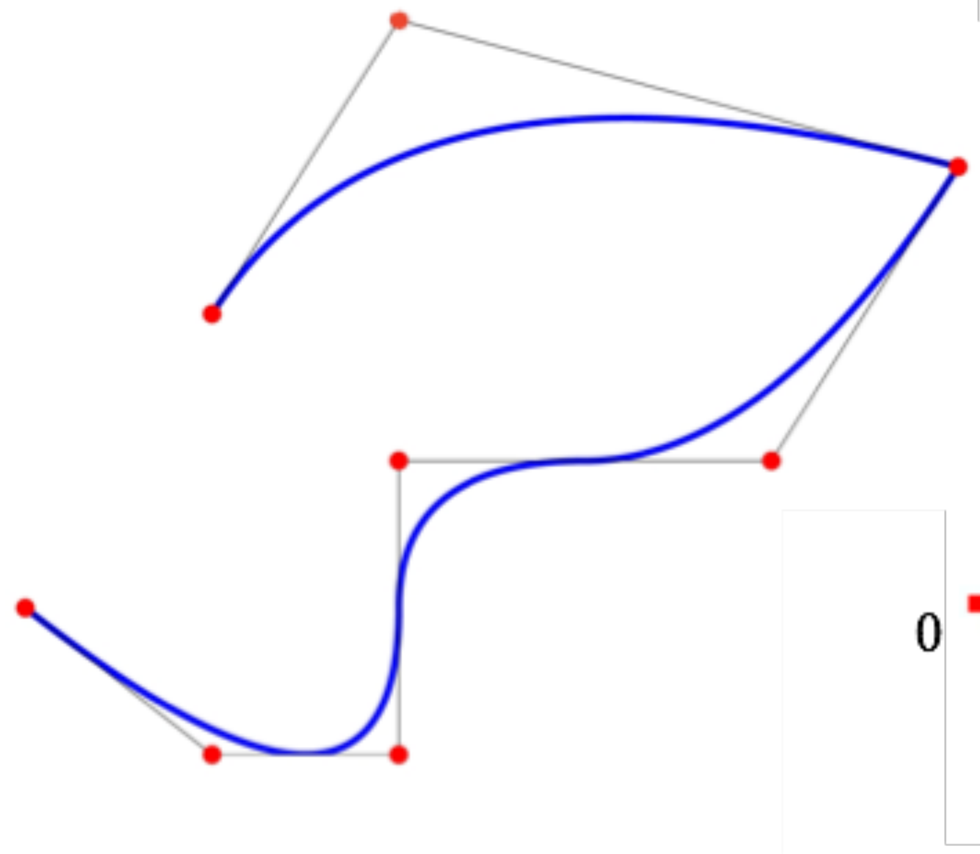
The *parametric space* is mapped into the *physical space*, constructed as the linear combination of the basis functions and the control points: the analogues of the *elements* are the *images of the knot sequence* and *3 refinement strategies* can be adopted: h-p-k refinements.

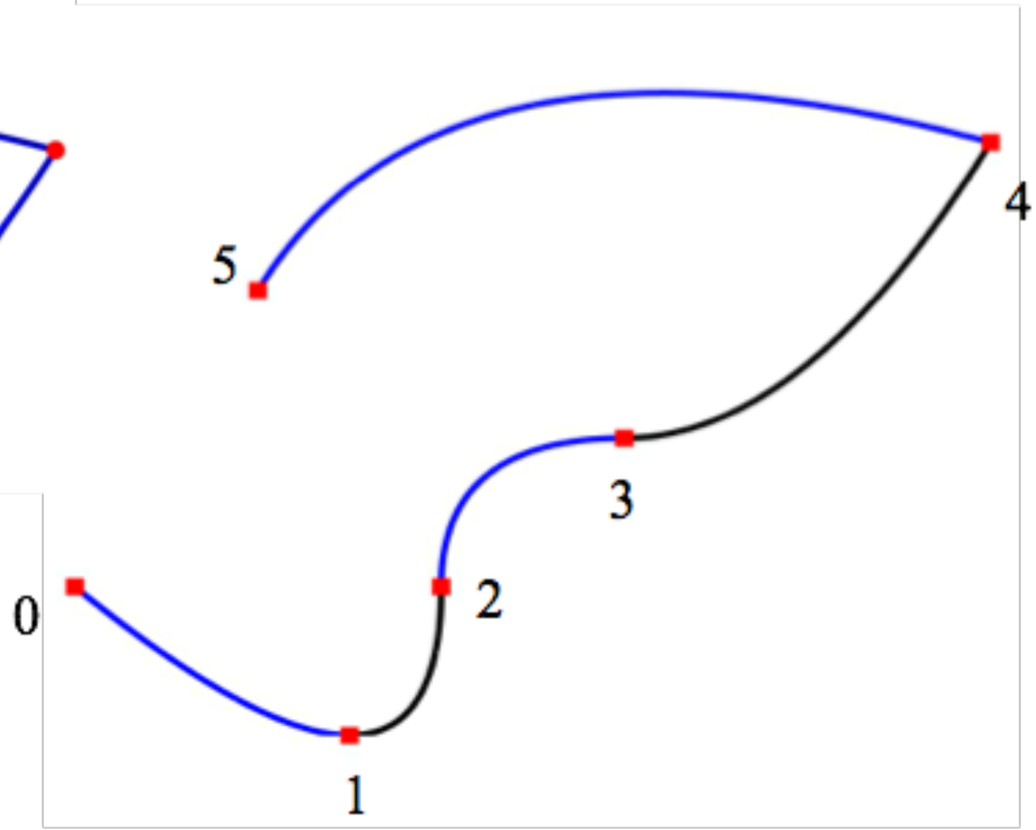


TERRIFIC
Enhancing Interoperability

SEVENTH FRAMEWORK PROGRAMME

Example of *h*-refinement: mesh refinement

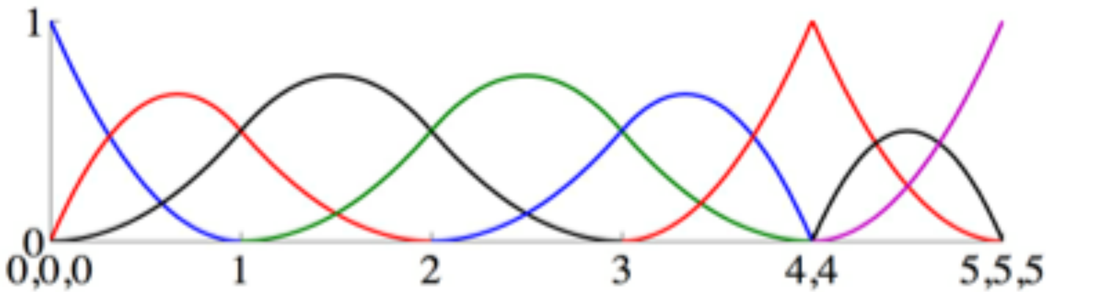




● - control points

■ - knots

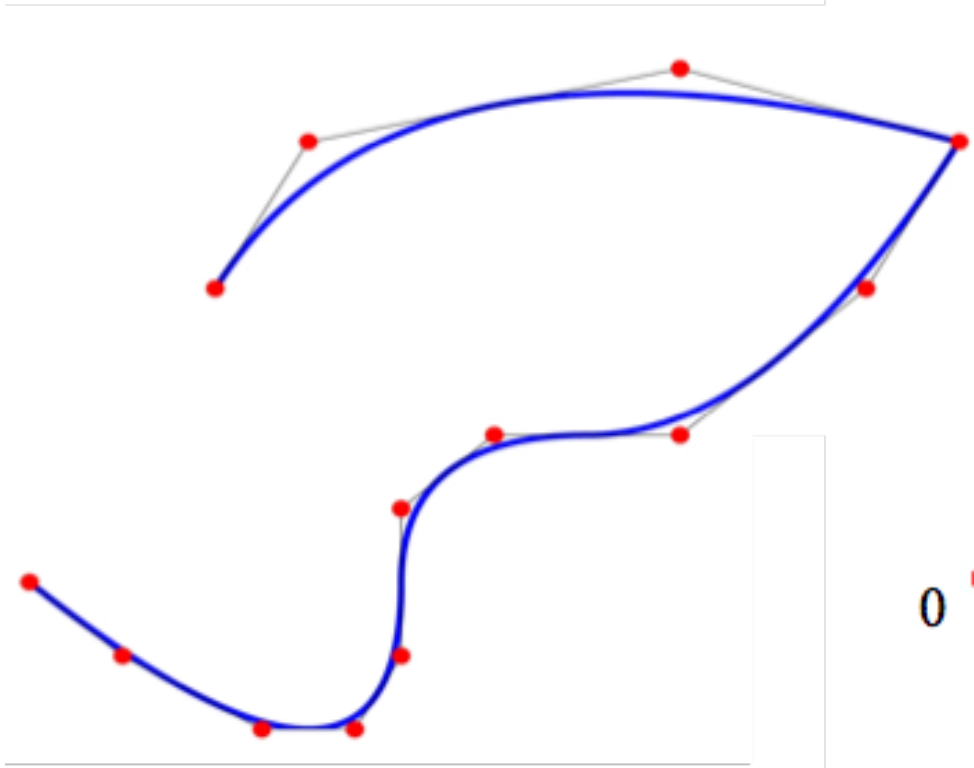
Quadratic basis

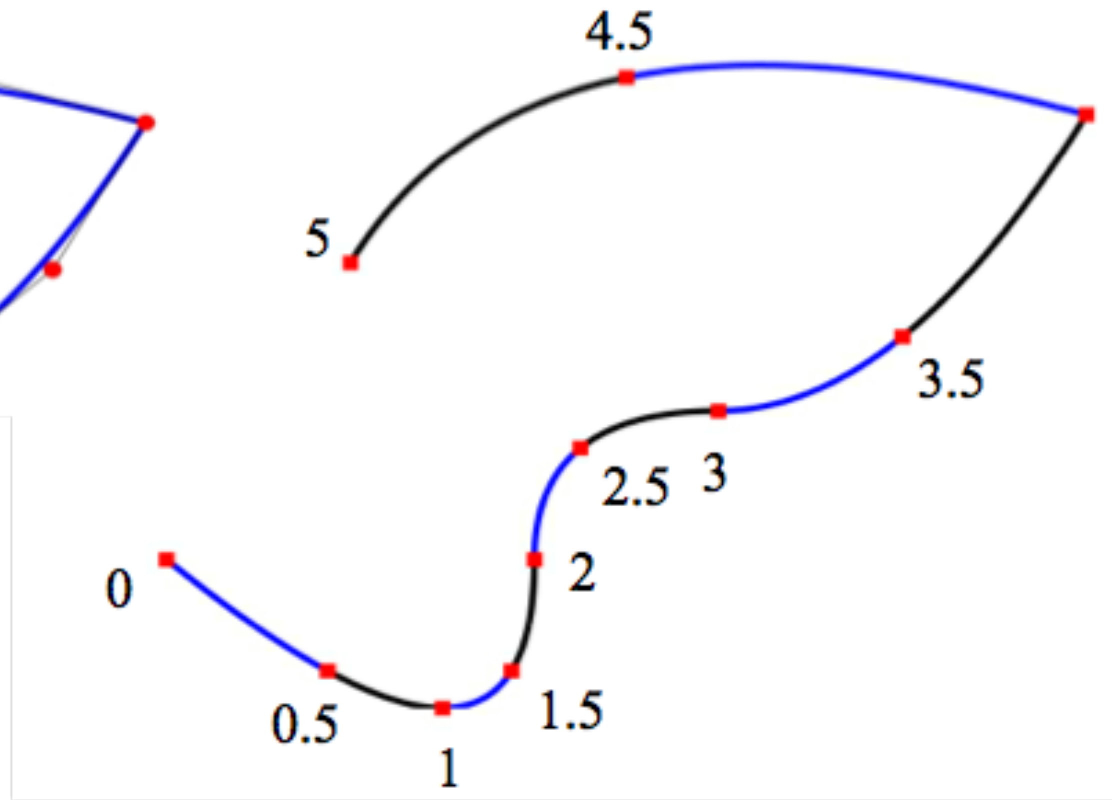


TERRIFIC
Enhancing Interoperability

SEVENTH FRAMEWORK PROGRAMME

Example of *h*-refinement: mesh refinement

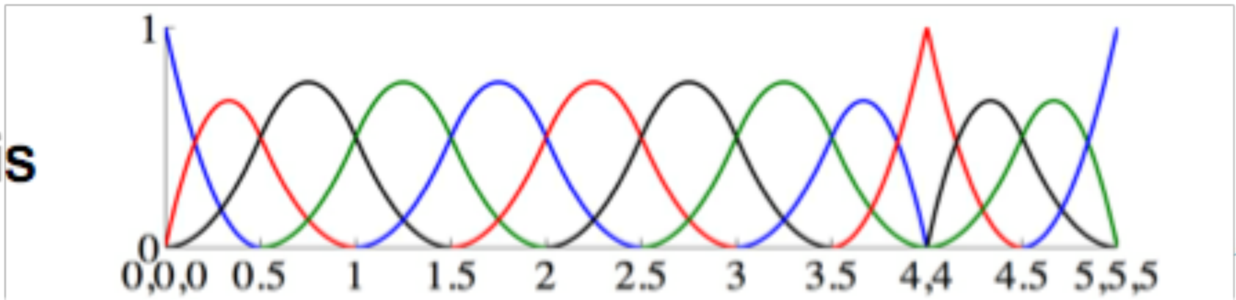




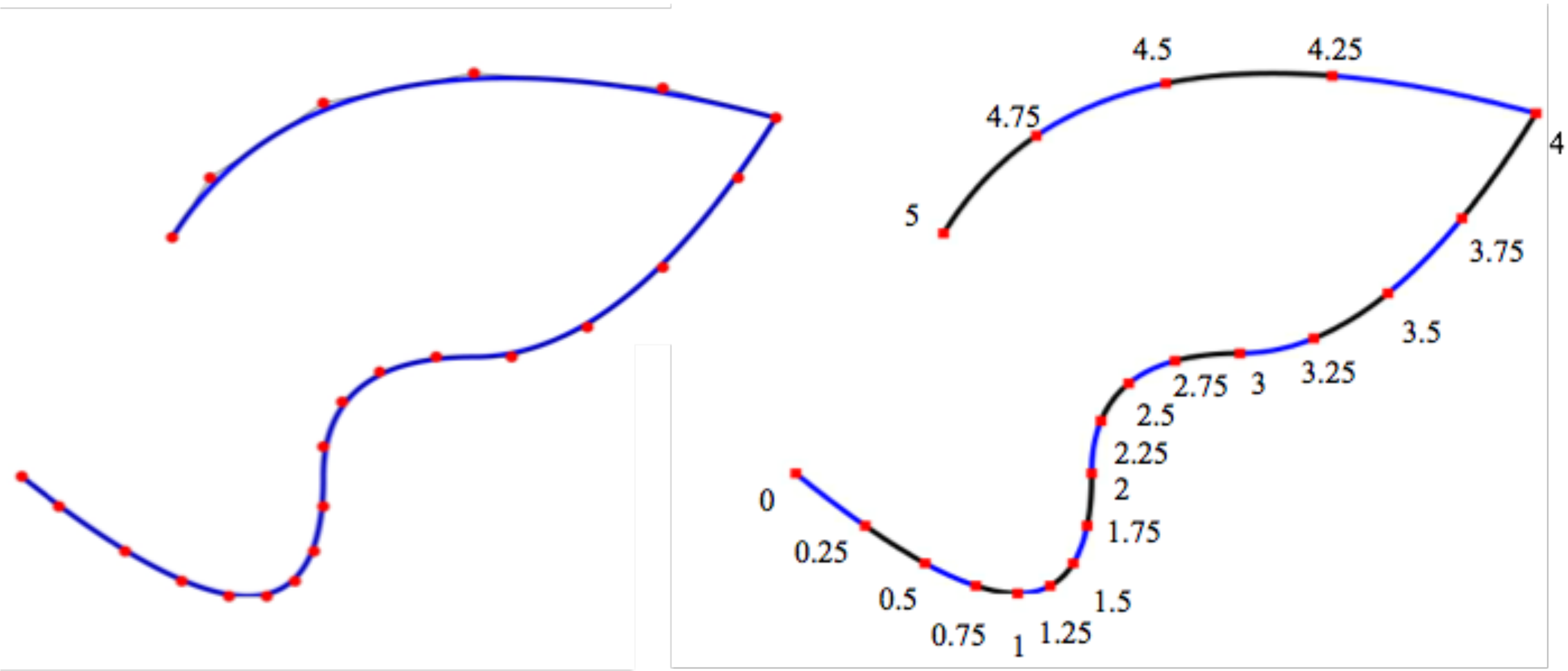
● - control points

■ - knots

Quadratic basis



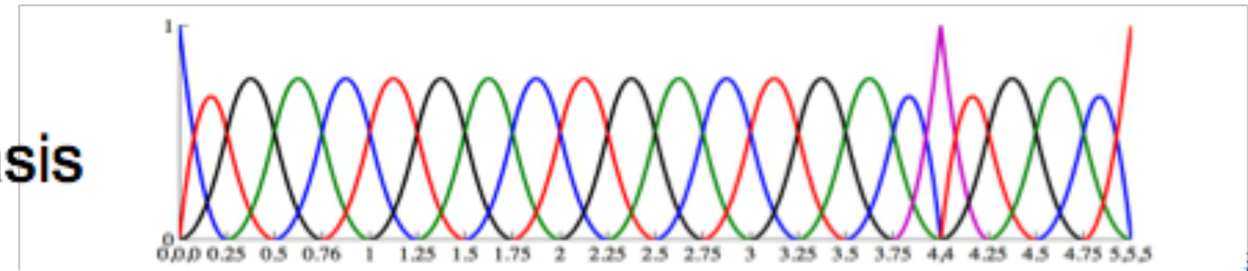
Example of *h*-refinement: mesh refinement



● - control points

■ - knots

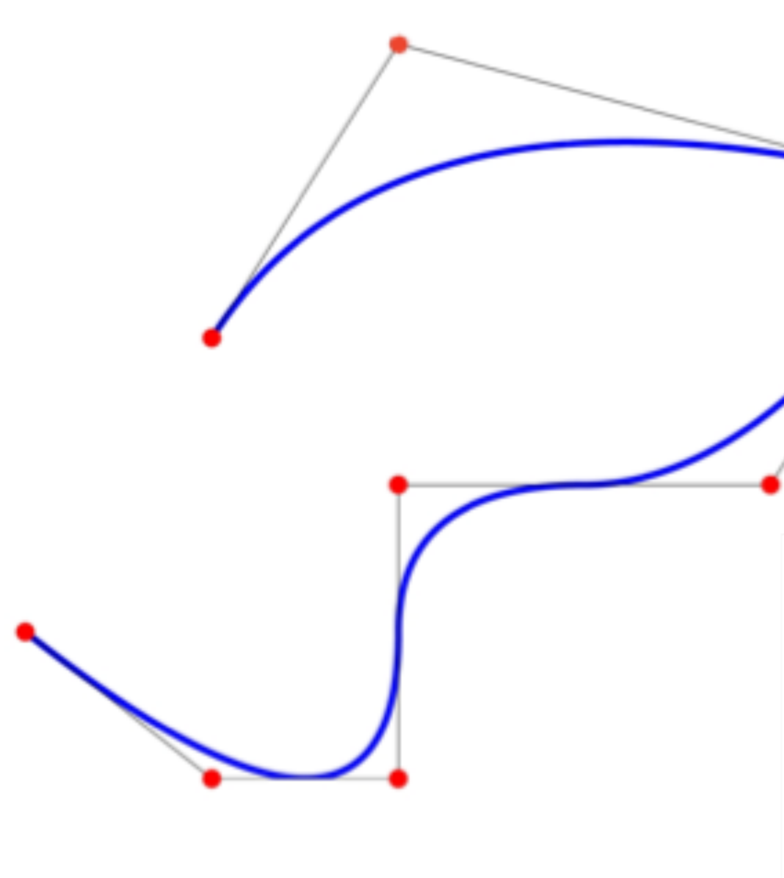
Quadratic basis

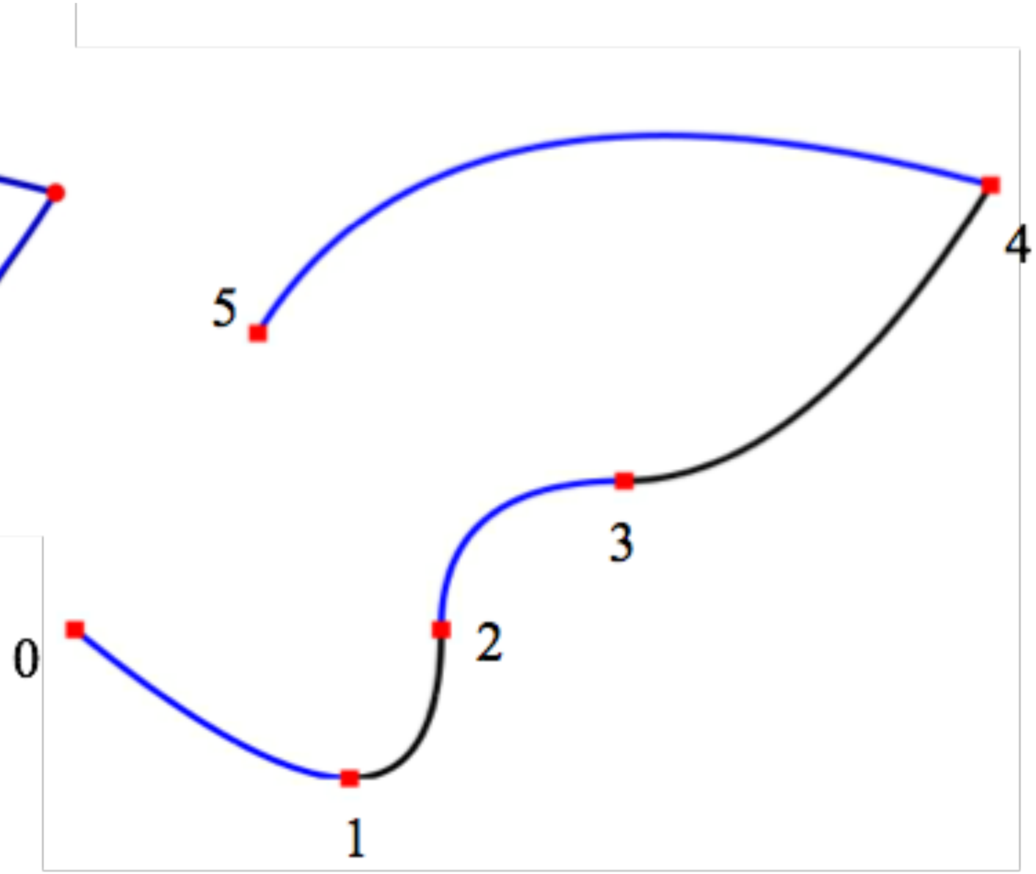


TERRIFIC
Enhancing Interoperability

SEVENTH FRAMEWORK PROGRAMME

Example of *p*-refinement: enlarge approximation space, same mesh

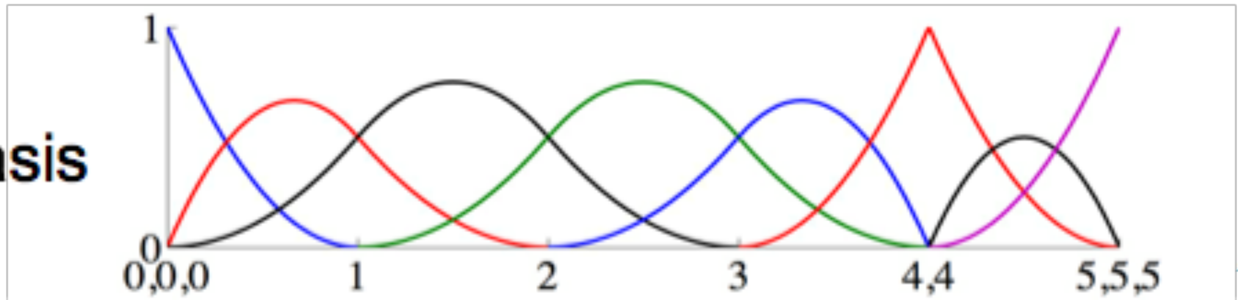




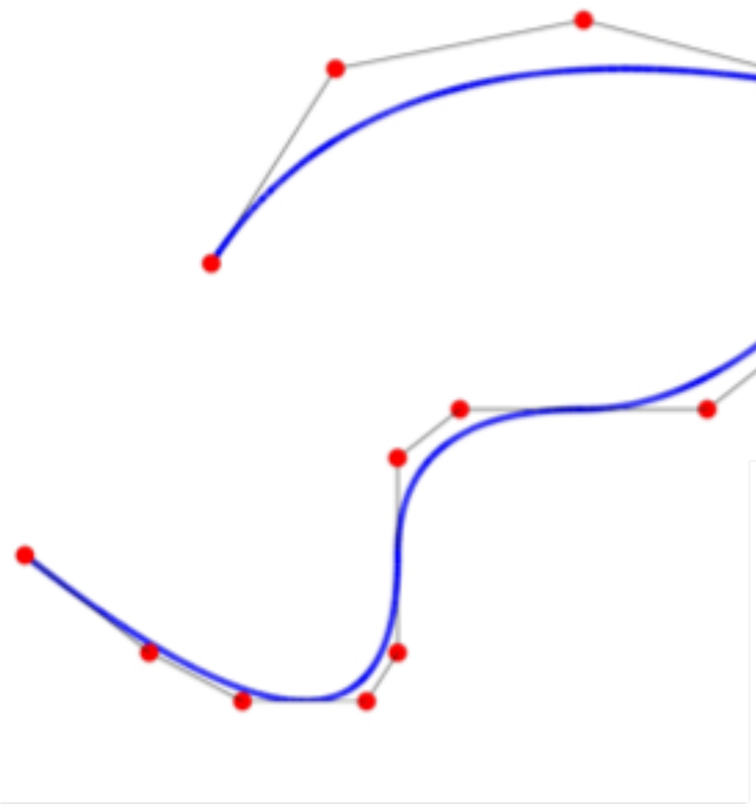
● - control points

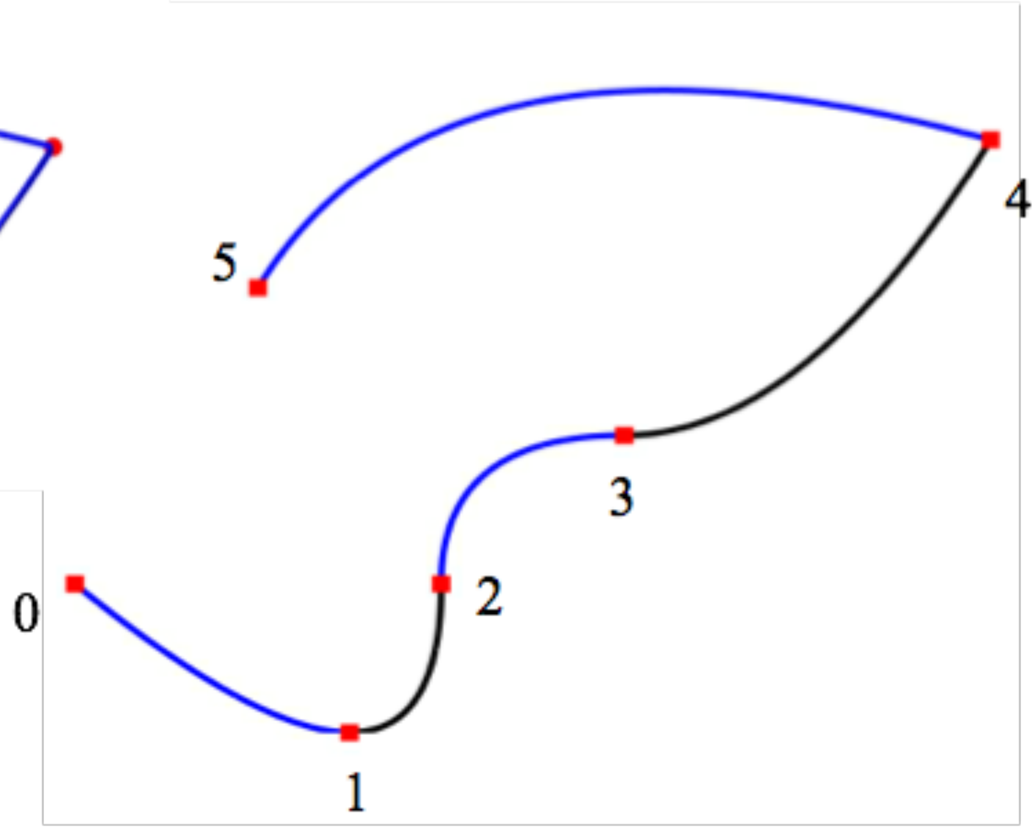
■ - knots

Quadratic basis



Example of *p*-refinement: enlarge approximation space, same mesh

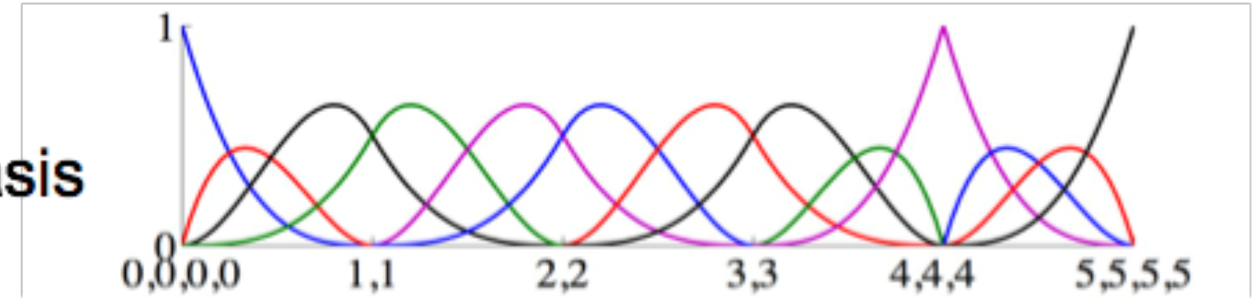




● - control points

■ - knots

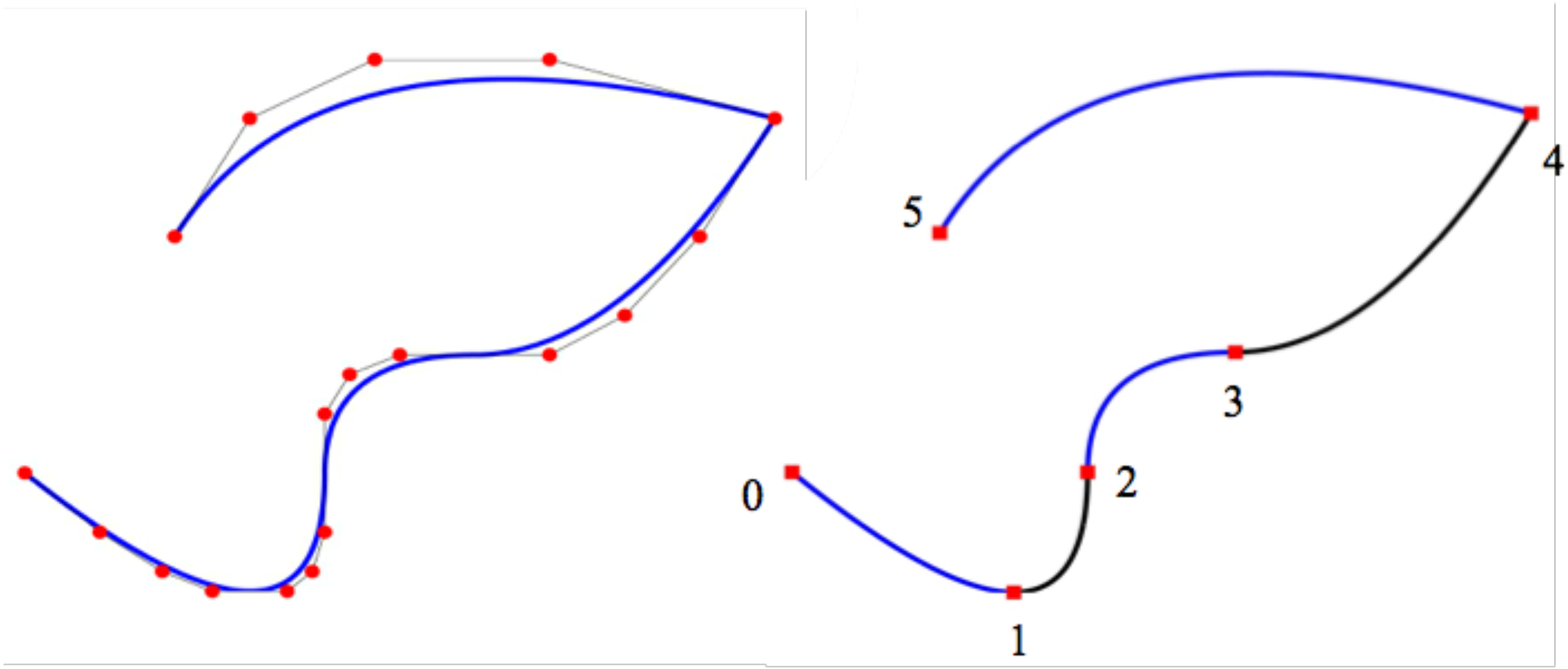
Cubic basis



TERRIFIC
Enhancing Interoperability

SEVENTH FRAMEWORK PROGRAMME

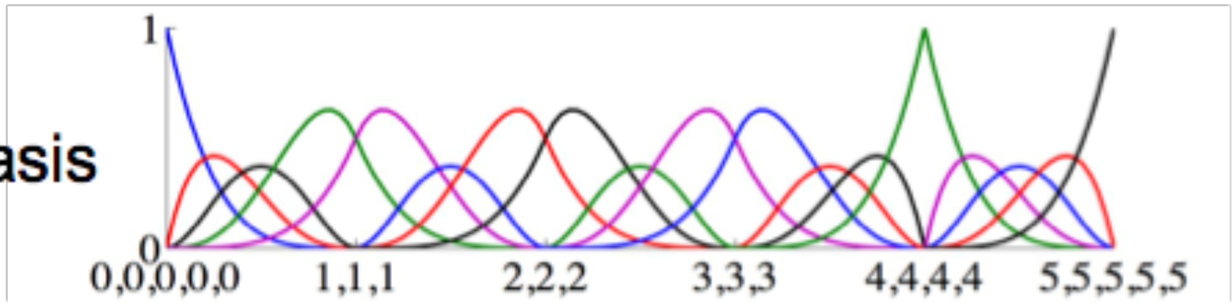
Example of *p*-refinement: enlarge approximation space, same mesh



● - control points

■ - knots

Quartic basis



A third refinement strategy:
k-refinement [no analogue in FEM]

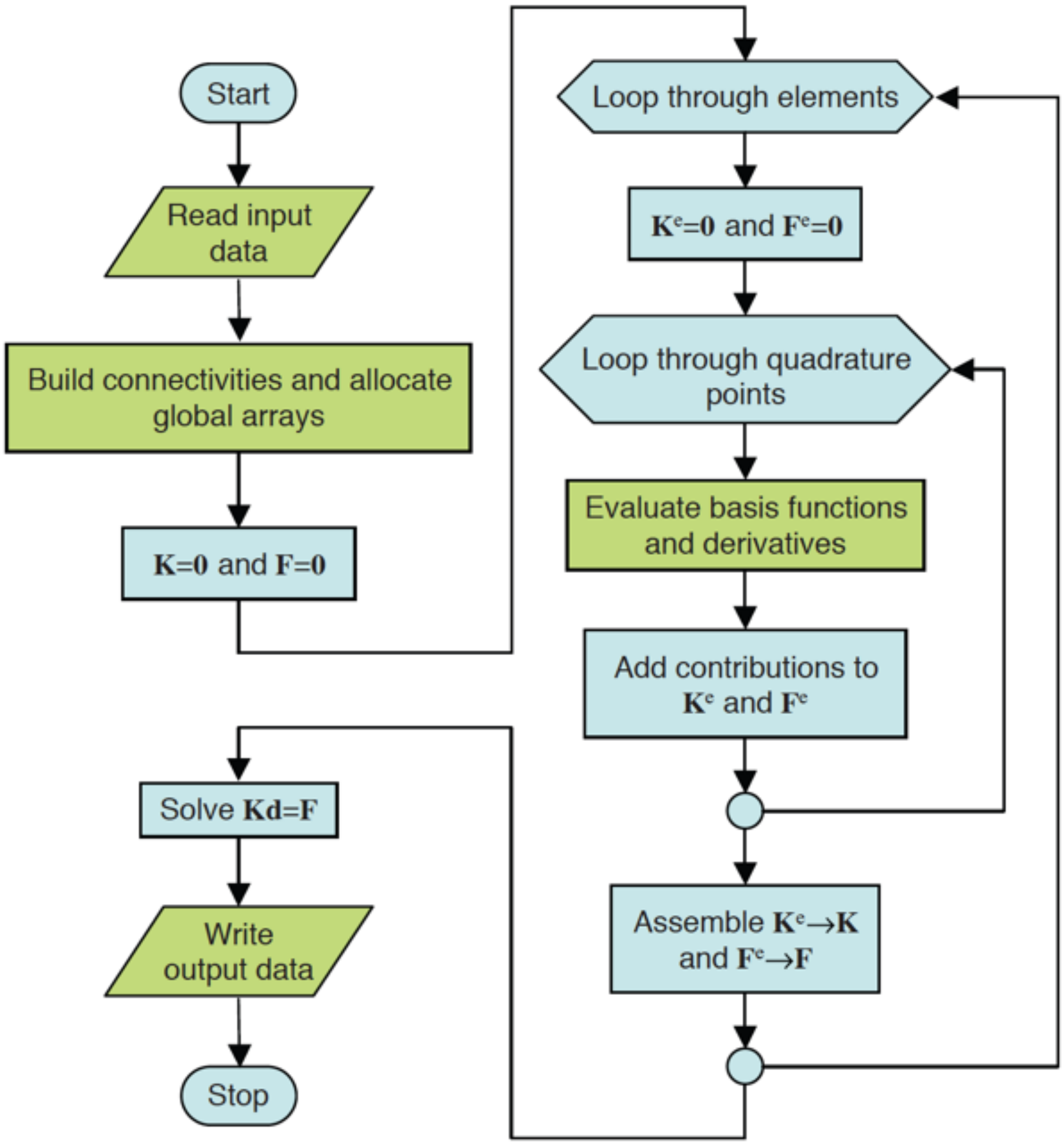
- Procedure in which the polynomial order and smoothness (differentiability) of the B-Spline basis functions are simultaneously increased
- No analogues in FEM
- Leads to possibilities previously unavailable in FEM:
  - Discretization of higher-order PDEs
  - Continuous stresses
  - Collocation methods
- Gives a sequence of “non-nested” spaces…

2D and 3D versions of h-p-k refinement procedures are available.

Implementation

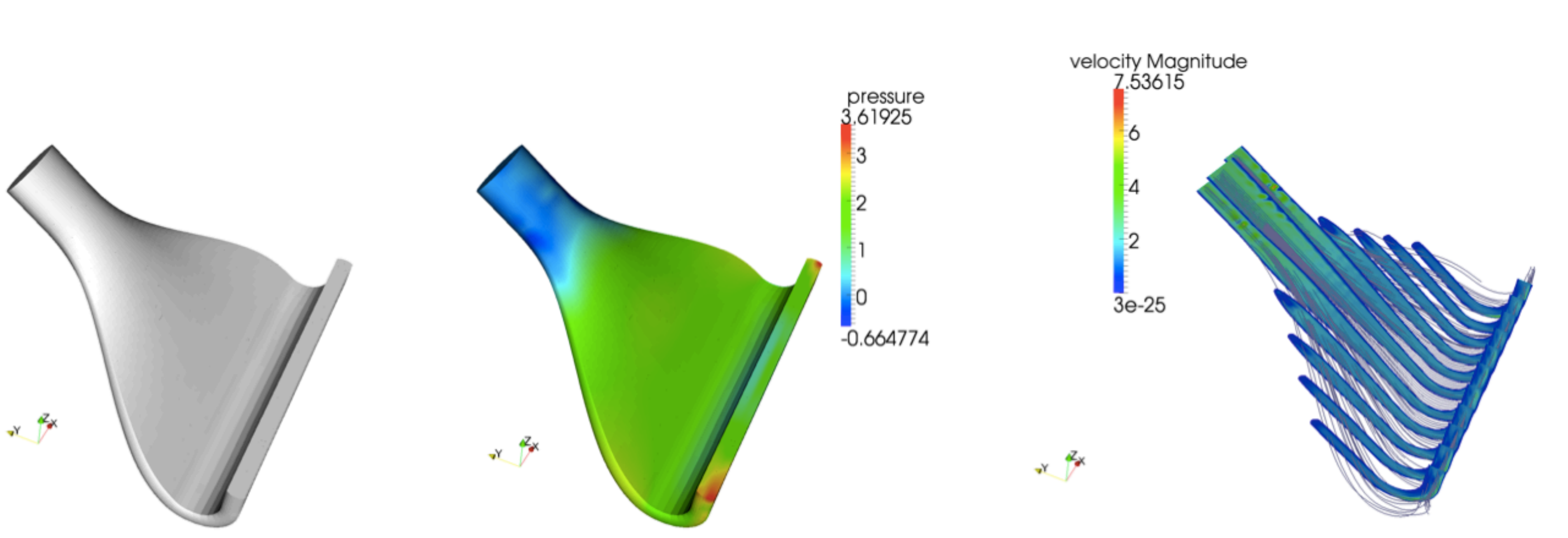
Flowchart of a classical finite element code. Such a code can be converted to a single-patch isogeometric analysis code by replacing the routines shown in green.

[Cottrell et al., 2009]



IGA for Navier-Stokes

$$\begin{cases} \rho\left(\partial_t \mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u}\right) - \mu\Delta\mathbf{u} + \nabla p = \mathbf{f} & \text{on } \Omega\times(0,T) \\ \nabla\cdot\mathbf{u} = 0 & \text{on } \Omega\times(0,T), \end{cases}$$

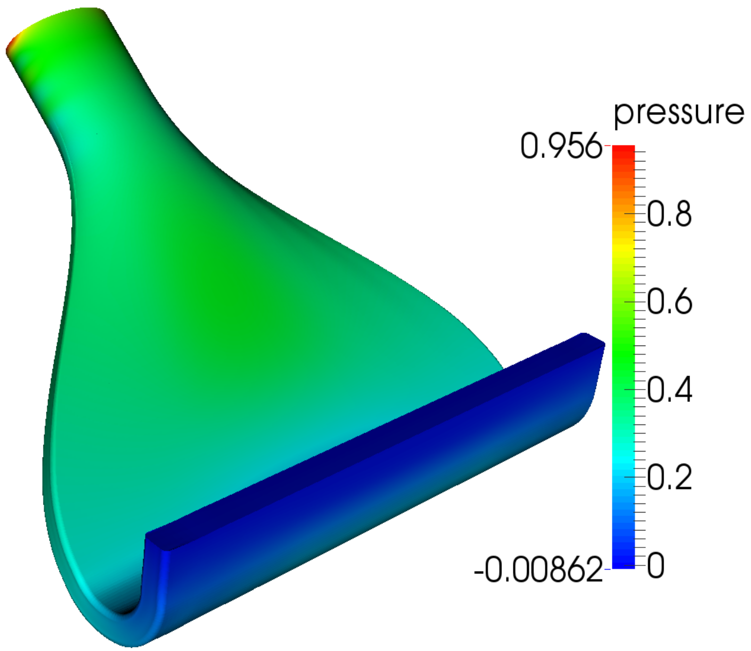


by N. Cavallini, in collaboration with Alenia S.p.A within TERRIFIC

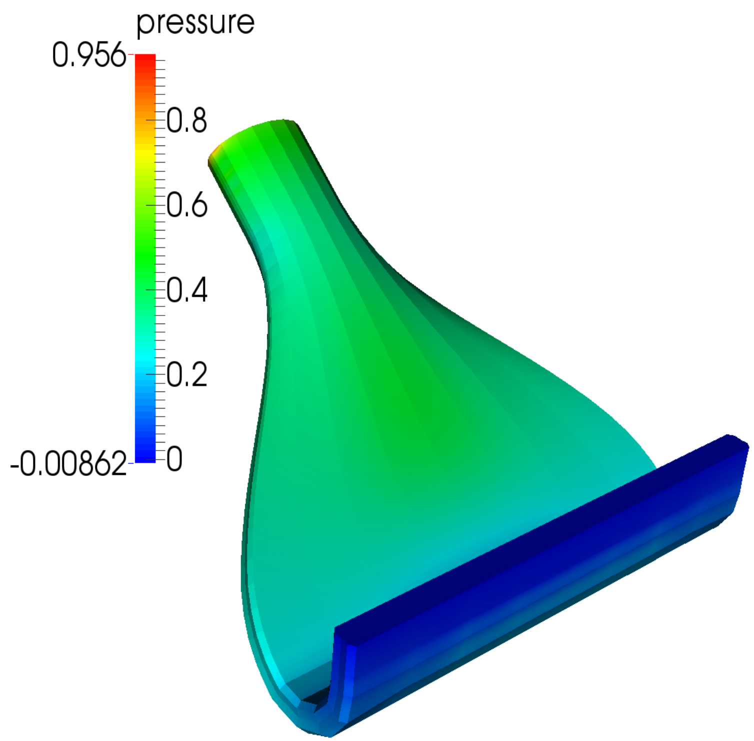
TERRIFIC
Enhancing Interoperability

SEVENTH FRAMEWORK PROGRAMME

Pressure profile: IGA



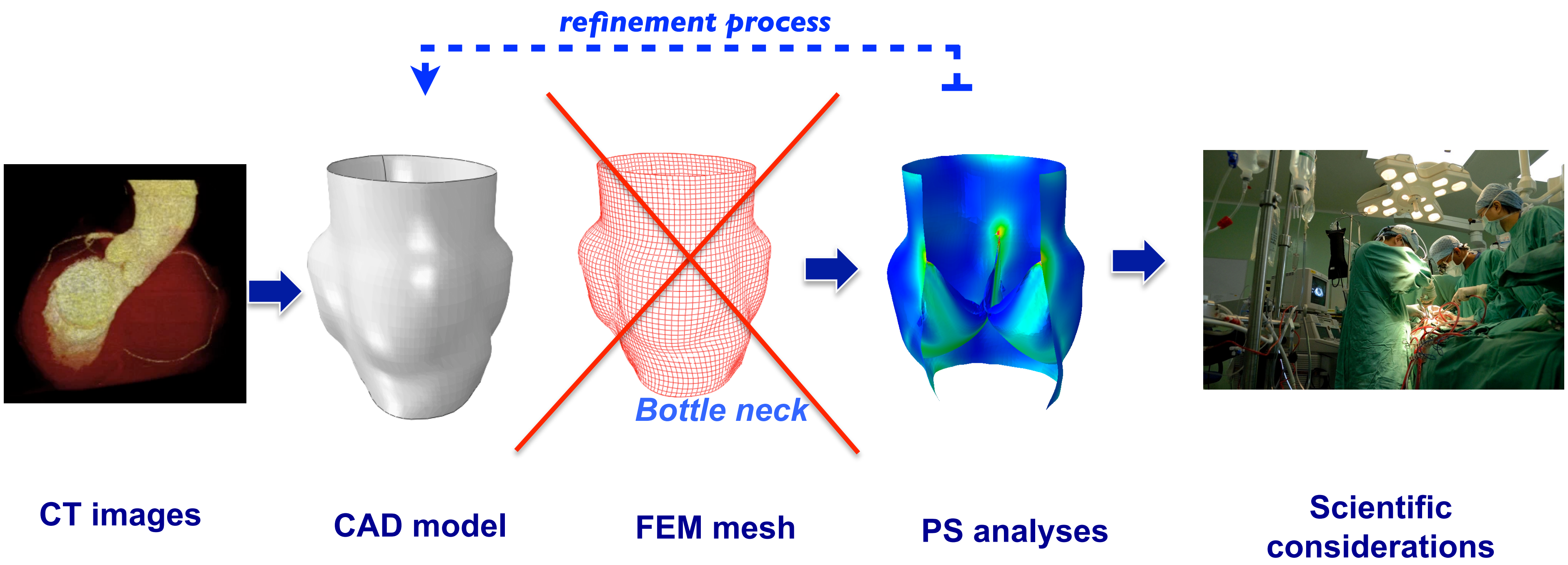
Pressure profile: FEM



TERRIFIC
Enhancing Interoperability

SEVENTH FRAMEWORK PROGRAMME

IGA for patient-specific structural analysis of aortic valve closure

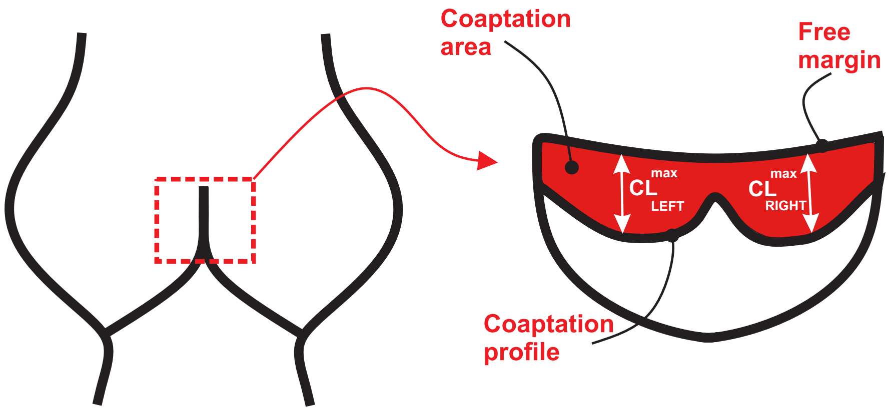


TERRIFIC
Enhancing Interoperability

SEVENTH FRAMEWORK PROGRAMME

IGA: R-M quadratic C1 shell for root and K-L quadratic shell for leaflets

FEA: Belytschko-Tsay 4-node R-M shell for root and leaflets

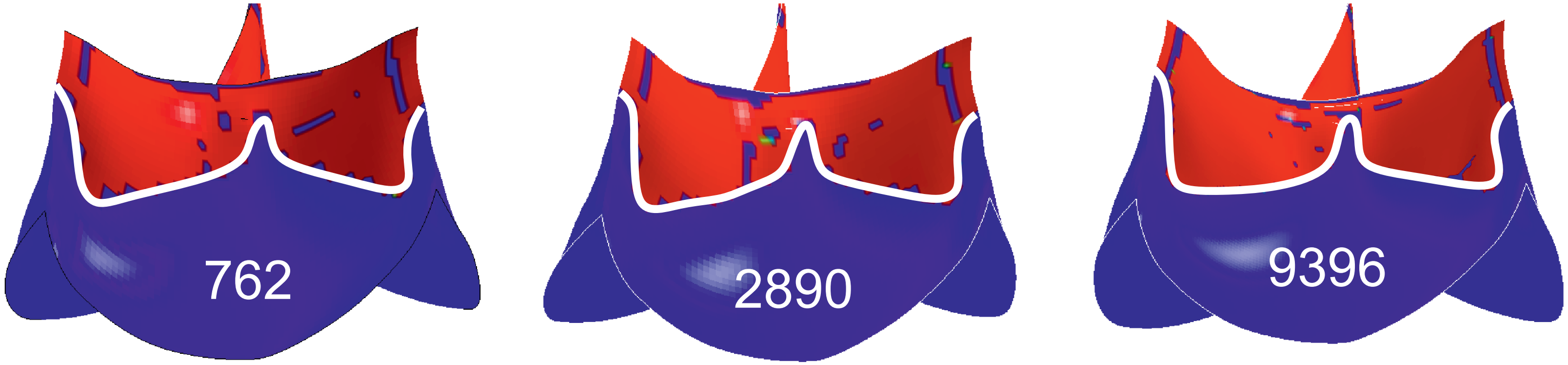


Simulation of valve closure: comparison with finite elements

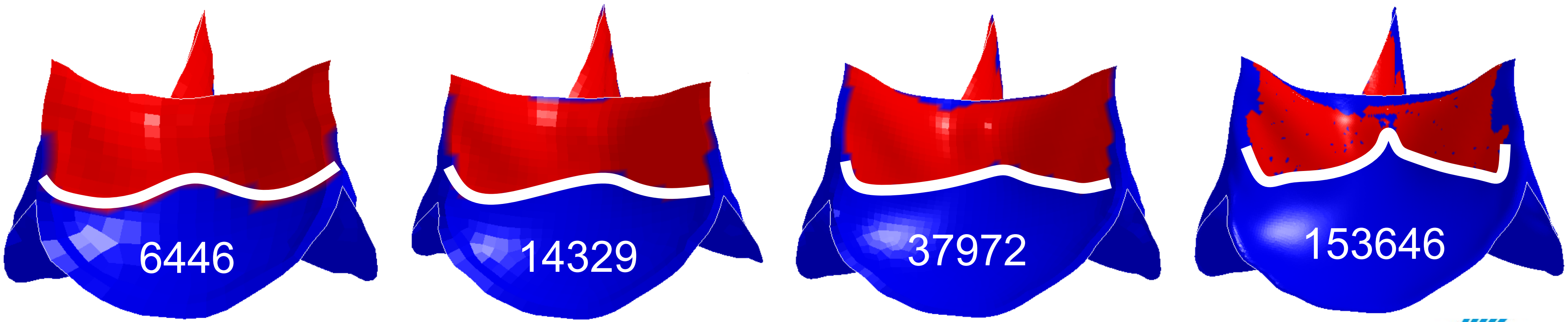
Analysis	# nodes	# DOF	Coaptation length	
			CL_{left}^{max}[mm]	CL_{right}^{max}[mm]
converged	762	3708	9.30	9.40
IgA	2890	19476	9.25	9.40
	9396	50496	9.30	9.35
	1112	6672	11.1	12.8
	3117	18702	10.8	10.2
	6446	38676	10.4	9.80
FEA	14329	85974	9.70	9.70
	37972	227832	9.45	9.50
converged	153646	921876	9.30	9.35

Coaptation profiles for different meshes (IGA and FEA)

IGA:

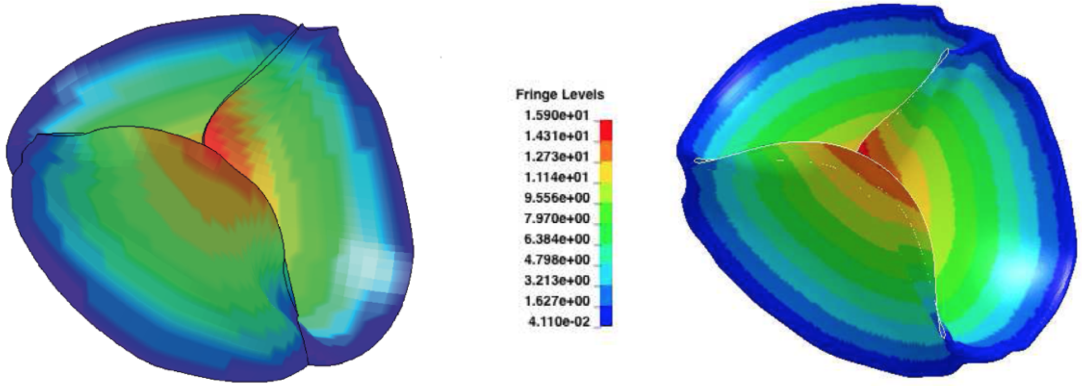


FEA:



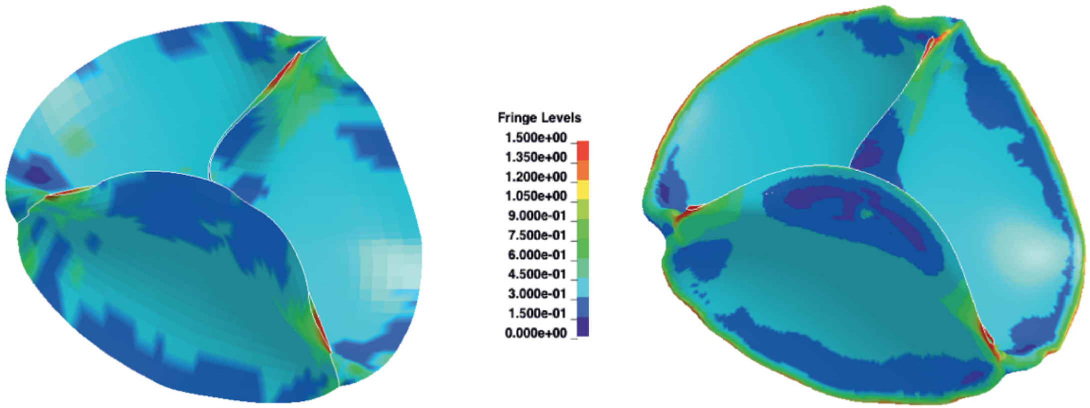
a) IGA (762 nodes) **b) FEA (153646 nodes)**

z-displacement contour map [mm]:



(a) (b)

von Mises stress contour map [MPa]:



(a) (b)

Comparison of computational time (IGA vs FEA)

Analysis	# nodes	# cpus	time step	# increments	total analysis time
IgA	762	12	2.30e-07	4347490	**1h 15m**
FEA	153646	12	2.65e-08	37787314	**550h 23m**

23 full days (24 hours)

Morganti S. et al., ICES Report 14-10,
and submitted to "*Computer Methods in Applied Mechanics and Engineering*"

CONCLUSIONS

Isogeometric Analysis is an emerging technology capable of:

- Directly interacting wtih the CAD systems
- Greatly simplifying the refinement processes
- Improving the solution accuracy
- Reducing the computational costs

CONCLUSIONS

Isogeometric Analysis is an emerging technology capable of:

- Directly interacting wtih the CAD systems
- Greatly simplifying the refinement processes
- Improving the solution accuracy
- Reducing the computational costs

If your applications demand high level quality…

CONCLUSIONS

Isogeometric Analysis is an emerging technology capable of:

- Directly interacting wtih the CAD systems
- Greatly simplifying the refinement processes
- Improving the solution accuracy
- Reducing the computational costs

If your applications demand high level quality…

TRY IT ! ! !

TERRIFIC
Enhancing Interoperability

SEVENTH FRAMEWORK PROGRAMME