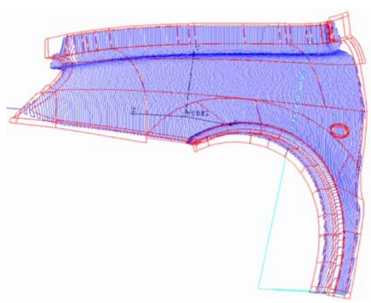
ABSTRACT TO BE PRESENTED

AT

COMPUTATIONAL METHODS
FOR ALGEBRAIC SPLINE SURFACES

COMPASS II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Hybrid Curve Fitting

By

Martin Aigner
Johannes Kepler University, Linz, Austria

Fitting a curve to a given set of data points is an essential problem in the fields of CAD/CAGD, computer vision, computer graphics and many more. There exist many algorithms for solving this problem but most of them are tied to a special representation of the curve. We introduce a novel curve fitting technique that is not linked to a special descriptive form, but may be used for different representations. For the cases of parametric and implicit curves this is explicitly shown. Furthermore we demonstrate how one may construct with this method hybrid curves that consist of an implicit and a parametric part.

The configurations of two real projective quadrics

By

Emmanuel Briand
Universidad de Cantabria, Spain

I will explain how to define properly the "configurations" of two smooth quadric surfaces, and I will explain how to list all the possible configurations, by giving one representant of each.

A numerical--symbolic method for computing the intersection between two bezier triangular patches

By

Fernando Carreras
Universidad de Cantabria, Spain

It will be shown how to determine the intersection curve between two bezier triangular patches, depending on the way that how the patches are defined: parametrically or implicitly. The intersection problem is formulated as an explicit and autonomous system of differential equations, coming from the algebraic computation of orthogonality condition between the intersection curve and the normal vector to the patches. The initial point of the intersection curve is found by analyzing how to travel along the gradient curves on the patches, until a common intersection point is achieved.

Combined subdivision and approximate implicitization method for computing the intersection of 2 grids of surface patches of bi-degree (2,2)

By

Stephane Chau
UNSA, Nice, France

The computation of surface-surface intersection is an important question in computer aided geometric design. Many numerical methods exist for that purpose, the most used are subdivision methods and tracing methods. However, in general case, this problem is difficult, because of the compromise has to make between accuracy, robustness and efficiency. Subdivision methods are robust but slow when a great precision is required, and tracing methods are efficient but the robustness depends on the detection of all intersection curve.

We present a combined methods using subdivision, approximate implicitization and tracing methods for the surface-surface intersection problem of two Bezier patches of bi-degree (2,2). The general idea is to rely on subdivisions to exclude domains where there is no intersection and then concentrate the complexe computation in relevant areas.

Mu-basis of rational curves and surfaces: Theory and Applications

By

Falai Chen
University of Science and Technology of China, Hefei, China

In this talk, I will review the theory and algorithms for the mu-bases of rational curves and surfaces-- a newly developed algebraic tool to handle rational curves and surfaces. Then I will talk about some applications of the mu-basis theory, such as implicitzation, reparameterization and computing singular points/locus of rational curves/surfaces.

Surfaces parametrized by their normals

By

Jens Gravesen
Technical University of Denmark, Denmark

For a surface with non vanishing Gaussian curvature the Gauss map is regular and can be inverted. This makes it possible to use the normal as the parameter, and then it is trivial to calculate the normal and the Gauss map. This in turns makes it easy to calculate offsets, the principal curvatures, the principal directions, etc.

Such a parametrization is not only a theoretical possibility but can be used concretely. One way of obtaining this parametrization is to specify the support function as a function of the normal, i.e., as a function on the unit sphere. The support function is the distance from the origin to the tangent plane and the surface is simply considered as the envelope of its family of tangent planes.

If the support function is the restriction to S^2 of a homogeneous polynomial of degree n then the surface is the image of S^2 by a homogeneous map of degree n+1 and we obtain a usual rational parametrization of degree 2(n+1). This class of surfaces is closed under rotation and translation if n is odd, and it is closed under rotation and offsets if n is even.

Suppose we are given points and normals and we want a C^n-surface interpolating these data. The data gives the value of the support function at certain points (the given normals) on the unit sphere, and the surface can be defined by determining the support function as a C^n interpolant to the given values.

Cube decompositions by eigenvectors of multivariate splines

By

Ioannis Ivrissimtzis
Max Planck Institut Informatik, Germany

A matrix is called g-circulant if its columns and rows are indexed by the elements of a group G. When G is cyclic we obtain the usual circulant matrices which appear in the study of linear transformations of polygons. In this paper we extend this polygonal theory into hypercubes and prisms, using g-circulant matrices where G is the direct product of cyclic groups. As application, we study the evolution of a single cell of an n-dimensional grid under the subdivision algorithm of the multivariate quadratic B-splines. Regarding the prism, we study its evolution under the tensor extension of the Doo-Sabin subdivision scheme by the univariate quadratic B-spline.

The Geometry of Monoid Surfaces

By

Pål Hermunn Johansen
CMA, University of Oslo, Norway

A monoid hypersurface is an algebraic surface with a point of multiplicity one less than the degree of the surface.

These surfaces are rational and therefore potentially interesting in CAGD applications. Properties of monoid hypersurfaces in general are explored, then monoid curves and surfaces are considered. We have a complete understanding of singularities on monoid surfaces away from the very singular point - all of them must be of the A series. Finally, an overview over monoid surfaces of low degree is given.

Rational Canal Surface

By

Rimvydas Krasauskas
Vilnius University, Lithuania

A canal surface with the rational spine curve and rational radius function is known to be rational. We study all such parametrizations with rational Gaussian image and present an algorithm that generates Bezier patches of minimal degree on such canal surfaces with given boundary curves. Applications include exact NURBS representations of several fixed or variable radius rolling ball blends between two natural quadrics in an arbitrary given position.

An Algorithm for Locating Interesting Examples within Families

By

Oliver Labs
Johannes Gutenberg Universitat Mainz, Germany

Often, one is interested in constructing an algebraic variety with certain properties. In many cases, it is not difficult to write down a family which probably contains such an interesting example. We describe an algorithm which can be used to locate such varieties within the family.
We apply the method to the classical subject of surfaces with many singularities. Our implementation in the computer algebra system SINGULAR allows us to find surfaces of degree 7 and 9 with 99 and 226 nodes, respectively. We illustrate the geometry of these constructions using many visualizations.

Classification of parametrized surfaces of bidegree (1,2): generic and non generic, real and complex cases

By

Thi Ha Le
UNSA, Nice, France

We present a classification and a geometric study of parametric surfaces of bidegree (1,2) over the complex field and over the real field. Patches of these surfaces are used as models in Computer Aided Geometric Design.

We first recall classical results (for the complex case) which go back to Cremona and to Cayley in the 19th century and were reviewed by Edge in 1931.

We provide another point of view by considering a dual scroll similar to the one introduced by Piene and Sacchiero in 1984. This allows a more algebraic complex classification, which eases the obtention of the classification of the real generic cases.
Then,we consider a list of non generic cases, classify and further study them geometrically and computationally.

Ridges and umbilics of polynomial parametric surfaces

By

Marc Pouget
INRIA, Nice, Francia

Given a smooth surface, a blue (red) ridge is a curve along which the maximum (minimum) principal curvature has an extremum along its curvature line. Ridges are curves of *extremal* curvature and therefore encode important informations used in segmentation,registration, matching and surface analysis.

State of the art methods for ridge extraction either report red and blue ridges simultaneously or separately --in which case need a local orientation procedure of principal directions is needed, but no method developed so far topologically certifies the curves reported.

First, for any smooth parametric surface, we exhibit the implicit equation $P=0$ of the singular curve ***P*** encoding all ridges of the surface (blue and red), and analyze its singularities.

Second, for a polynomial surface this equation defines an algebraic curve. We develop the first certified algorithm to produce a topologically certified approximation of the curve ***P***. The algorithm exploits the singular structure of ***P*** --umbilics and purple points, and reduces the problem to solving zero dimensional systems using Rational Univariate Representations and isolate roots of univariate polynomials. An experimental section illustrates the efficiency of the algorithm on a Bezier patch.

Sparse parametrization of algebraic curves and surfaces

By

Josef Schicho
Johann Radon Institute for Computational and Applied Mathematics, Linz, Austria

There are well-known algorithms for deciding rationality and computing rational parametrizations (if exist) for implicit curves in the projective plane, and for implicit surfaces in projective 3-space. In some examples, the equation has specific structure one can exploit in order to reduce the computational costs. The stepping stone in this new technique is (once more) the Newton polygon/polyhedron of the implicit equation.

Implicitization and numerical stability

By

Ibolya Szilagyi
Research Institute for Symbolic Computation, Linz, Austria

When approximate implicitization is used we are faced with questions of the following types: How stable is the computation of the coefficient vector of an approximate implicitization? How robust (geometrically) is the resulting implicit representation with respect to small perturbations of its coefficients?

To answer to above questions we introduce the condition number of the implicitization problem. This condition number depends not only on the input, but also on the estimation of the degree of the implicit form. For hypersurfaces with a high condition number, the computation of the coefficients of an approximate implicitization is numerically unstable, no matter which numerical method for implicitization is chosen.

Using this condition number the distance between the two coefficient vectors of two approximate implicitizations can be estimated. The condition number allows us to give a stability test for various implicitization techniques.