

**Bergen, Norway**  
**BNAM 2010**  
**May 10-12**

**Finite element analysis and measurements of ultrasonic piezoceramic transducers in air and water**

Magne Aanes, Espen Storheim, Magne Vestrheim

University of Bergen, Department of Physics and Technology, P.O box 7803, N-5020 BERGEN, Norway, [magne.aanes@ift.uib.no](mailto:magne.aanes@ift.uib.no),  
[espen.storheim@ift.uib.no](mailto:espen.storheim@ift.uib.no), [magne.vestrheim@ift.uib.no](mailto:magne.vestrheim@ift.uib.no)

Per Lunde

University of Bergen, Department of Physics and Technology, P.O box 7803, N-5020 BERGEN, Norway  
Christian Michelsen Research AS (CMR) P.O box 6301 Postterminalen, N-5892 BERGEN, Norway, [per.lunde@ift.uib.no](mailto:per.lunde@ift.uib.no)

Advantages, limitations and problems in applying finite element modeling of ultrasonic piezoceramic transducers are discussed, with examples from air and water. Finite element simulations of transducers are compared to measurements and simplified one-dimensional Mason-type models for piezoceramic materials with emphasis on deviations. With the accurate finite element methods achieved in recent years the standardized methods for determining material data for simulations do not provide adequate precision. Comparison of finite element simulations with electrical measurements are done using adjusted material data. Finite element modeling is used for both modal and direct harmonic analysis. Interactions between electrical and acoustical properties, including displacement on the surfaces of the transducer are studied. Source sensitivity is simulated using finite element methods and compared to measurements. Thus, agreement and disagreement for electrical and acoustical transducer properties are discussed on basis of the finite element methodology and material data.

## **1 Introduction**

In design and examination of ultrasonic piezoceramic transducers accurate simulation tools such as the finite element (FE) method can be valuable in the designing process [1, 2, 3]. The advances in computer technology has made the FE method one of the most used simulation techniques in recent years. FE simulations will, in the present paper be compared to electrical measurements in air and water, and acoustical measurements in air. The comparison of simulations and measurements will also be discussed on basis of the material data used in the simulations. Two solution methods for calculating transducer response functions is considered, concerning advantages and disadvantages of each method. From simulations displacement on the surface of the disk will be demonstrated and discussed together with source sensitivities in air and water. Advantages, limitations and problems in applying the finite element method to piezoceramic transducers are addressed on basis of the finite element methodology, material data and the comparison with electrical and acoustical measurements.

In transducer construction models such as the simplified one-dimensional Mason-type [4, 5] models for radial (R) and thickness-extensional (TE) modes have been extensively used. In Figure 1, a comparison of these models for a circular piezoceramic disk with a D/T-ratio of 6.216. The finite element method reveals only fair agreement for the Mason radial mode model for frequencies around the first series resonance frequency, defined in [4]. Poor agreement is otherwise demonstrated for the whole frequency range. The material data from Ferroperm A/S [6] for Pz27 has been used for each simulation. Poor agreement between the Mason TE-model and finite element is presented for the whole frequency range. The finite element method reveals a more elaborate structure than the simplified models, since this model also includes

other modes than radial and TE.

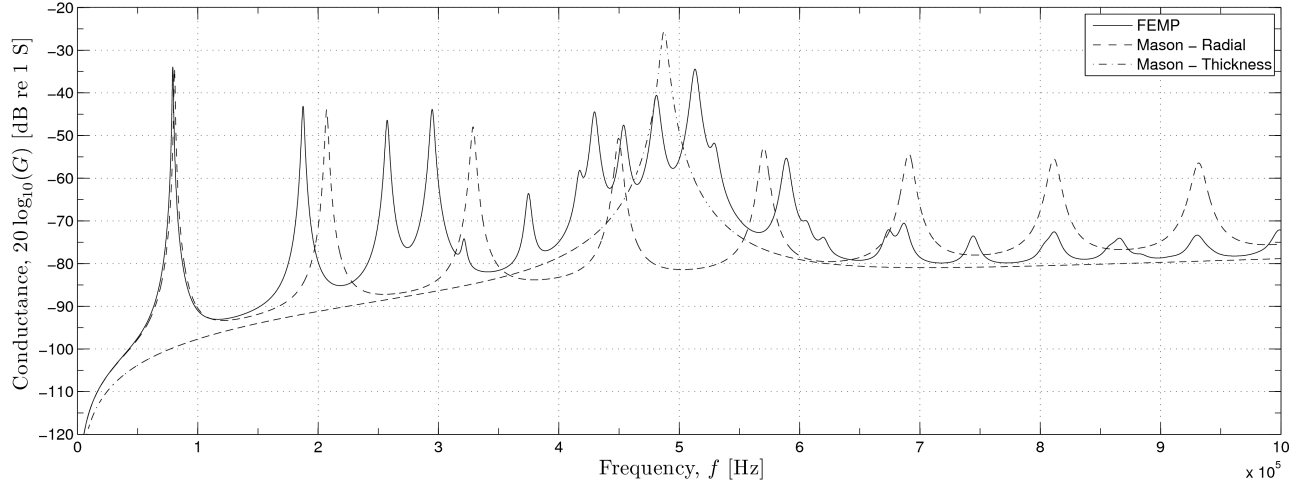


Figure 1: Conductance for a Ferroperm Pz27 disk in vacuum with  $D/T = 6.216$  using finite element modeling, Mason radial mode and Mason TE-model with material data from the manufacturer.  $D = 24.94$  mm,  $T = 4.0123$  mm.

## 2 Measurements and simulations

In the present paper, a circular Pz27 piezoceramic disk from Ferroperm A/S[6] with a diameter of 24.94 mm and thickness of 4.0123 mm, i.e. a  $D/T$ -ratio of 6.216, is used. One of the electrodes have two flaps of size 2x6 mm on opposite sides in the thickness direction. These flaps are meant as solder points when adding a front layer to the piezoceramic disk. These flaps are not taken into consideration when using an axisymmetric finite element method. The electrical admittance of the disk was measured with a HP 4192 LF impedance analyser [7] in air and water. For the measurements in water, the disk was fully submerged in de-ionized water to reduce current leakage. The electrical impedance between the electrodes in the de-ionized water was measured over the same frequency range, and for the measurements presented here, the current leakage through water has been compensated for. Source sensitivity in air has been measured using a B&K microphone type 4138 [8]. Measurements are limited to 200 kHz since the measurement microphone calibration data are available up to this frequency only [8].

The finite element program used in the present work is FEMP U3.2 [1, 9]. A problem using finite element modeling is to achieve accurate material data for materials used in construction of the transducer. To minimize this problem only the piezoceramic disk will be used in this discussion. Simulations of more complex transducer constructions using FEMP are presented in [2, 3]. The finite element simulations of the Pz27 disk has been conducted with 5 elements per shear wavelength both in the radial- and the thickness direction at the highest frequency in the simulations. The total number of elements in a simulation is limited by the computer memory (RAM), which limits the maximum frequency that can be simulated with a reasonable uncertainty in the simulation. Infinite elements are included to ensure that the transducer radiates in an infinite medium, thus satisfying Sommerfelds radiation condition [10]. The uncertainty in the finite element simulations depend on the total number of elements used, and when acoustical properties are calculated, also on the distance between the transducer and the infinite elements [1]. A computer with 32 GB RAM has been used for the simulations and a maximum of approximately 40.000 elements can be used, including elements for the piezoceramic disk (finite) and fluid (finite and infinite elements). The source sensitivity in air is for that reason limited to 550 kHz. Since the infinite elements have to be implemented further away from the piezoceramic disk as frequency is increased to ensure reasonable uncertainty, element division increases more for air than water due to the difference in speed of sound.

### 3 Results

#### 3.1 Electrical admittance and material data

The electrical admittance for the Pz27 disk is calculated for vacuum using the finite element method using material data from the manufacturer. Comparing the conductance in air to this simulation is presented in Figure 2. For frequencies above the first series resonance frequency at approximately 80 kHz, notable disagreements between measurement and simulation is demonstrated for the whole frequency range.

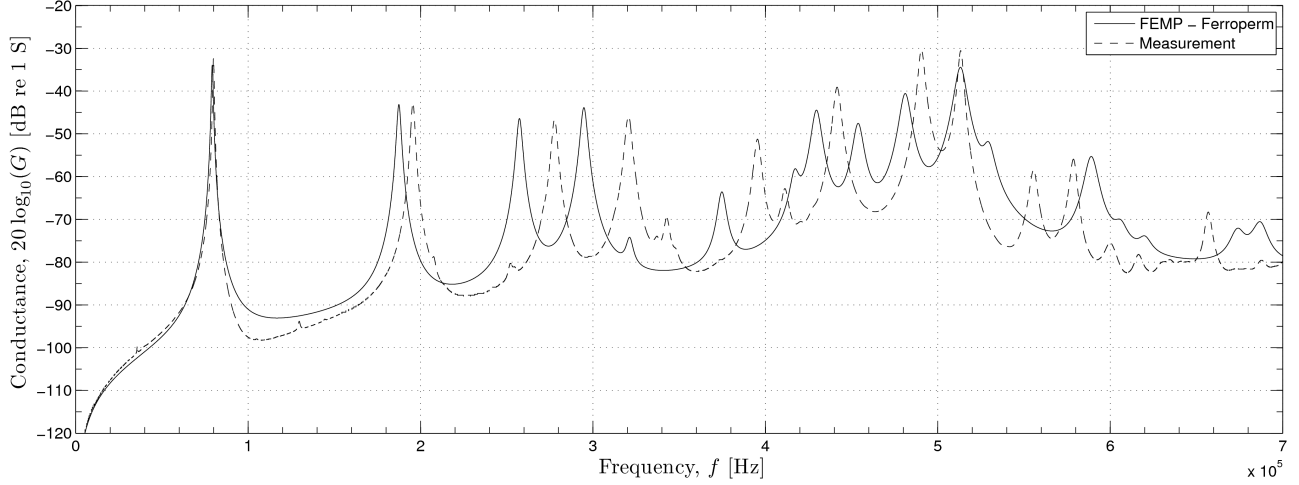


Figure 2: Conductance for a Pz27 disk in vacuum with D/T = 6.126 using the finite element method with material data from Ferroperm A/S, compared with electrical measurement in air.

With the development of accurate finite element methods in recent years, material data supplied by the manufacturer may not provide the optimal comparison between simulations and measurements. The standardized methods [4] for determining material data also may not provide acceptable precision since the requirements in these methods, based on one-dimensional models, are not sufficiently satisfied for low D/T-ratios. A new set of constants have been calculated for the Pz27 disk by applying Sherrit-methods [11, 12], and then adjusting them to measurements in a similar way as described in [13]. These new material data provides an improved agreement with measurements on this particular disk. This set is presented in Table 1 and compared to the original material data for Pz27 from the manufacturer. The constants from Ferroperm A/S do not include loss factors for each constant, but two loss factors, one elastic and one dielectric loss constant,  $Q_M$  and  $\tan \delta$ , respectively. The losses in the adjusted set is represented by the imaginary part of complex constants as used in [14].

Table 1: Material data for Pz27. Material data from manufacturer and adjusted from electrical measurements in air.

	Ferroperm	Adjusted		Ferroperm	Adjusted
$c_{11}^E [10^{10} \text{ N/m}^2]$	14.7	$12.0(1 + i/110)$	$e_{31} [\text{C/m}^2]$	-3.09	$-5.4(1 - i/70)$
$c_{12}^E [10^{10} \text{ N/m}^2]$	10.5	$7.43(1 + i/250)$	$e_{33} [\text{C/m}^2]$	16.0	$16.0389(1 - i/200)$
$c_{13}^E [10^{10} \text{ N/m}^2]$	9.37	$7.5(1 + i/200)$	$e_{15} [\text{C/m}^2]$	11.64	$11.0(1 - i/200)$
$c_{33}^E [10^{10} \text{ N/m}^2]$	11.3	$11.4(1 + i/177.99)$	$\epsilon_{11}^S [10^{-9} \text{ F/m}]$	10.005	$8.110436208(1 - i/50)$
$c_{44}^E [10^{10} \text{ N/m}^2]$	2.30	$2.105(1 + i/75)$	$\epsilon_{33}^S [10^{-9} \text{ F/m}]$	8.0927	$8.14585296(1 - i/80)$
$\rho [\text{kg/m}^3]$	7700	7700	$\tan \delta$	0.017	-
			$Q_M$	74	-

The finite element method is then used to calculate conductance in air with the new adjusted material data. In Figure 3, an improved agreement for the overall response, both in frequency and amplitude can be observed between the finite element

simulation and electrical measurements in air.

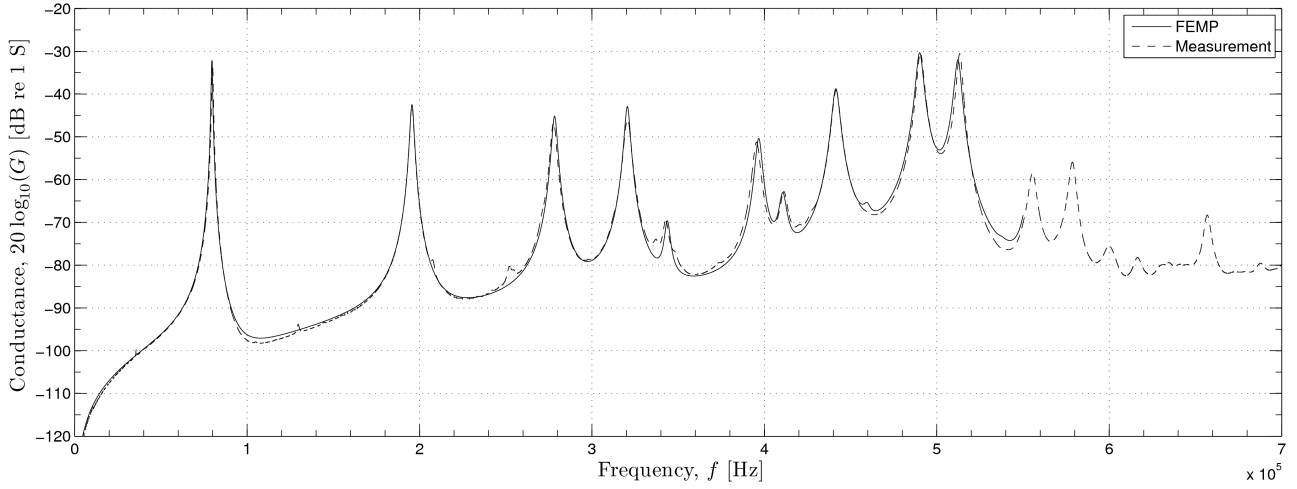


Figure 3: Conductance for a Pz27 disk in air with  $D/T = 6.216$  using the finite element simulations with adjusted material data compared to electrical measurements.

In Figure 4 the conductance from FE simulations is compared to electrical measurements for the Pz27 disk in water. Good agreement is displayed around the different resonance frequencies. The effect of fluid loading can also be seen by the dampening of resonances compared to in air.

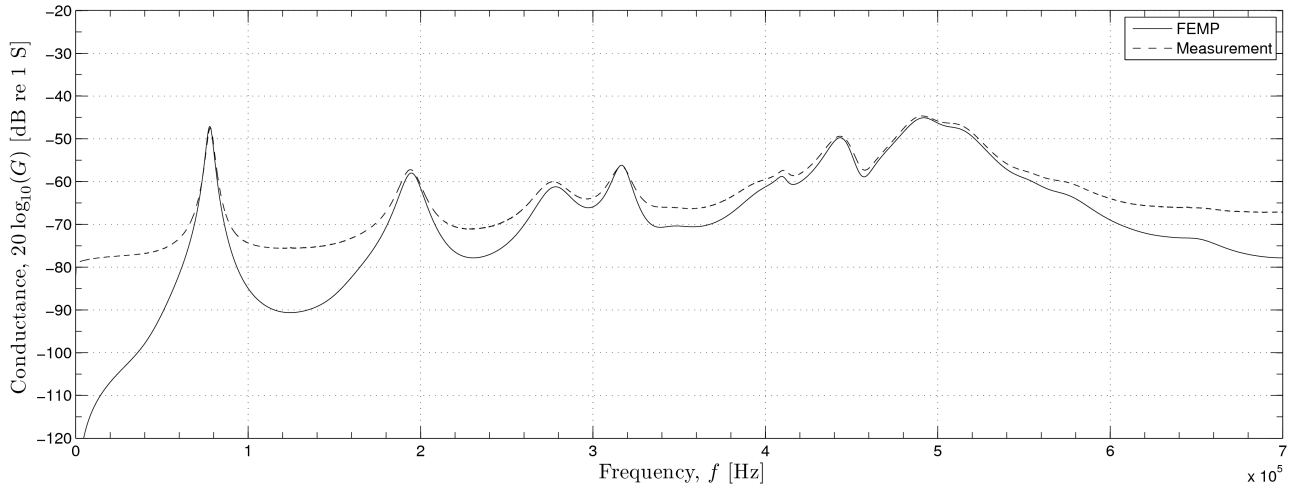


Figure 4: Conductance from finite element simulations with the adjusted material data compared to measured conductance in water on the Pz27 disk with  $D/T = 6.216$ .

### 3.2 Solution methods in the finite element method

The uncertainty of the finite element method also depends on the solution method and the element division. In the FE implementation, used two solution methods are available to calculate transducer response functions; a modal analysis (or mode superposition) method, and a direct harmonic method [1, 9]. The mode superposition method solves the FE-equations using eigenmodes and eigenfrequencies. This method provides information regarding the distribution of eigenmodes and their influence on the electrical and acoustical transducer response functions. It can be used in mode classification and in calculation of eigenmode frequency spectra. One disadvantage of the mode superposition method in the FE implementation used, is that it cannot be used for radiation in an infinite fluid [1]. Further on, the accuracy of the method is dependent on the total number of eigenmodes included, which corresponds to the total number of elements used in the finite element simulation[1, 15]. The direct harmonic analysis calculates response functions directly from the FE-equations, and also have

the ability to include an infinite fluid. The disadvantage of this method is that it does not provide any information regarding eigenmodes and eigenfrequencies, and when response functions for many frequencies are calculated, it is computationally more heavy than the mode superposition method [1].

The two solution methods provide the same result when all eigenmodes in the modal analysis are included [1, 15], but including all eigenmodes for large element divisions is computationally demanding. The effect of not including all eigenmodes in the mode superposition method can be seen in Figure 5. Here both the solution methods are used for the Pz27 disk in vacuum with the adjusted material data. In this case conductance has been compared, calculated from mode superposition method and the direct harmonic analysis with exactly the same number of elements. This is done to ensure that differences observed is not related to element division. Difference of conductance from mode superposition method and direct harmonic analysis in % is shown in Figure 5. Three cases are presented, with eigenmodes up to 400 kHz, 1 MHz and 2 MHz, respectively. For this particular disk the TE1 mode is at approximately 500 kHz and TE2 is at approximately 1.6 MHz. The deviations between these two solution methods increases when approaching frequencies near the highest calculated eigenmode. This is because some eigenmodes which influences conductance around these frequencies, is not taken into consideration while performing the modal analysis. Figure 5 demonstrates the effect of the TE-mode area on the conductance calculated from the mode superposition method [16].

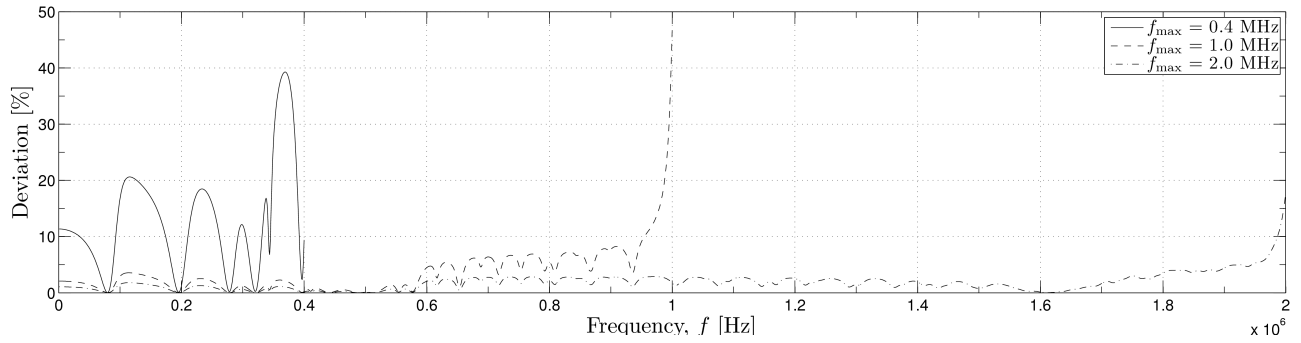


Figure 5: Difference of conductance using the mode superposition method and the direct harmonic analysis in %, using different number of eigenmodes in the mode superposition method.

### 3.3 Displacement and source sensitivity

The displacement in the thickness direction on the surface of the disk with air and water loading with 1 V peak drive voltage using the adjusted material data is calculated. Figure 6 demonstrates displacement calculated for the first series resonance frequency of 79.58 kHz and 77.81 kHz, in air and water, respectively. Large dampening of the displacement in water compared to in air is observed, as expected because of the larger mechanical loading in water.

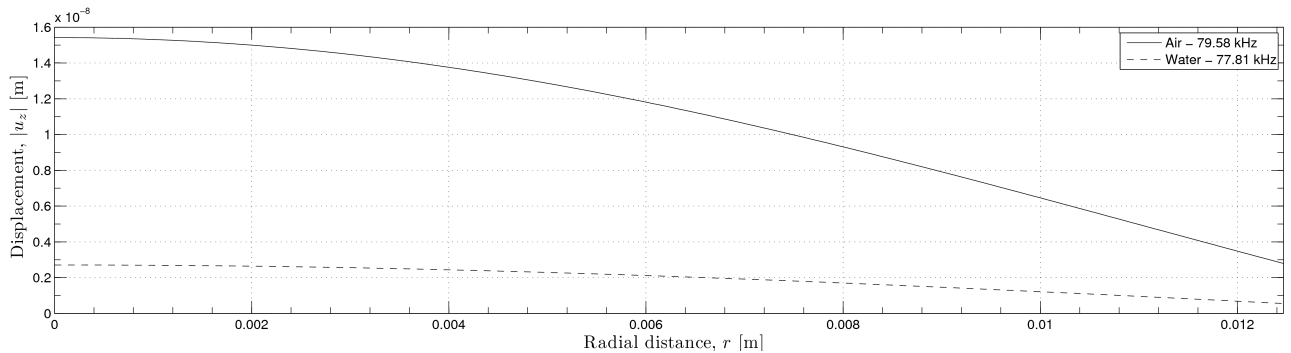


Figure 6: Displacement in the thickness direction on the surface of the element for the first series resonance frequency in air and water. 79.58 kHz and 77.81 kHz, respectively. 1 V peak drive voltage.

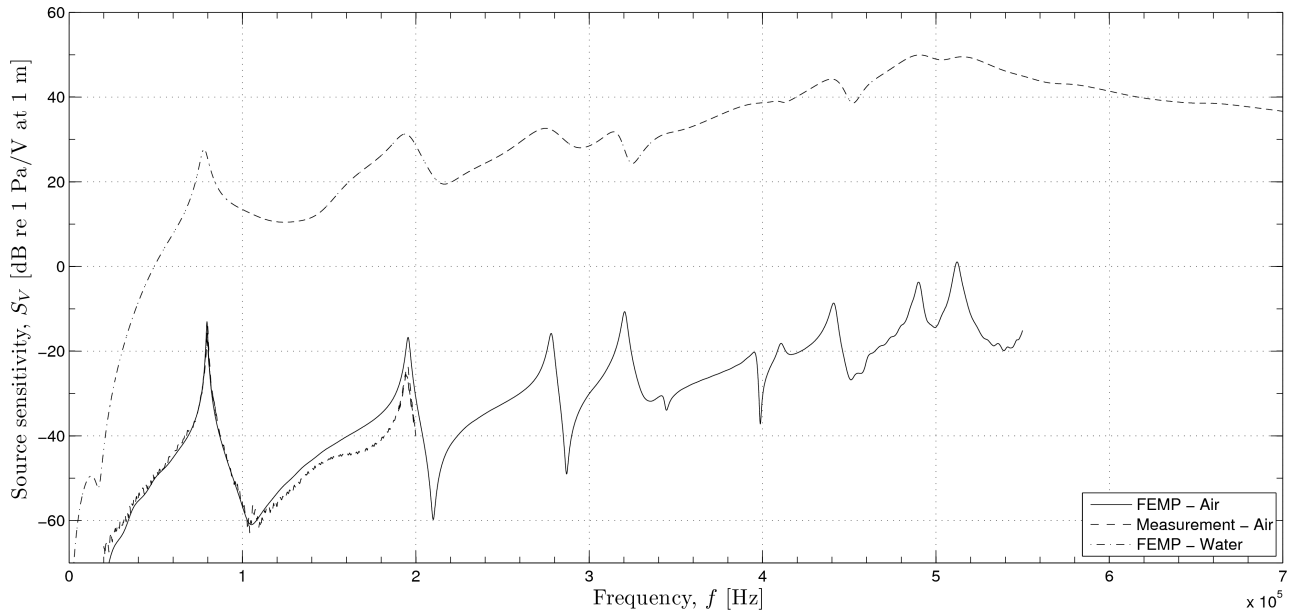


Figure 7: Source sensitivity of the Pz27 disk with  $D/T = 6.216$  in air and water. Finite element simulations and measurements.

In Figure 7 the source sensitivity is calculated for air and water using the adjusted material data. The source sensitivity in water is higher than in air due to the higher density in water. The bandwidth in water has also increased compared to air, due to the additional fluid loading. Comparing the finite element simulation of source sensitivity in air to acoustical measurements of the piezoceramic disk demonstrates good agreement for frequencies from 20 kHz to 200 kHz. Non-linear effects around the two resonance frequencies can be observed when exciting the disk with 20 V peak-to-peak. Better agreement around these frequencies are obtained when reducing the voltage to 2 V peak-to-peak, but the SNR (signal-to-noise)-ratio decreases.

## 4 Summary

FE-simulations are known to better describe the complicated vibrations and the electrical and acoustical properties of piezoelectric transducers including piezoelectric ceramic disks than more traditional simplified Mason type of models. However, accurate knowledge of material data are required for accurate modeling. An adjusted set of material data have been used in the present work, although systematic and physically correct standardized methods to obtain such data are still required. It is also pointed out how choice of method and limitations in number of elements affect simulation accuracy. With such limitations in addition to the more inherent assumptions in the method, the powerful capabilities in simulating electrical and acoustical response functions are to be further investigated and utilized for specific transducer designs for various media.

## 5 Acknowledgements

This work has been supported through two Ph.D.-projects: The 4-year project "Full wave propagation of acoustic beams in viscoelastic media; finite element modeling and measurements", financed by the University of Bergen, and the 3-year project "High-precision sound velocity cell technology for gas characterization", financed through the Michelsen Centre of Industrial Measurement Science and Technology by The Research Council of Norway and Statoil ASA.

## References

- [1] J. Kocbach, "Finite element modeling of ultrasonic piezoelectric transducers", Ph.D. Dissertation, Department of Physics, University of Bergen, September 2000.
- [2] P. Lunde, K.-E. Frøysa, R.A. Kippersund and M. Vestrheim, "Transient diffraction effects in ultrasonic meters for volumetric, mass and energy flow measurement of natural gas", *Proc. of 21st North Sea Flow Measurement Workshop*, Tønsberg, Norway, 28-31 October 2003.
- [3] P. Lunde, R.A. Kippersund and M. Vestrheim, "Signal modelling using the *FLOSIM* system model in ultrasonic instrumentation for industrial applications", *Proc. of NORSIG 2003, Norwegian Symposium on Signal Processing 2003, Bergen, October 2-4, 2003*, Norwegian Society of Signal Processing (NORSIG) (October 2003), 6 p. (CD issue only, ISBN 82-993158-5-9).
- [4] IEEE Standard on Piezoelectricity 176-1987. The Institute of Electrical and Electronics Engineers, Inc 345 East 47th Street, New York, NY 10017, USA, 1988.
- [5] A. H. Meitzler, J. Henry M. O'Bryan, and H. F. Tiersten, "Definition and measurement of radialmode coupling factors in piezoelectric ceramic materials with large variations in Poissons ratio", *IEEE Transactions on sonics and ultrasonics*, vol. SU-20, No. 3, pp. 233–239, July 1973.
- [6] (15. Mars 2010) Ferroperm piezoceramics A/S. [Online]. Available: <http://www.ferroperm-piezo.com/>
- [7] Operation and Service Manual: HP 4192 LF Impedance Analyzer, Hewlett Packard, 1981.
- [8] (15. Mars 2010) Brüel & Kjær Measurement microphone type 4138 [Online]. Available: <http://www.bksv.com/doc/bp2030.pdf>
- [9] J. Kocbach, P. Lunde, and M. Vestrheim, "FEMP - Finite Element Modeling of Piezoelectric Structures. Theory and Verification for Piezoceramic Disks." Department of Physics, University of Bergen, Norway, Tech. Rep. 1999-07, August 1999.
- [10] R. J. Astley, G. J. Macaulay, J.-P. Coyette, and L. Cremers, "Three-dimensional wave-envelope elements of variable order for acoustic radiation and scattering. Part I. Formulation in the frequency domain", *J. Acoust. Soc. Am.* 103, 49–63 (1998).
- [11] S. Sherit, N. Gauthier, H. Wiederick, and B. Mukherjee, "Accurate evaluation of the real and imaginary material constants of piezoelectric resonator in the radial mode", *Ferroelectrics*, vol. 119, pp. 17–32, 1991.
- [12] S. Sherit, H. Wiederick, and B. Mukherjee, "Non-iterative evaluation of the real and imaginary material constants of piezoelectric resonators", *Ferroelectrics*, vol. 134, pp. 111–119, 1992.
- [13] R. Fardal, "Endelig element analyse av elektriske egenskaper til piezoelektriske skiver", Master thesis, Department of Physics, University of Bergen, 9. January 2002 [in Norwegian].
- [14] R. Holland, "Representation of dielectric, elastic, and piezoelectric losses by complex coefficients", *IEEE Transactions on sonics and ultrasonic*, vol. SU-14, no. 1, pp. 18-20, January 1967.
- [15] K. J. Bathe, "Finite element procedures in engineering analysis", Department of Mechanical Engineering Massachusetts Institute of Technology, Prentice-Hall, Inc. Englewood Cliffs, New Jersey 07532, 1982.
- [16] M. Aanes, J. Kocbach and M. Vestrheim, "Modal and direct harmonic solution methods in FE modeling of piezoceramic disks", Paper presented at 33rd Scandinavian Symposium on Physical Acoustics, Geilo, Norway, February 7-10, 2010.