

*Executive summary of the final report of GAIA*

*IST-1999-29010*

*“Application of approximate algebraic geometry in industrial computer aided geometry”*

The main objective of GAIA has been to assess to which extent approximate algebraic techniques can be used to find better ways of solving challenging problems within Computer Aided Geometric Design (CAGD). The focus has been on intersection algorithms, especially self-intersection algorithms for curves and surfaces. Such algorithms are almost non-existent in CAGD-systems.

The project has developed prototype self-intersection algorithms that works well on the industrial examples supplied by the industrial partner early in the project. The prototype software has been integrated into the think3 system through a COM-interface.

The traditional approach to curve and surface intersection and self-intersection has been to regard the problem as the intersection of separate objects. Self-intersection has been approached by splitting a curve or surface in pieces and then regards the problem as the intersection of these. In GAIA we have considered the intersection of two curves or two surfaces as the intersection of separate branches or a loop on an algebraic surface. By making an implicit approximation of these two surfaces, intersections and self-intersections can be detected by looking for singularities on the approximate algebraic curve or surface. Singularities are points where the gradient of the algebraic curve and surface vanish. To look for singularities we have checked for sign changes in the gradient of the implicit approximation relative to the normal of the parametric curve or surface.

The project partners consider the project results as promising and have identified a number of other applications where such a combination of approaches can be useful:

- Detect loops on 3D curves. Use: Ensure that curves can be used for surface design, detect loop on thread in surgical simulators.
- Closest point calculations between geometric objects by using approximate algebraic geometry on line coordinates and higher dimensional spaces.
- Ray tracing of surfaces as described by [Sederberg 99]

Consortium composition and partner roles:

- SINTEF Applied Mathematics, Norway. The RTD Institute SINTEF has been the project coordinator and responsible for developing the software for approximate implicitization and using this to make prototype software for detection of self-intersections in NURBS curves and surfaces.
- think3 SPA, Italy. think3 has as the industrial partner of the consortium had the role to define industrial requirements of the project, specify how the software has to be integrated into the industrial CAD-system of think3, develop a test environment in the think3 system and test the software on industrial examples delivered early in the project, as well as testing the software on numerous additional industrial problems.
- University of Nice Sophia Antipolis, France. The focus of UNSA has been the development of a sampling based reference method for self-intersections. In addition the focus has been to understand better approximate implicitization from the algebraic geometry viewpoint.
- University of Oslo, Norway. UO has focused on the literature about classification of real algebraic curves and surfaces, in order to possibly speed up the process of detecting self-intersections in the algebraic approximation to NURBS curves and surfaces.