Convex optimization

François Glineur

francois.glineur@uclouvain.be

Université catholique de Louvain - EPL/INMA & CORE

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in two slides

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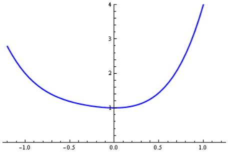
where f is 

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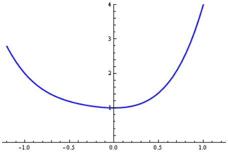
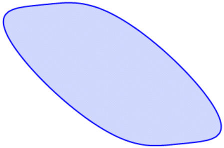


and X is



$$\min_{x \in \mathbb{R}^n} f(x) \text{ such that } x \in X \subseteq \mathbb{R}^n \quad \text{with } f \text{ convex, } X \text{ convex}$$

What do we get in return ?

- Recognizing (global) optimal solutions becomes easy
- Efficient algorithms (both in theory and in practice): *interior-point methods*
  Polynomial-time algorithmic complexity for (nearly) all common convex optimization problems:
  $$\mathcal{O}\left(\sqrt{\nu} \log \frac{1}{\epsilon}\right) \text{ iterations to achieve } \epsilon \text{ accuracy}$$
  where ν is a measure of the problem size
- Duality: existence of another strongly related dual problem,
  a maximization problem with the same optimal value
  - How to check that solution to $Ax = b$ is correct ? Plug x into equation
  - How to check that a solution to an optimization problem is correct ? . . .
    Use duality : exhibit a solution of the dual → checkable optimality certificate

Examples: Linear optimization, (convex) quadratic optimization, semidefinite optimization, geometric optimization, sum-of-norm optimization, etc.

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Thanks for you attention