A continuously differentiable turbine layout optimization model for offshore wind farms

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Wind farm layout design / turbine micro-siting

- Layout problem
- Optimal placement of turbines within an offshore wind farm
- Wind slows down behind (in the “wake” of) a wind turbine
- Other turbines in the wake experience lower wind speeds and thus produce less power

(Credit: Vattenfall)

Outline

- Problem definition
- Optimization model
- Preliminary experimental results
- Open problems

Problem definition

Aim

- Model suitable for gradient based optimization methods
- Investigate performance

Approach

- Set up of optimization model
  - continuous variables
  - differentiable
  - non-convex
- Computations with wind data of real wind farm sites

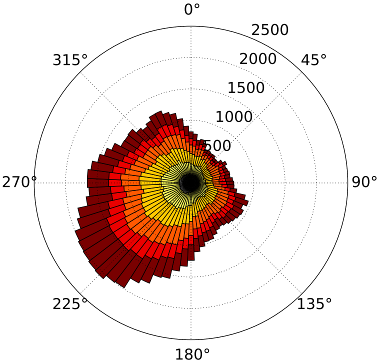
Wind turbine locations

- Set of turbines $\mathcal{T}$ with Turbine locations as independent variables $r_t = \begin{pmatrix} x_t \\ y_t \end{pmatrix} \in \mathbb{R}^2$, $t \in \mathcal{T}$
- Given parameters
  - Number of turbines
  - Allowed convex area for turbine placement
  - Wind rose
  - Turbine parameters
- All turbine locations have the same polyhedral constraint

$$Ar_t \leq b \quad \forall t \in \mathcal{T}$$

Wind information

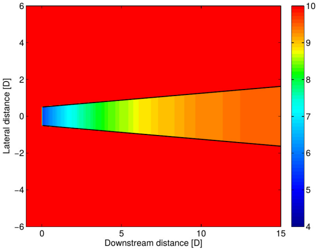
- Wind rose
- Expected wind data over lifetime of wind farm
  - historical data over 10 years to capture variations
- Discretized set W of wind data, $w \in W$
  - undisturbed wind velocity v_w
  - direction ϕ_w
  - frequency of occurrence f_w



Basis for wake model

- Calculates wind velocity deficit in wake of a turbine
- Based on widely used Jensen wake model (Jensen 1986)
  - only defined in wake of turbine
  - non-differentiable

- Extension by Haugland (2012), Park and Law (2015)
- Differentiable in radial direction
- Still non-smooth in downwind direction



(Credit: Renkema 2007)

Extension of wake model

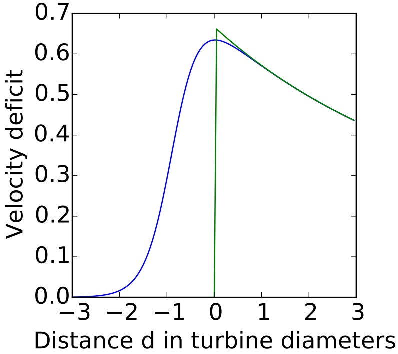
- Approximation of Heaviside step function in downwind direction
- Wake function g continuously differentiable on $\mathbb{R}^2$
- d and s projection of vector between turbine $i, j \in T$ on wind direction ϕ_w, $w \in W$
- g_{ijw} velocity deficit from turbine $i \in T$ on turbine $j \in T$ for wind vector $w \in W$

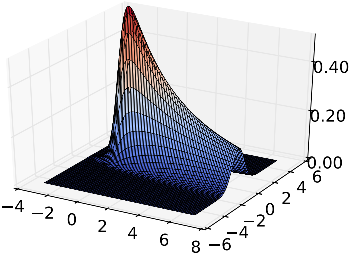
$$d_{ijw} = \begin{pmatrix} x_j - x_i \\ y_j - y_i \end{pmatrix} \begin{pmatrix} \sin(\phi_w) \\ \cos(\phi_w) \end{pmatrix}$$

$$s_{ijw} = \begin{pmatrix} x_j - x_i \\ y_j - y_i \end{pmatrix} \begin{pmatrix} \sin(\phi_w - \frac{\pi}{2}) \\ \cos(\phi_w - \frac{\pi}{2}) \end{pmatrix}$$

$$g_{ijw} = \frac{\frac{2}{3}\left(\frac{R}{R+\kappa d_{ijw}}\right)^2 \exp\left(-\left(\frac{s_{ijw}}{R+\kappa d_{ijw}}\right)^2\right)}{1 + \exp\left(-1.75\left(\frac{d_{ijw}}{R} + 1.7\right)\right)}$$

Extension of wake model II





Left: Jensen (green) and our model (blue) on $s = 0$.

Right: Visualization of model in 3d

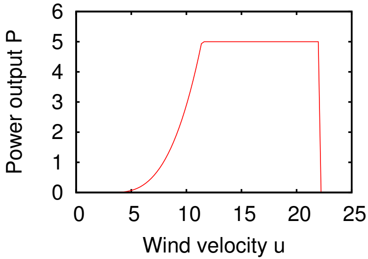
Wake combination model

- Total wind velocity deficit Δu_{tw} and wind velocity u_{tw} for a turbine $t \in \mathcal{T}$ with undisturbed wind vector v_w, $w \in \mathcal{W}$.
- Combination of all wake deficits for a given wind vector
- Constraints for $t \in \mathcal{T}, w \in \mathcal{W}$

$$u_{tw} = v_w \left(1 - \sqrt{\sum_{k \in \mathcal{T}, k \neq t} (g_{ktw})^2} \right)$$

Power curve

- Power production of turbine as function of wind velocity
- Characteristic of turbine
- Rated power P^{rated} and wind speed u^{rated}, cut-in wind speed $u^{\text{cut-in}}$, cut-off wind speed $u^{\text{cut-off}}$



$$C(u) = \begin{cases} 0 & \text{if } u < u^{\text{cut-in}} \\ a(u - u^{\text{cut-in}})^3 & \text{if } u^{\text{cut-in}} \leq u < u^{\text{rated}} \\ P^{\text{rated}} & \text{if } u^{\text{rated}} \leq u < u^{\text{cut-off}} \\ 0 & \text{if } u^{\text{cut-off}} \leq u \end{cases}$$

Power curve

- Remove wind velocities higher than $u^{\text{cut-off}}$ from set $\mathcal{W}$
- Add additional constraints to remove non-differentiable function
- For each turbine $t \in T$ and wind vector $w \in W$

$$P_{tw} \leq \begin{cases} 0 & \text{if } u_{tw} \leq u^{cut-in} \\ (u_{tw} - u^{cut-in})^3 & \text{if } u_{tw} \geq u^{cut-in} \end{cases}$$

$$P_{tw} \leq P^{\text{rated}}$$

Total power production

- Objective function is total power production
- Sum over turbines and wind vectors, weighted with frequencies

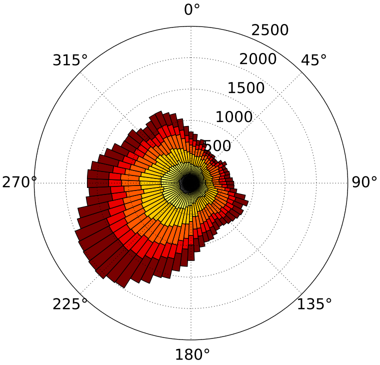
$$\max \sum_{w \in \mathcal{W}} \left(f_w \sum_{t \in \mathcal{T}} P_{tw} \right)$$

Solution method

- Model formulated in AMPL
- Solver Ipopt (Interior Point OPTimizer)
- Multistart with grid and random initial turbine locations
- Computations on Intel Xeon E5-2699, 72 logical cores, 256 GB Ram
  - Each optimization runs on a single core, parallel computations possible

Wind data

- Simulated wind data from 07/1999 to 12/2009
  - Lorenz and Barstad, 2015
- 5-10 minute time resolution
- Aggregated in $2m/s$ and 1° and 5° bins
- Locations
  - Dogger Bank
  - Dudgeon
  - Greater Gabbard
  - Gunfleet Sands
  - Horns Rev
  - Race Bank
  - Sheringham Shoal

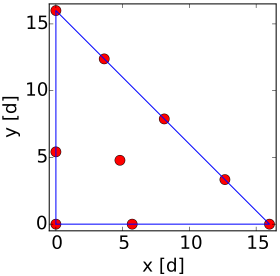


Data for experiments

- Reference 5MW wind turbine (Jonkman 2009, NREL)
  - $u^{cut-in} = 3m/s$, $u^{rated} = 11.4m/s$, $u^{cut-off} = 25m/s$, $P^{rated} = 5MW$
- 9, 16, 25 turbines with rotor diameter d
- Minimal turbine spacing $3d$
- Grid turbine spacing $5d$ to $20d$

Preliminary experimental results

- Quadratic farm boundaries
  - grid layout is optimal for wind data of all farms, for 9, 16, 25 turbines, for 5*d* and 7*d* turbine spacing
  - multistart with 400 random initial locations for 9 turbines, 32 for 16 and 25 turbines.
- Algorithm behaves well placing turbines in other shapes



Open problems

- Speed of model/solver
  - Approximation of power curve with splines
- Validation of results
  - Applying other wake models
- Optimizing shape of farm, number of turbines
- Investigating uncertainty in wind information

Thank you!