## Cross-Border Transfer of Electric Power Under Uncertainty: A Game of Incomplete Information

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Cross-border transfer of electric power promotes collaboration in power generation between integrated electricity markets. It as well resolve grid reinforcement issues in existing transmission networks. Because of that, researchers have given higher attention to the field and have conducted various studies on the subject using technical simulation approaches. Yet, substantial works have to be done for quantifying the socioeconomic benefits of the mechanism. This paper intends to fill the gap by introducing a method for analyzing the mechanism by representing it as a game of incomplete information. The subject is modeled as Bayesian game in which the type of marginal generators located within one (or more) external market area is not known. Based on that, the Bayesian equilibrium which represent the state where all marginal generators would incline to converge is found. The authors suggest that the method is robust and can be used for quantifying the performance of a market coupling mechanism because it realistically consider all marginal generation scenarios.



A slightly modified IEEE 30-bus test system

Formally, let  $\theta_i$  be the type of player i in a game, and  $p(\theta_{-i}|\theta_i)$  represents first order belief owned by player i towards the type of his opponent (given that the type of him is  $\theta_i$ , which is known only to himself). The set of all types of player i is  $\Theta_i$  and  $\theta_i \in [0, 1)$  for all  $\theta_i \in \Theta_i$ . Under such conditions, a player would choose his action based on his types. Based on that, he owns a strategy,  $s_i$  that maps  $\Theta_i$  to  $A_i$ . Hence,  $s_i : \Theta_i \to A_i$ . Because Bayesian game theory suggests that the choice of a player's action follows  $\theta_i$  and  $p(\theta_{-i}|\theta_i)$ , the expected payoff of by player i in the game becomes:

$$\begin{split} E[u_i(s_i|s_{-i},\theta_i)] &= \\ \sum_{\theta_i \in \Theta_{-i}} u_i(s_i,s_{-i}(\theta_i),\theta_i,\theta_{-i})p(\theta_i|\theta_{-i}) \end{split} \tag{1}$$

where,  $s_{-i}(\theta_i)$  is the strategy taken by players except player i, given that the type of player i is  $\theta_i$ . A Bayesian equilibrium (BE) is the Nash equilibriums of the Bayesian game, formulated as follows.

$$E[u_i(s_i|s_{-i},\theta_i)] \ge E[u_i(s_i'|s_{-i},\theta_i)]$$

Upon achieving BE, player *i* receives lower expected utility if he uses a strategy other than  $s_i$  (denoted by  $s'_i$ ). The existence of BE is guaranteed because of the proven existence of NE.



Matrix game I Matrix game I

The Bayesian game in cross-border trade of electric power simulated in this work.

Matrix game II

	G41					G61					G41,61			
G4	41–G31	Low	Medium	High		G61-G31	Low	Medium	High		G41-G31	Low	Medium	High
	Low	700, 1000	700, 800	700, 600		Low	700, 875	700, 700	700, 525		Low	1400, 466	1400, 373	1400, 592
М	fedium	525,1000	525,800	525,600		Medium	525, 875	525, 700	525, 525		Medium	1050, 466	1050, 373	1050, 600
	High	350,1000	350, 800	350, 600		High	350, 875	350, 700	350, 525		High	700, 466	700, 373	700, 280

G31

(2)

" ... In order to decide what we ought to do to obtain some good or avoid some harm, it is necessary to consider not only the good or harm in itself. But also the probability that it will or will not occur, and to view geometrically the proportions all these things have when taken together ... "

Matrix game III

Payoff received by area 2: 910, area 3: 858