Using Heterogeneous Computing for Solving Vehicle Routing Problems

GPU based local search for CVRP with REFs

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SINTEF ICT

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Outline

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2. Motivation: Why heterogeneous computing
3. Introduction to GPU
4. CVRP and REFs
5. Three-opt on GPU
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Motivation

Transportation management

- Goal: good solutions computed fast, based on thorough exploration of search space
- Increase in computing power $\Rightarrow$ existing methods faster or more exploration
- Better algorithms, methodological improvements

Variety of methods for solving VRP

- Metaheuristics
- Heuristics based on exact methods and hybrid methods
- Variants and hybrids of large neighbourhood search
- Variable neighbourhood search
- Iterated local search
Motivation cont’d

Parallelism often occurs naturally in the methods

- Algorithmic level, metaheuristics
- Iteration level, neighbourhood evaluation (generation)
- Solution level

Parallel platforms

- Traditional supercomputers: Cluster (large number) of CPUs
  High level of independence, can perform basically independent tasks ⇒ Task parallelisation
- Parallel methods in optimisation not new, but most focus on task parallelisation (according to Crainic 2008)
- What about the new multi-core CPUs
- What about the GPUs
Moore’s law

The number of transistors that can be placed inexpensively on an integrated circuit doubles every two years.

- 1971 (4004), 2300 transistors, 1 x 0.000740 GHz
- 2004 (Pentium 4 Prescott), 125 000 000 transistors, 1 x 4 GHz
- 2008 (Core i7 Quad), 731 000 000 transistors, 4 x 3.33 GHz

Picture from http://en.wikipedia.org/wiki/Moore's_Law
What happened?

Increasing frequency hits three major problems (walls): Memory, ILP, Power density (power/area)

Memory

- Memory speeds did not increase as fast as core frequencies
  Processor can wait hundreds of clock cycles for data/instructions from main memory
- Wait can be reduced by larger caches and instruction level parallelism

Instruction level parallelism

- Difficult to find enough parallelism in instructions stream of single process to keep cores busy
Multi-core

Power density (heat)

- Increase in frequency leads to increase in power density
- CPU has higher power density than a cooking plate
- Using about 80% of frequency halves power consumption

⇒ Use of 2 cores with ~ 80% of frequency: same power consumption, ~ 160% performance

But: Deep pipelines, heavy ILP use and huge caches drain a lot of power
⇒ no 100 core processor

Acceleration cores

- Shallow pipelines, low or no ILP, small or no caches
- Power efficient
Heterogeneous computer

Classical supercomputer consist of many processors, maybe with dual/quad core
⇒ Consume lot of power, maintenance, expensive

But: Commodity PCs nowadays have multi-core CPUs and one (or more) GPU (has acceleration cores)
⇒ Cheap, high performance if it can be harnessed

Heterogeneous computer: Tightly coupled system of processing units with distinct characteristics
GPU

- Background: Computer graphics
- Nowadays: General purpose GPU
- Massively parallel: 512 cores
- High memory bandwidth
- Typical speedup: 10-50 (to CPU)
- Data parallelism: Typically same task performed by each core on different pieces of data
- NVIDIA Fermi:
  - IEEE 754-2008 floating point standard
  - Improved double precision performance (now half of single precision)
Programming GPU

Direct Compute

- Part of Microsoft DirectX
- Debugger (NVIDIA) on Windows

OpenCL (AMD, NVIDIA)

- Extension of C, reminiscent of GLSL
- Relatively immature, but improves as we speak

Cuda (NVIDIA)

- Large subset of C++, can share code with CPU code
- Mature
- Debugger on Linux and Windows
GPU in Science

GPU usage in other Sciences/Industry
- PDE / Simulation: Shallow water
- Medicine: Automated ultrasound imaging system
- Finance: Analyses the entire U.S. equity options market in real time

GPU in Optimisation
- Knapsack
  M. Scherger, Two Parallel Algorithms to Solve the 2D Knapsack Problem Using GPUs, 2008  
  D. M. Quan, and L. T. Yang, Solving 0/1 Knapsack Problem for Light Communication SLA-Based Workflow Mapping Using CUDA, 2009

- Evolutionary algorithms
  Harding, S. and W. Banzhaf, Fast Genetic Programming on GPUs, 2007  

- Neighbourhood evaluation
  Luong, T.V., N. Melab, and E.-G. Talbi, Parallel Local Search on GPU, 2009

⇒ Good point in time to start using it
Capacitated Vehicle Routing Problem

Given:
- A depot and number of customer nodes
- Length/Cost $c_{ij}$ between nodes
- Capacity of vehicle(s) $C$
- Demand of customers $d_i \leq C$
Capacitated Vehicle Routing Problem

Given:
- A depot and number of customer nodes
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Wanted: Route(s)
- Each customer is visited once
- Each route visits depot
- Minimal length/cost
- Capacity feasible
Model


- Solution represented as a giant tour

- Use of classical resource extension functions to model capacity constraint
Simple method: Local search with 3-opt move

Initial solution
- Star solution: A single route to each customer

3-opt move
- Remove 3 connections/edges ⇒ 4 segments
- Reconnect in all possible ways ⇒ 7 possibilities
  \[1 - 3 - 2 - 4, \ 1 - 3 - \bar{2} - 4, \ 1 - \bar{3} - 2 - 4, \ 1 - \bar{3} - \bar{2} - 4, \ \\
  1 - 2 - \bar{3} - 4, \ 1 - \bar{2} - 3 - 4, \ 1 - \bar{2} - \bar{3} - 4\]

⇒ Nearly \((7/6)(n - 1)(n - 2)(n - 3)\) moves
  \(n: \text{number of nodes in solution}\)
Classical Resource extension function

Resource constraints modeled by resource consumption

- Resource: cost, time, load, distance, ...
- Resource vector \( \mathbf{t} \in \mathbb{R}^n \)
- Each node has a associated resource interval \([a_i, b_i]\)
- Change of resource consumption from \(i\) to \(j\): \(f_{ij} : \mathbb{R}^n \to \mathbb{R}^n\)
- A path is feasible if for each node \(i\) there exists a resource vector \(\mathbf{T}_i \in [a_i, b_i]\) s.th.
  \[
  f_{i,i+1}(\mathbf{T}_i) \leq \mathbf{T}_{i+1}
  \]

- Classical REF: 
  \[
  f_{ij}(\mathbf{T}) = \mathbf{T} + \mathbf{t}_{ij} \quad \text{or} \quad f_{ij}(\mathbf{T}) = \max(a_j, \mathbf{T} + \mathbf{t}_{ij})
  \]

CVRP (capacity): Classical REF with
\[
  a_i = 0, \quad b_i = C,
  \quad t_{ij} = d_j \text{ for } j \text{ a customer}, \quad t_{ij} = -C \text{ for } j \text{ depot}
\]
Why classical REF? Simple, can build segment hierarchy

Segment - Hierarchy

Aggregation:
- \([3-6]\) contains: \(3 \rightarrow 5\), \(3 \rightarrow 6\) and \(4 \rightarrow 6\) and inverse
- \([0-9]\) contains: \(0 \rightarrow 6\), \(0 \rightarrow 9\) and \(3 \rightarrow 9\) and inverse
Segment - Hierarchy cont’d

- Why segment hierarchy? Gives constant time feasibility check

  Example: Exchange two nodes, e.g. 5 and 20:
  
  path up to first: 0 → 4: 0 → 3, 3 → 4
  reconnect first: 4 → 20:
                       20 → 6:
  path to second: 6 → 19: 6 → 9, 9 → 18, 18 → 19
  reconnect second: 19 → 5:
                       5 → 21:
  path to end: 21 → 32: 21 → 27, 27 → 32

- Maximum number of segments in one path: 2^l-1 (l: depth of hierarchy)
- How to do feasibility check with segments, see paper(s) by Irnich
- Effort to create hierarchy: $O(n^{2^l}/(2^l-1))$
Parallel local search

Why parallelize local search

- Local search is an essential part of more advanced strategies such as metaheuristics
- Embarrassingly parallel: Moves independent from each other
  ⇒ Potential for significant speed up
What we do on the GPU

Transfer of data GPU ↔ CPU slow ⇒ try to minimize/avoid it

On GPU

- Once:
  - Create neighbourhood

- Each iteration:
  - Create hierarchy
  - Evaluation of capacity constraint and length objective for each move
  - Choosing best move

- Neighbourhood and hierarchy live whole time on GPU, no transfer

- Transfer once: constraint & objective data

- Transfer per iteration: move, solution (for now)
Does it pay?

Early timing, only gives indication:

- CPU code is not optimized
- GPU code is not optimized

GPU is fast is known, real task: Efficient usage of GPU hardware
Why optimize GPU code

Example reduction, taken from NVIDIA CUDA SDK whitepaper

<table>
<thead>
<tr>
<th>Kernel</th>
<th>Time (2^{22} ints)</th>
<th>Bandwidth</th>
<th>Step Speedup</th>
<th>Cumulative Speedup</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kernel 1:</td>
<td>8.054 ms</td>
<td>2.083 GB/s</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interleaved addressing with divergent branching</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kernel 2:</td>
<td>3.456 ms</td>
<td>4.854 GB/s</td>
<td>2.33x</td>
<td>2.33x</td>
</tr>
<tr>
<td>Interleaved addressing with bank conflicts</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kernel 3:</td>
<td>1.722 ms</td>
<td>9.741 GB/s</td>
<td>2.01x</td>
<td>4.68x</td>
</tr>
<tr>
<td>Sequential addressing</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kernel 4:</td>
<td>0.965 ms</td>
<td>17.377 GB/s</td>
<td>1.78x</td>
<td>8.34x</td>
</tr>
<tr>
<td>First add during global load</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kernel 5:</td>
<td>0.536 ms</td>
<td>31.289 GB/s</td>
<td>1.8x</td>
<td>15.01x</td>
</tr>
<tr>
<td>Unroll last warp</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kernel 6:</td>
<td>0.381 ms</td>
<td>43.996 GB/s</td>
<td>1.41x</td>
<td>21.16x</td>
</tr>
<tr>
<td>Completely unrolled</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kernel 7:</td>
<td>0.268 ms</td>
<td>62.671 GB/s</td>
<td>1.42x</td>
<td>30.04x</td>
</tr>
<tr>
<td>Multiple elements per thread</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Local Search with 3opt - Results

<table>
<thead>
<tr>
<th>Problem</th>
<th>Our</th>
<th>Best</th>
<th>#Lt.</th>
<th>Time(s)</th>
<th>Nbh size</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-n16-k8</td>
<td>473.782</td>
<td>451.335</td>
<td>7</td>
<td>0.109</td>
<td>28420</td>
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<tr>
<td>P-n20-k2</td>
<td>233.995</td>
<td>217.416</td>
<td>18</td>
<td>0.327</td>
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<td>P-n23-k8</td>
<td>560.598</td>
<td>531.174</td>
<td>13</td>
<td>0.281</td>
<td>92708</td>
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<tr>
<td>E-n30-k3</td>
<td><strong>508.139</strong></td>
<td>535.797</td>
<td>31</td>
<td>1.013</td>
<td>215992</td>
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<tr>
<td>B-n35-k5</td>
<td><strong>1403.96</strong></td>
<td>956.294</td>
<td>32</td>
<td>1.432</td>
<td>350812</td>
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<tr>
<td>P-n40-k5</td>
<td>506.039</td>
<td>461.726</td>
<td>37</td>
<td>2.290</td>
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<tr>
<td>F-n45-k4</td>
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<td>723.541</td>
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<td>3.598</td>
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<td>B-n50-k7</td>
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<td>744.228</td>
<td>44</td>
<td>4.890</td>
<td>1064672</td>
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<td>A-n60-k9</td>
<td>1407.09</td>
<td>1355.800</td>
<td>56</td>
<td>10.731</td>
<td>1868412</td>
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<td>P-n70-k10</td>
<td>915.380</td>
<td>829.933</td>
<td>60</td>
<td>18.301</td>
<td>2999752</td>
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<tr>
<td>A-n80-k10</td>
<td>1833.49</td>
<td>1766.500</td>
<td>75</td>
<td>34.391</td>
<td>4514692</td>
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<tr>
<td>E-n101-k8</td>
<td>990.737</td>
<td>828.737</td>
<td>97</td>
<td>90.523</td>
<td>9193800</td>
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<tr>
<td>M-n151-k12</td>
<td>1124.44</td>
<td>1043.410</td>
<td>144</td>
<td>475.321</td>
<td>31185700</td>
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<tr>
<td>M-n200-k16</td>
<td><strong>1402.67</strong></td>
<td>1499.780</td>
<td>190</td>
<td>1585.751</td>
<td>72998772</td>
</tr>
</tbody>
</table>

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Summary & Future Work

Summary

- Your office PC is a heterogeneous computer
- Proper algorithms can harness CPU+GPU power
- Early results in local search for CVRP promising

Future Work

- Optimise code
- Larger solutions: memory, number of tasks
- More advanced strategies such as metaheuristics
- Keep CPU and GPU busy
Thank you for your attention!